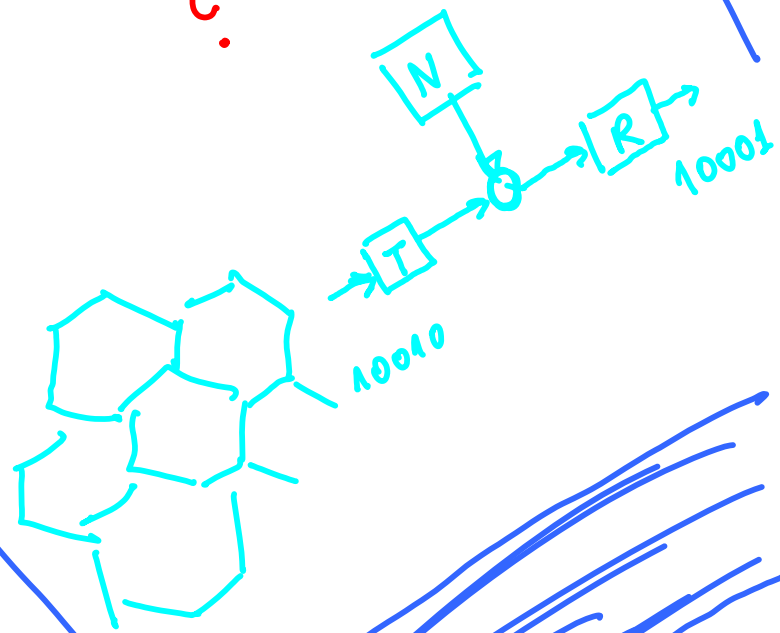
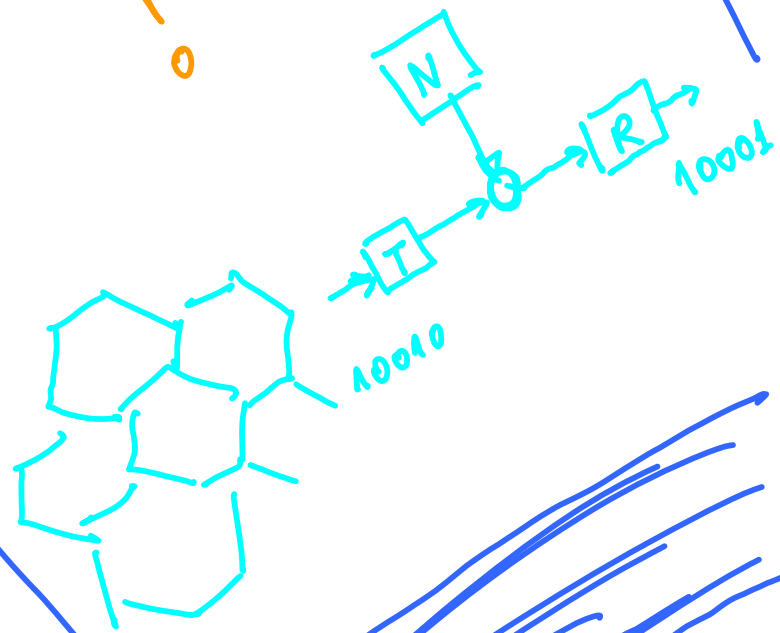
Why to Write a Book (about lattice codes) ?



Ram Zamir

Why NOT to Write a Book (about lattice codes)?

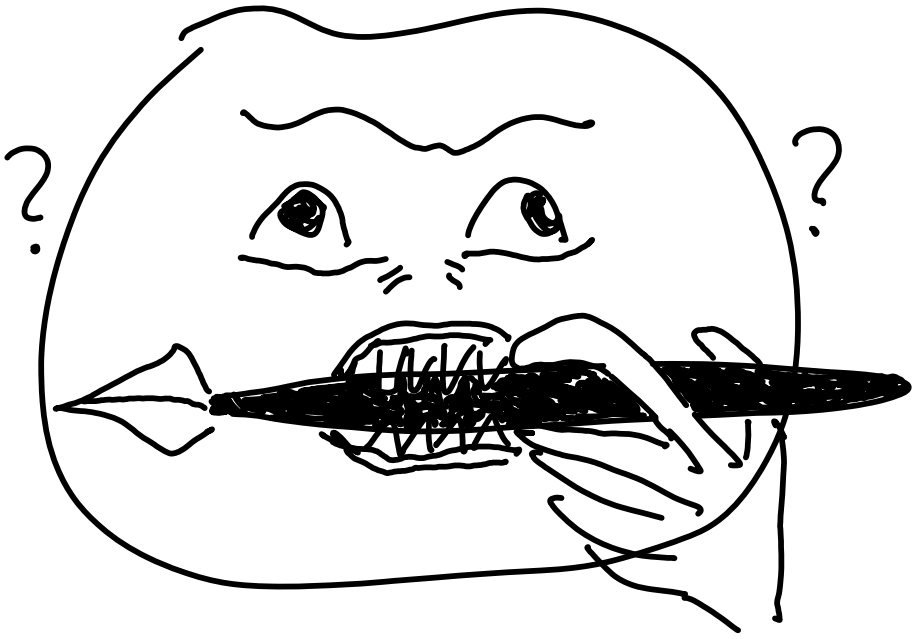


Ram Zamir

Writer's Misery

"קנאת סופרים תרבה חכמה" [תלמוד בבלי בבא בתרא]

(≈ Jealousy among writers increases wisdom)



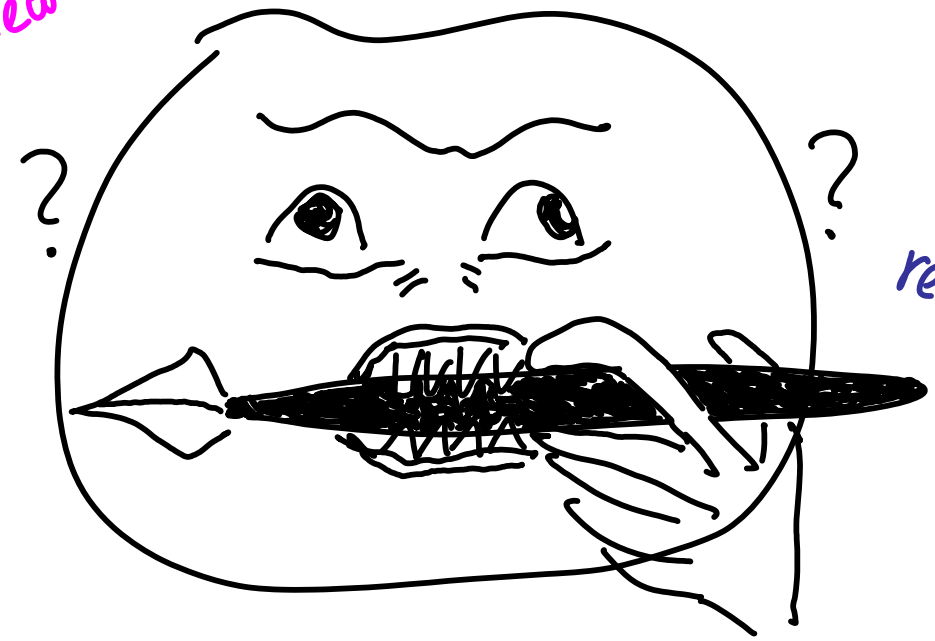
Writer's Misery

Where to start?

Alone or with...?

Question of style: formal or friendly?

Old or new material?



Textbook or research monograph?

Whose notation to use?

How many figures?

What do I (really) want to say?

Where / When to finish?...

Competition...?

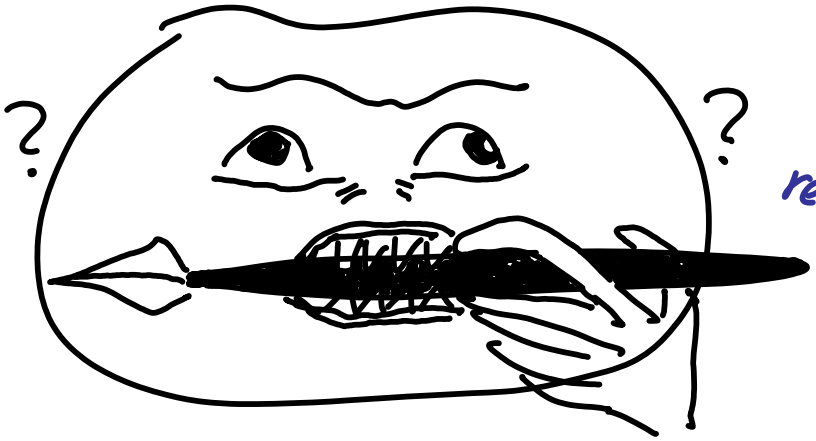
Writer's Misery

Where to start?

Alone or with...?

Question of style: formal or friendly?

Old or new?



Textbook or research monograph?

Whose notation to use?

How many figures?

What do I want to say?

Where / When to finish?...

Competition...?

Why did I start all this??...

Complexity Theory & Statistics

* writing complexity $= 1 + 2 + \ldots + n = O(n^2)$

where $n = 422$ pages ...

* 4 + 1 years (writing: March 2009 ÷ Oct. 2013
editing + printing: one more year)

* ~ 16 drafts (4 years, 1 every 3 months)

* ~ 5 deadline extensions with publisher

* 13 chapters + 1 Appendix,
(originally 10 + 4 chapter splits - 1 cancelled)

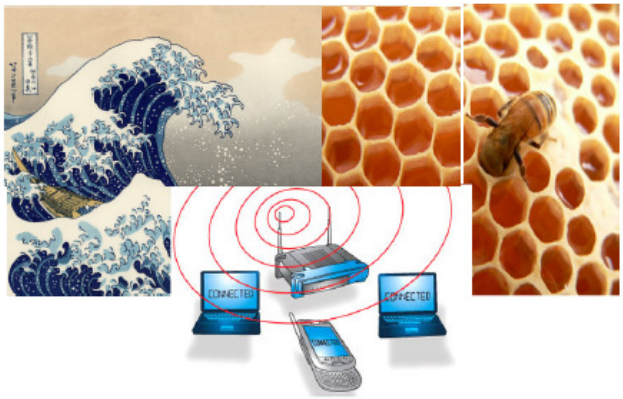
134 figures, __ examples

__ definitions, __ theorems

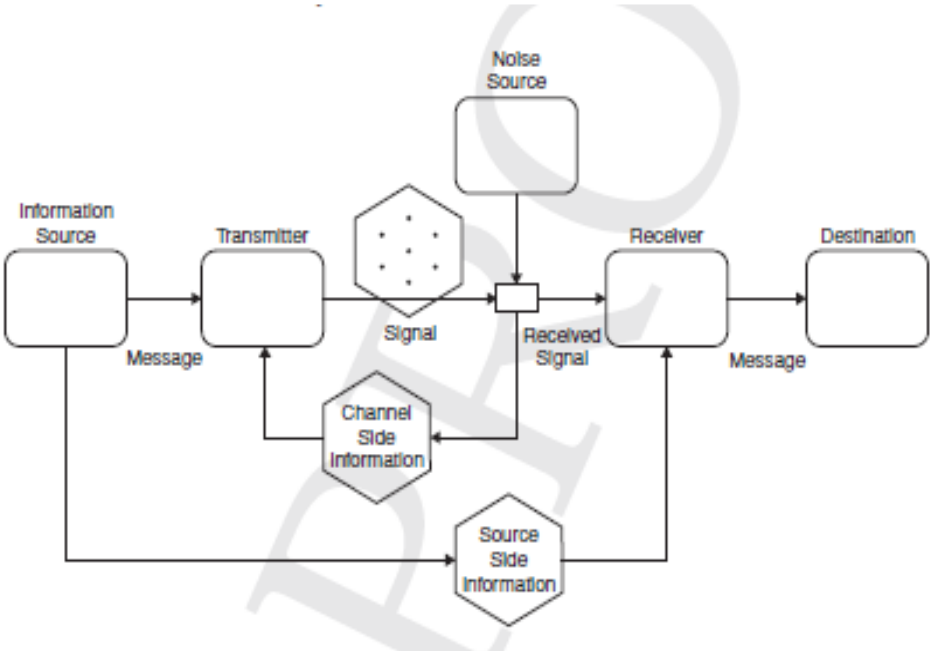
__ references, __ index terms

Cover page designs ...

Lattice Coding for Signals and Networks



Ram Zamir

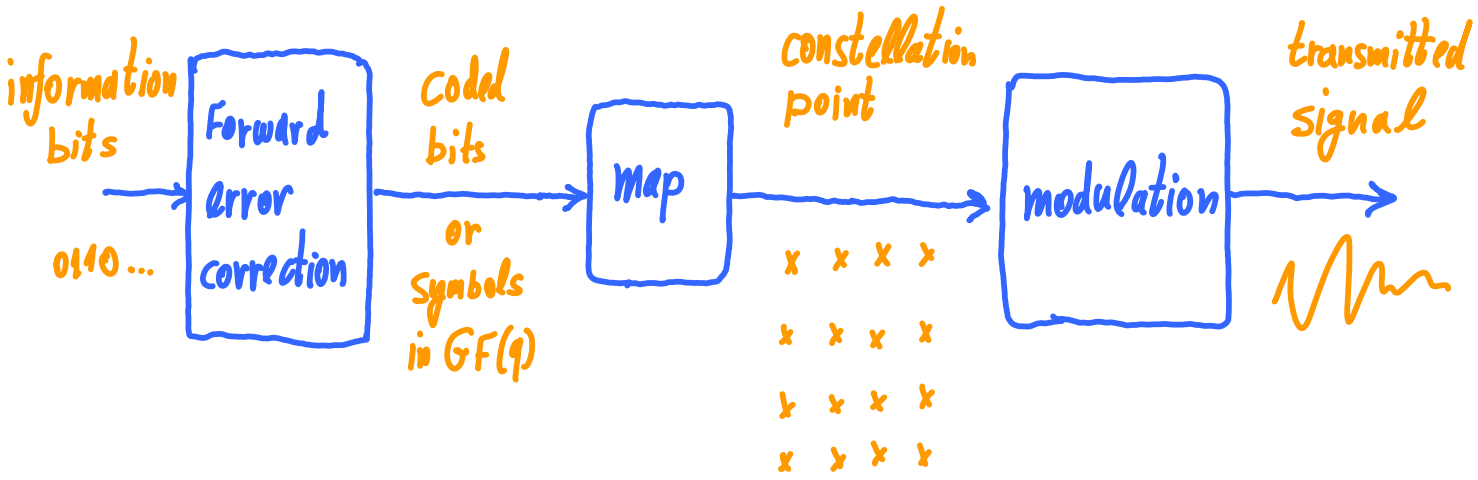


Many Cons, what about pros?

- Beauty of lattices
- History
- Circumstances
- Teaching (basis for a course)
- Anecdotes, Future ...

Lattice Codes as an "Alternative Language" for Digital Communication

1. Error correction in signal space (AWGN channel):



Block diagram of a digital modulator

Lattice Codes as an "Alternative Language" for Digital Communication

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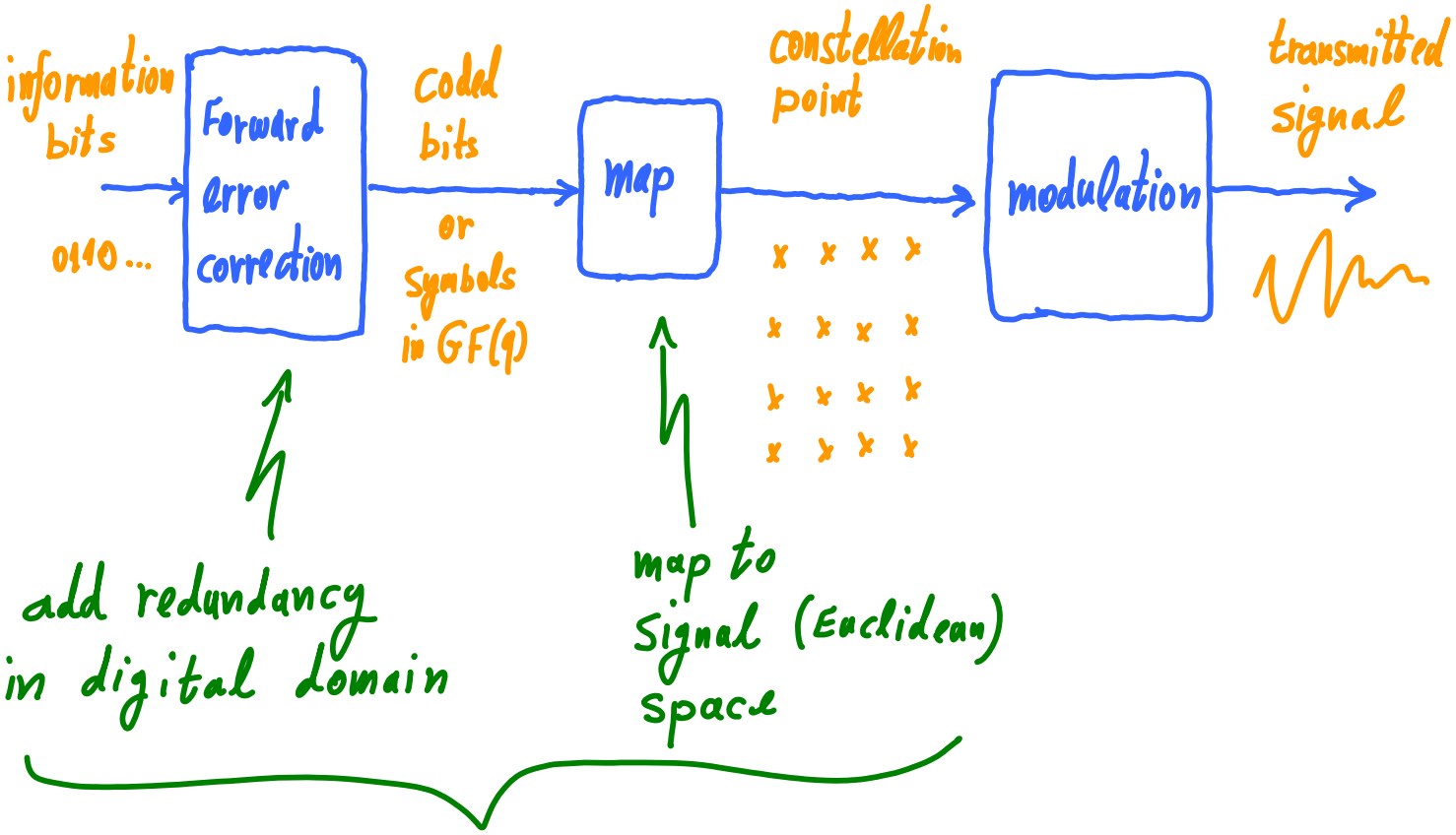
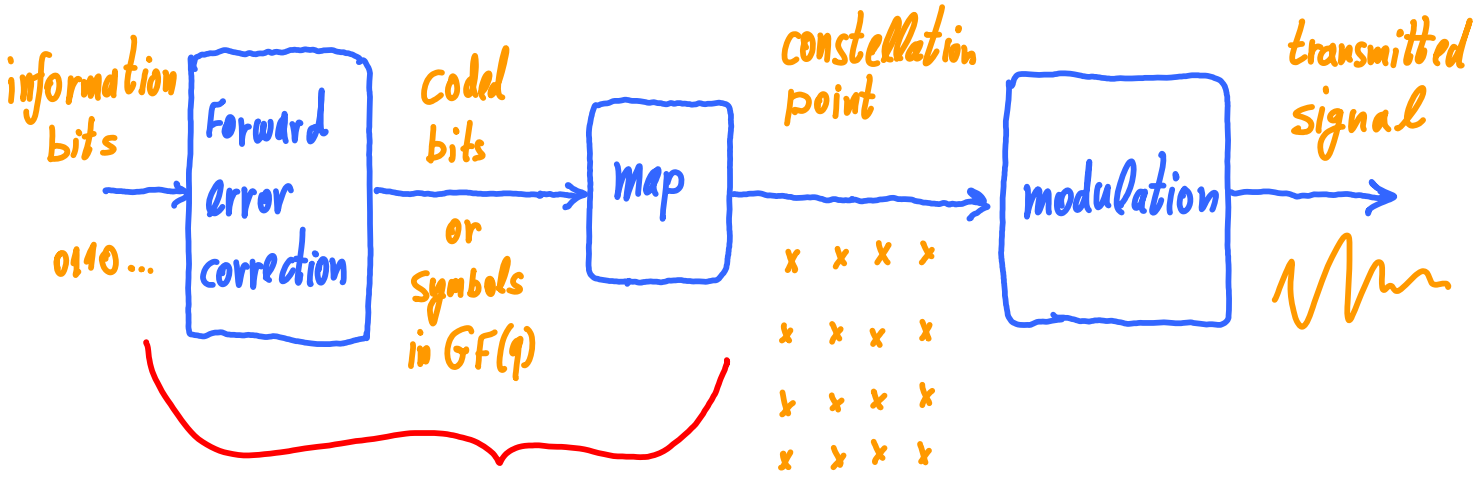


$\Rightarrow$ immunity to AWGN

Block diagram of a digital modulator

Lattice Codes as an "Alternative Language" for Digital Communication

1. Error correction in signal space (AWGN channel):



Replace by a lattice code:

info bits → Λ

Block diagram of a digital modulator

What is a Lattice ?

* Picture editing by Kessem Zamir

n-dimensional lattice: Definition

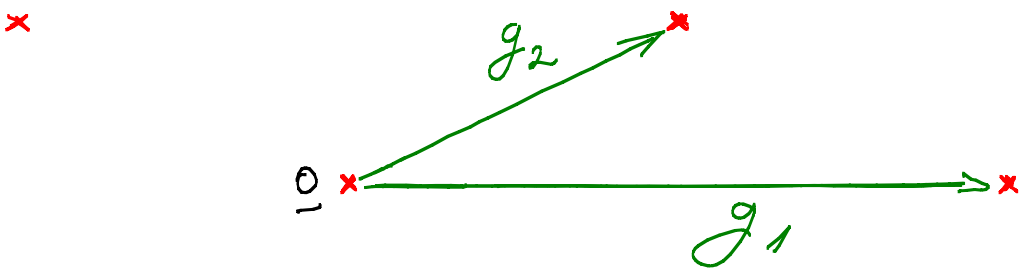
Let $\underline{g}_1, \ldots, \underline{g}_n$ – linearly independent vectors in $\mathbb{R}^n$

$$\underline{\underline{G}} = \left(\underline{g}_1 \,\middle|\, \cdots \,\middle|\, \underline{g}_n \right) = \text{generator matrix}$$

$$\Lambda(G) = \left\{ i_1 \cdot \underline{g}_1 + \ldots + i_n \cdot \underline{g}_n : \; i_1 \ldots i_n \in \mathbb{Z} \right\}$$

$$= \left\{ \underline{\underline{G}} \cdot \underline{i} : \; \underline{i} \in \mathbb{Z}^n \right\}$$

$$= \underline{\underline{G}} \cdot \mathbb{Z}^n$$



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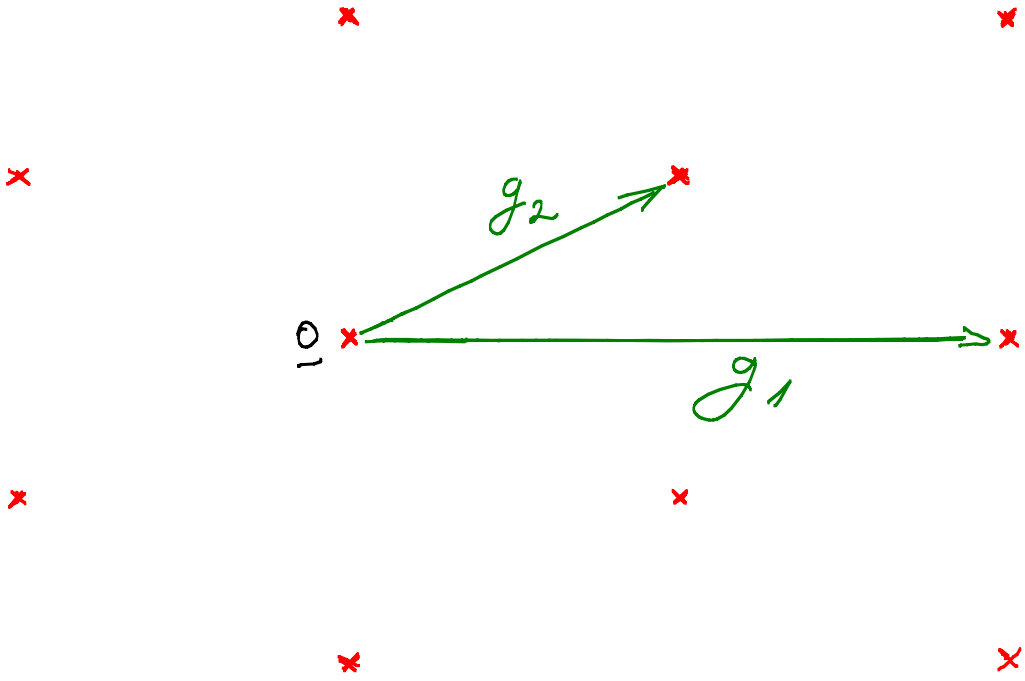
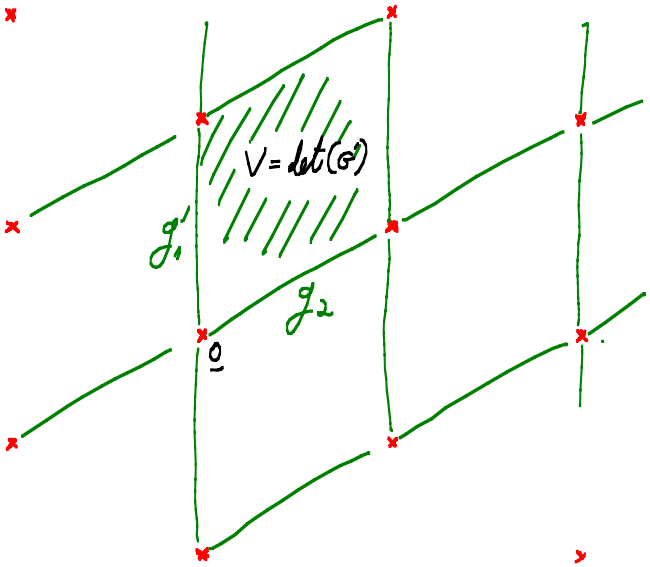


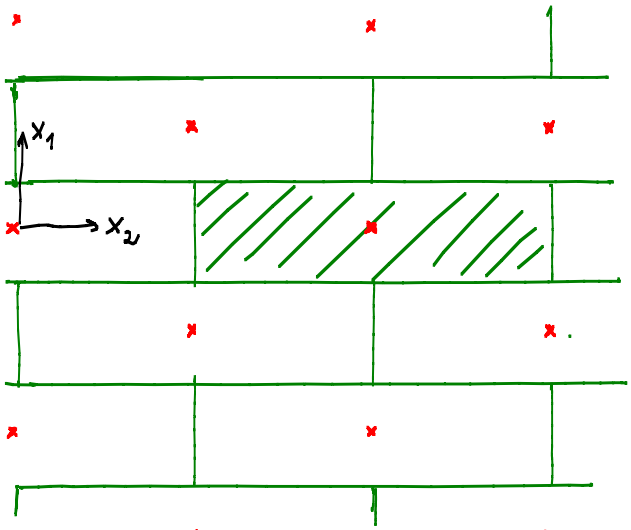
Linearity:

$$\lambda_1, \lambda_2 \in \Lambda \Rightarrow \lambda_1 \pm \lambda_2 \in \Lambda$$

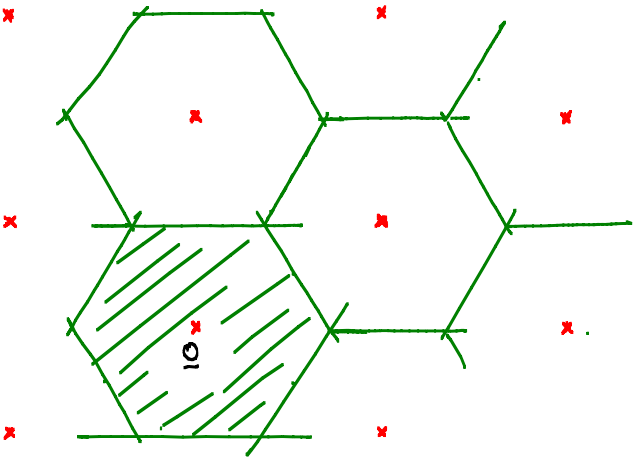
Partitions, Fundamental Cells



Other Basis ⇒
other Parallelepiped
⇒ Cell Volume V is
invariant of partition



Sequential Quantization

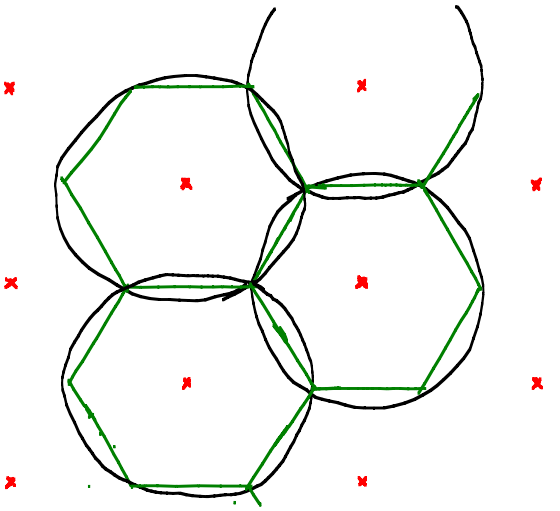


Voronoi Partition

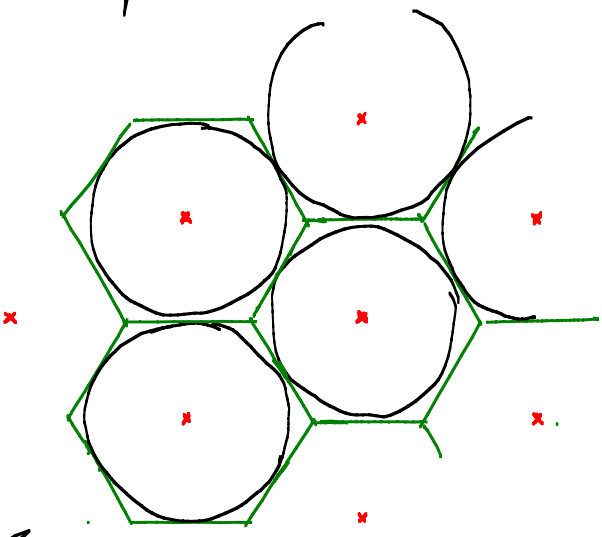
$$P_0 = \{ x : \|x\| \leq \|x - l_i\| \quad \forall l_i \in \Lambda \}$$

Covering, Packing, kissing Number & More....

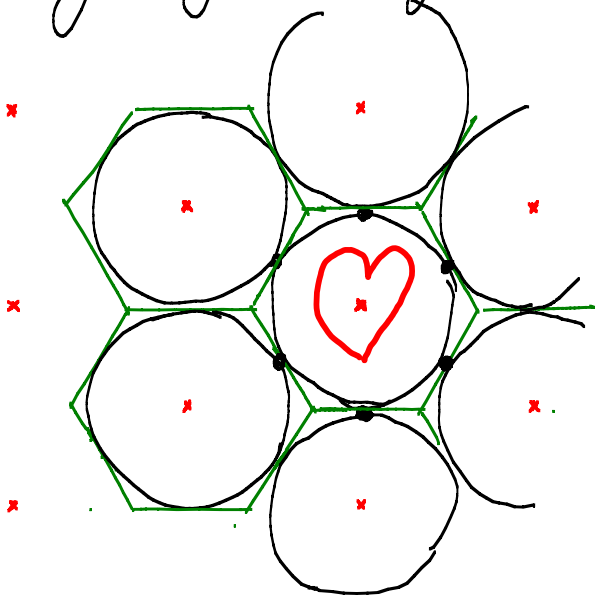
Covering $\mathbb{R}^n$ with (few) spheres



Packing (many) spheres in $\mathbb{R}^n$



Kissing by (many) Spheres

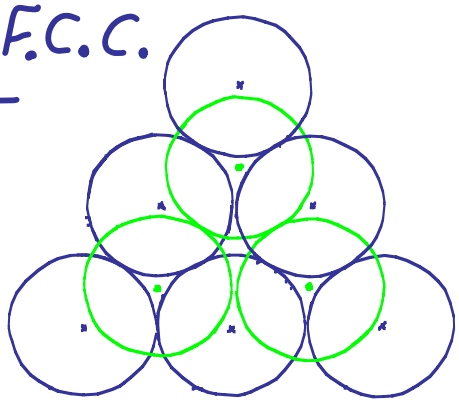


&

good arrangements for quantization and AWGN channel coding

Not an "All-Purpose" Lattice!

* **Best 3-dim Packing: F.C.C.**



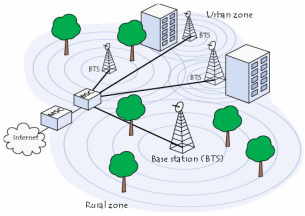
each layer = hexagonal Λ

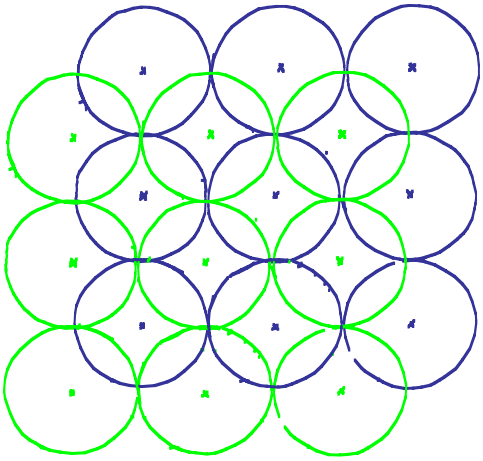
layers are staggered

* **Best 3-dim Covering: B.C.C.**

each layer = cubic Λ

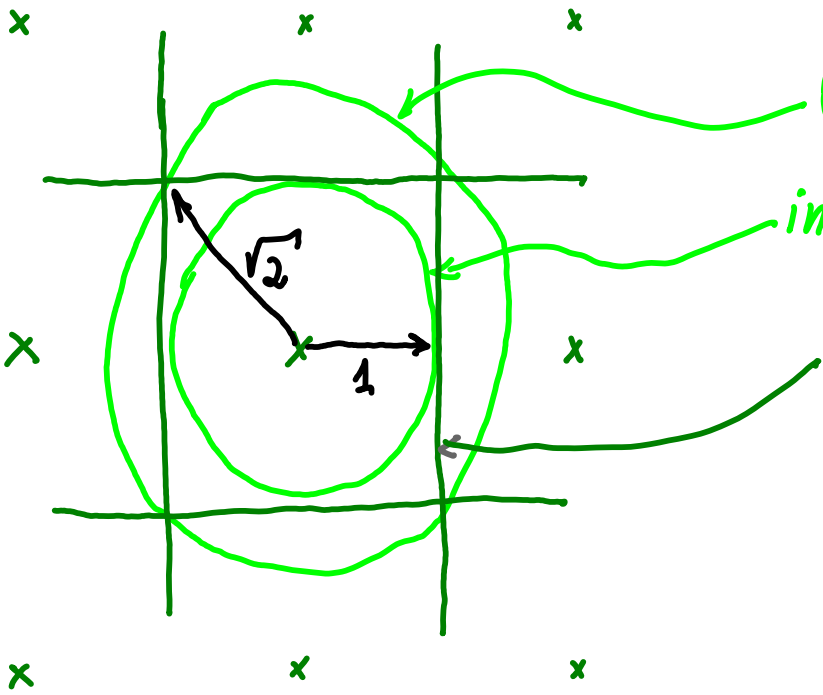
layers are staggered





As $n \to \infty$, most lattices are "good" for almost everything!

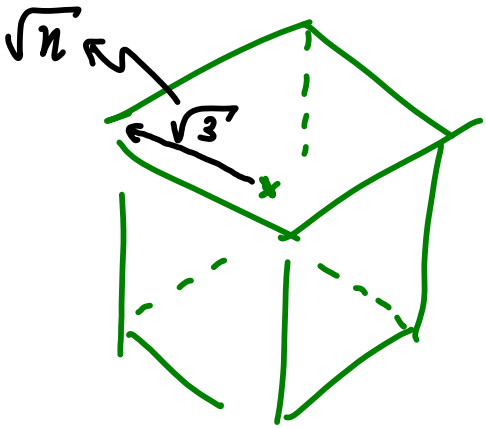
Packing & Covering efficiency of the Cubic lattice



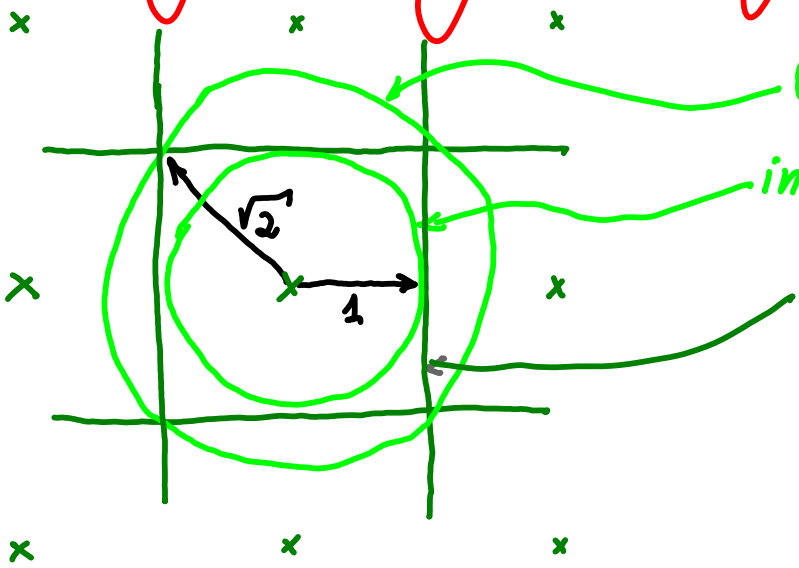
Outer circle area = $\pi(\sqrt{2})^2$

inner circle area = π

square area = 2^2



Packing & Covering efficiency of the Cubic lattice



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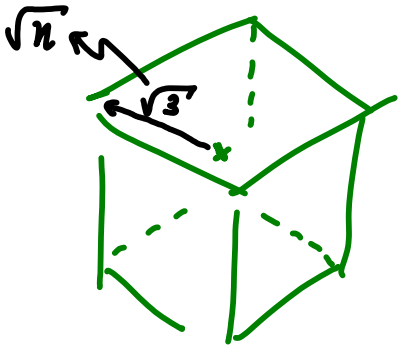
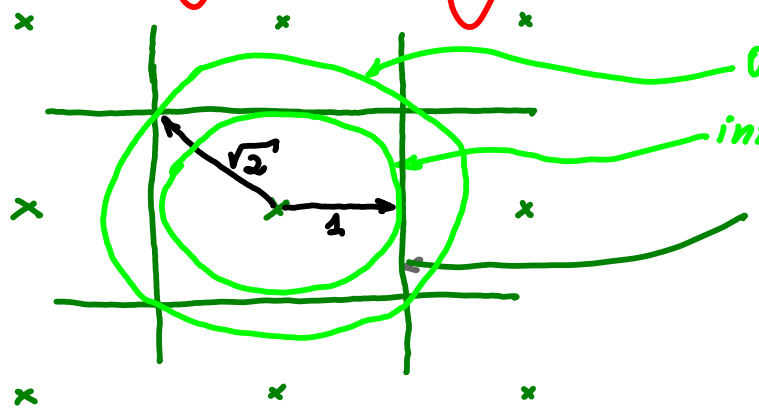
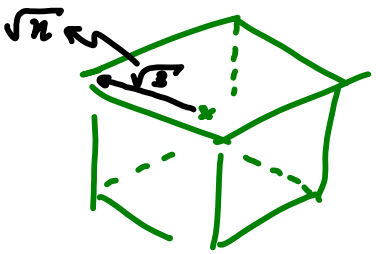


Behavior @ general dimension n:

* $n = 1, \quad 2 \quad 3, \ldots \quad n$
* cubic cell volume = $2, \quad 2^2, \quad 2^3, \ldots \quad 2^n$ ↗
* inner ball " = $2, \quad \pi, \quad \frac{4\pi}{3}, \ldots \quad V_n$ ↘
* outer ball " = $2, \quad \pi(\sqrt{2})^2, \quad \frac{4\pi}{3}\cdot(\sqrt{3})^3, \ldots \quad V_n \cdot (\sqrt{n})^n$ ↗
* where $V_n \triangleq$ volume of unit radius n-dim ball $\sim \left(\sqrt{\frac{2\pi e}{n}}\right)^n$

Packing & Covering efficiency of the Cubic lattice





Behavior @ general dimension n:

$n =$	1	2	3	...	n	
cubic cell volume =	2	2^2	2^3	...	2^n	↗
inner ball " =	2	π	$\frac{4\pi}{3}$	...	V_n	↘
outer ball " =	2	$\pi(\sqrt{2})^2$	$\frac{4\pi}{3}(\sqrt{3})^3$	....	$V_n \cdot (\sqrt{n})^n$	↗

* where $V_n \triangleq$ volume of unit radius n-dim ball $\backsim \left(\sqrt{\frac{2\pi e}{n}}\right)^n$

$\Rightarrow$

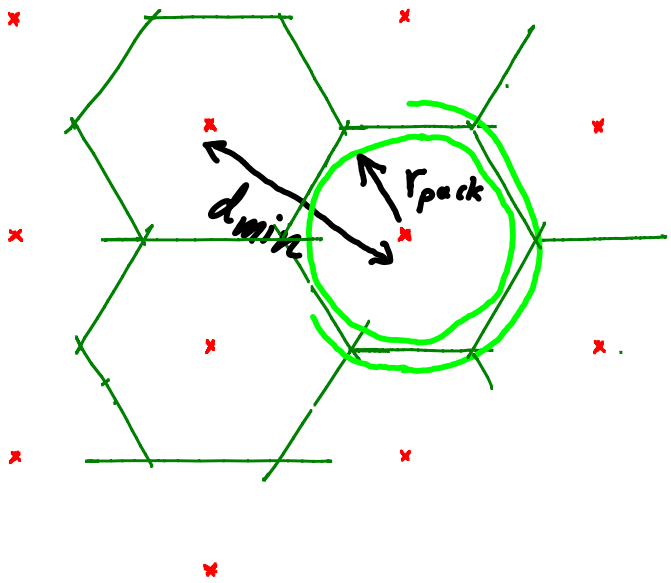
$$\text{Packing efficiency} \triangleq \sqrt[n]{\frac{\text{Packed balls volume}}{\text{space volume}}} \to 0 \Rightarrow \text{bad...}$$

while

$$\text{Covering efficiency} \triangleq \sqrt[n]{\frac{\text{Covering balls volume}}{\text{space volume}}} \to \sqrt{\frac{\pi e}{2}} = 2$$

$\Rightarrow$ quite good!

The nominal coding gain of a (good) lattice



$$d_{min} = 2 \cdot r_{pack}$$

$d_{min}(\text{hexagonal}) \simeq 1.074 \times d_{min}(\text{square})$ @ same cell volume $\Rightarrow \Gamma_c^{nom} \simeq 0.62\ dB$

$d_{min}(\text{FCC}) \simeq 1.122 \times d_{min}(\text{cubic})$ @ same cell volume $\Rightarrow \Gamma_c^{nom} \simeq 1\ dB$

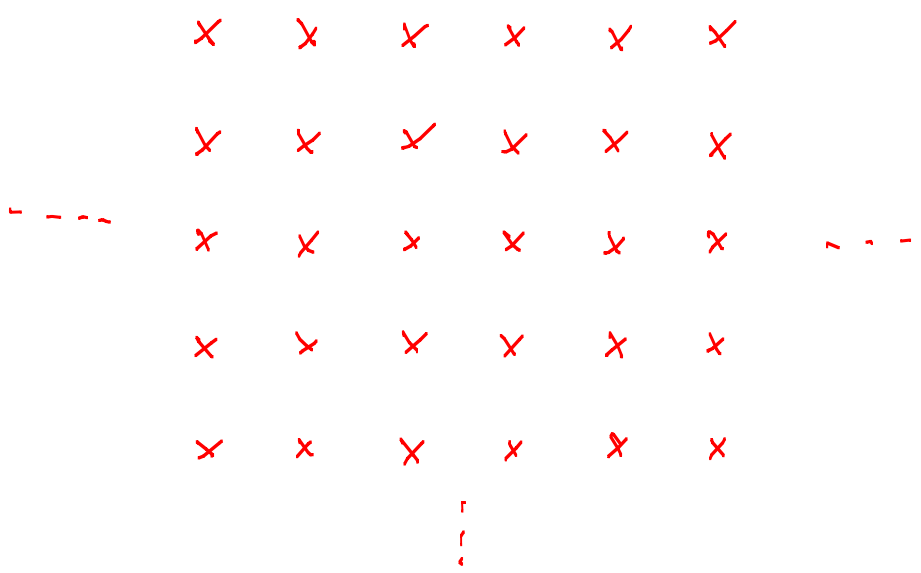
Why is Γ_c^{nom} important?

It's a dominant factor in determining error probability in the presence of Additive White Gaussian Noise

Lattice Codes in Signal Space

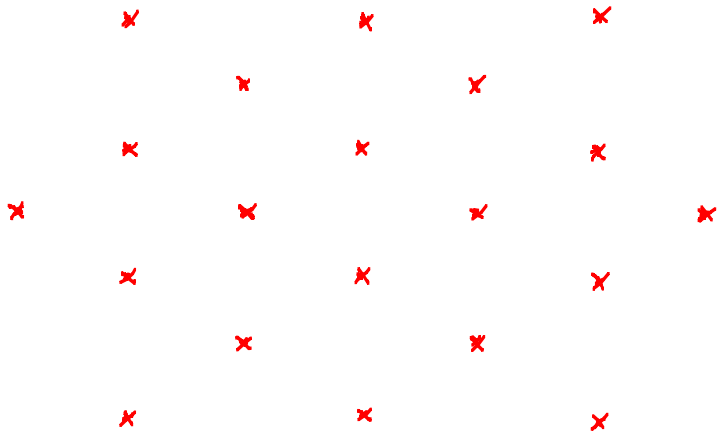
square ($\mathbb{Z}$)-lattice $\Rightarrow$ uncoded constellation

("bad lattice")



More "interesting" lattice $\Rightarrow$ coded constellation

("good lattice")



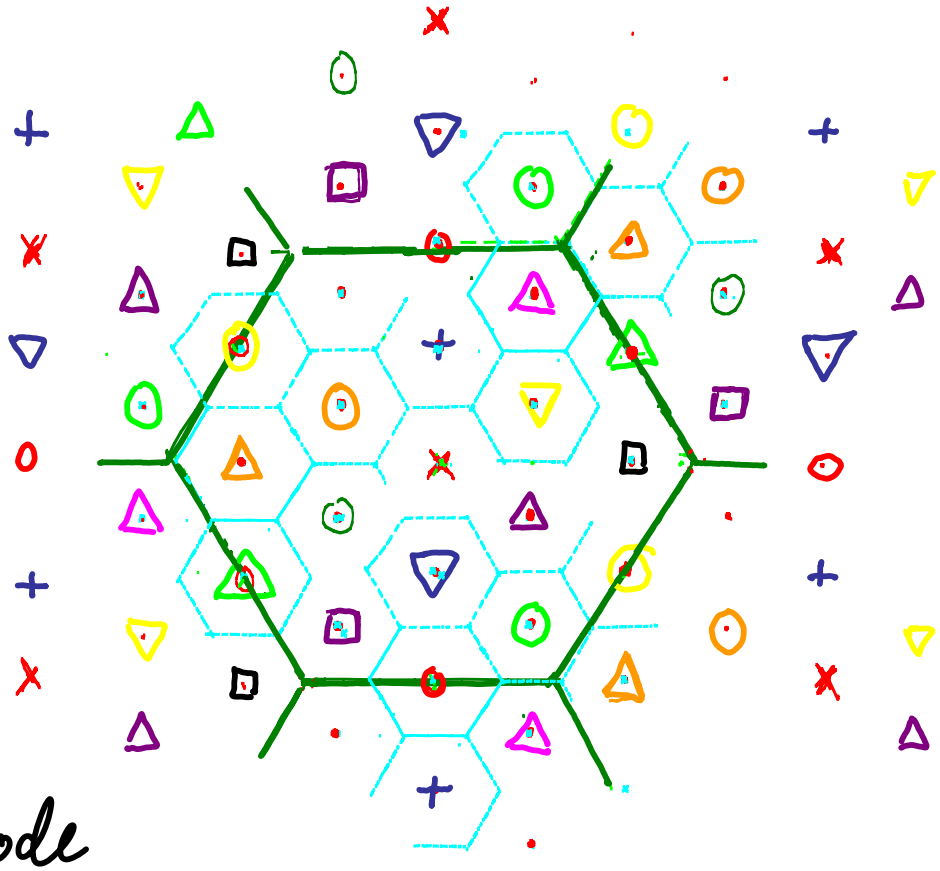
Nested Lattices

$$\Lambda_2 \subset \Lambda_1 \Rightarrow \underline{\underline{G_2}} = \underline{\underline{G_1}} \cdot \underline{\underline{J}}$$

Relative Cosets $= \Lambda_2 / \Lambda_1$

Coset $\triangleq \ell_1 + \Lambda_2$, for some $\ell_1 \in \Lambda_1$

$$|\Lambda_2 / \Lambda_1| = V_2 / V_1 = |\det(J)|$$



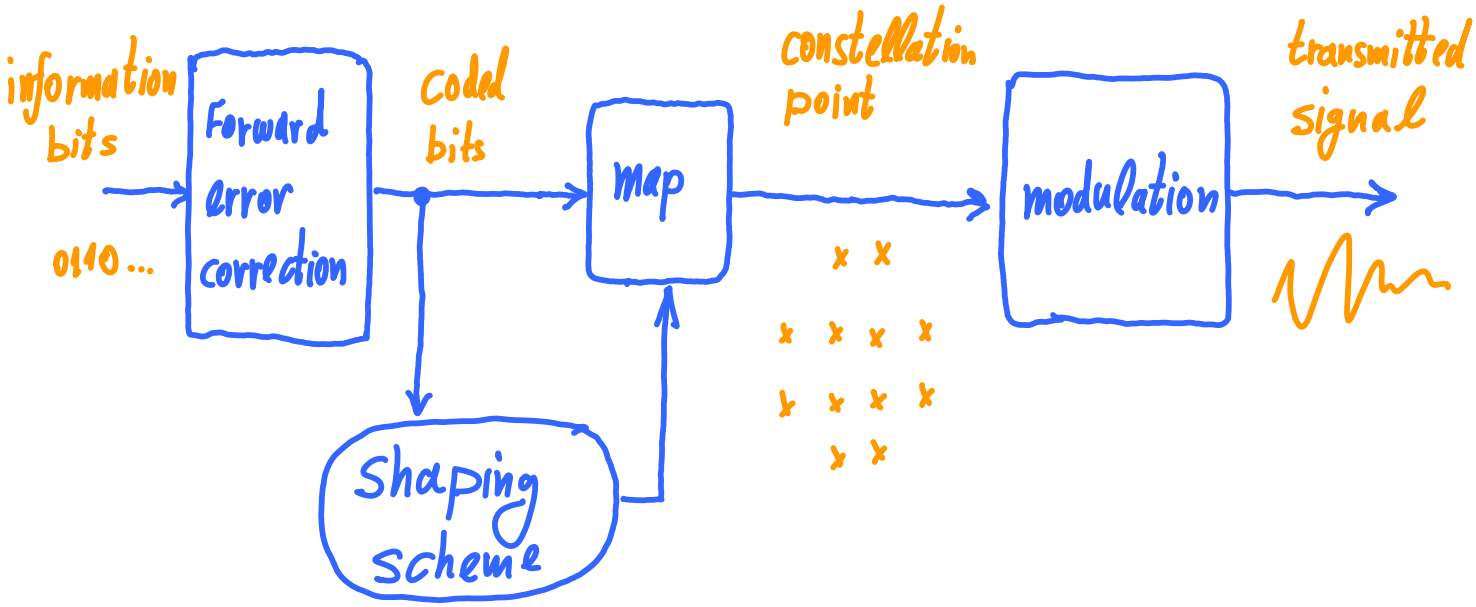
$\Rightarrow$ Lattice Code

Back to ...

Lattice Codes as an "Alternative Language" for Digital Communication

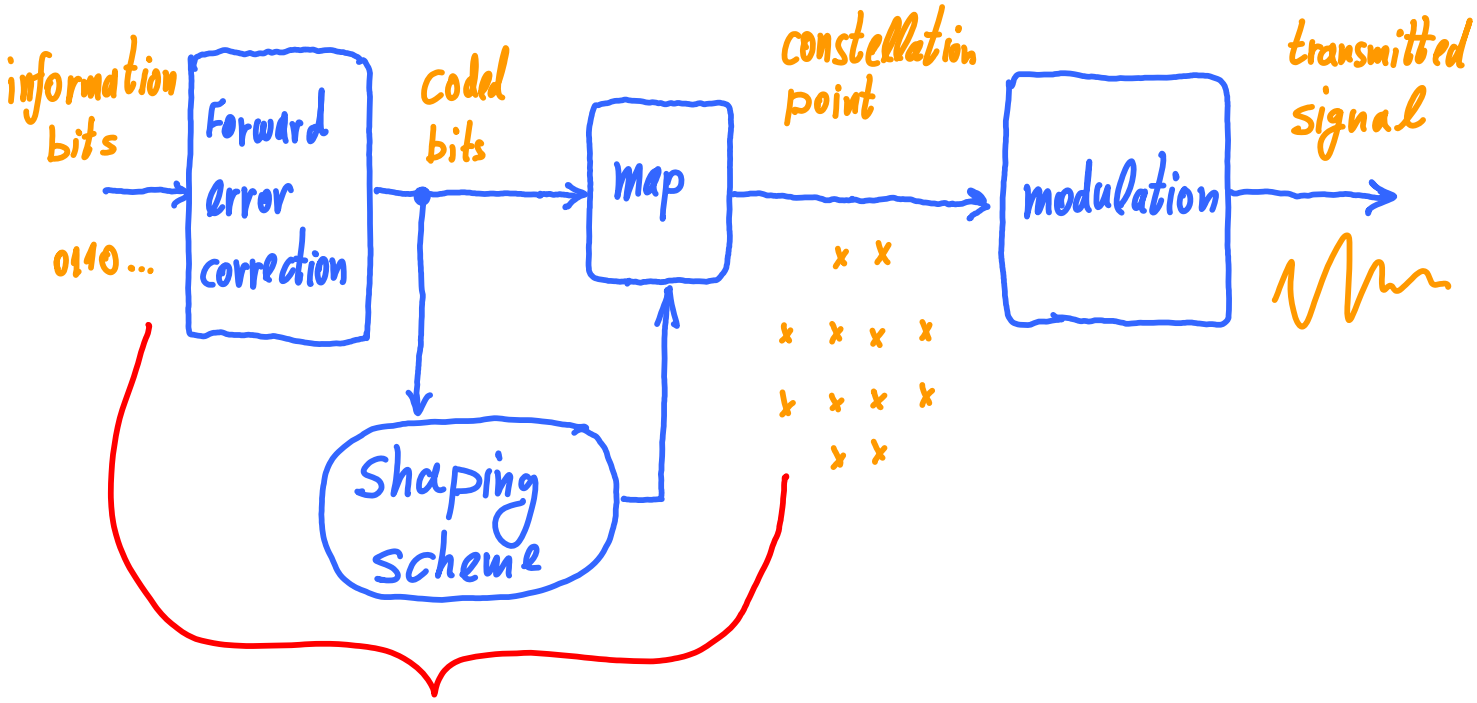
Lattice Codes as an "Alternative Language" for Digital Communication

2. Shaping : power & peak amplitude control :



Block diagram of a digital modulator

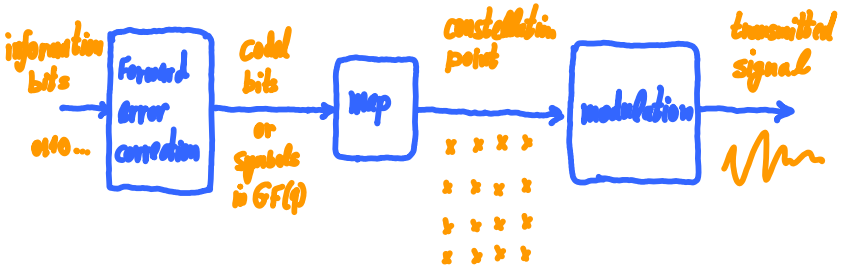
2. Shaping : power & peak amplitude control :

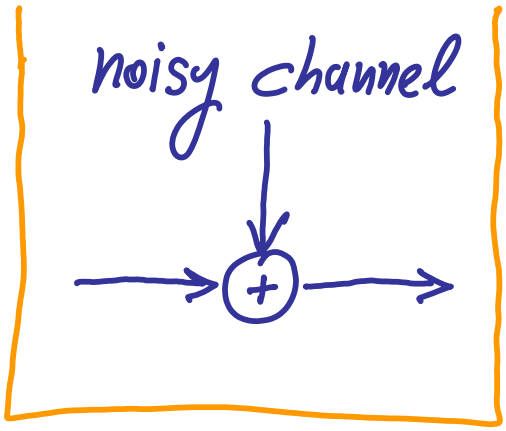


Replace by a Voronoi constellation :

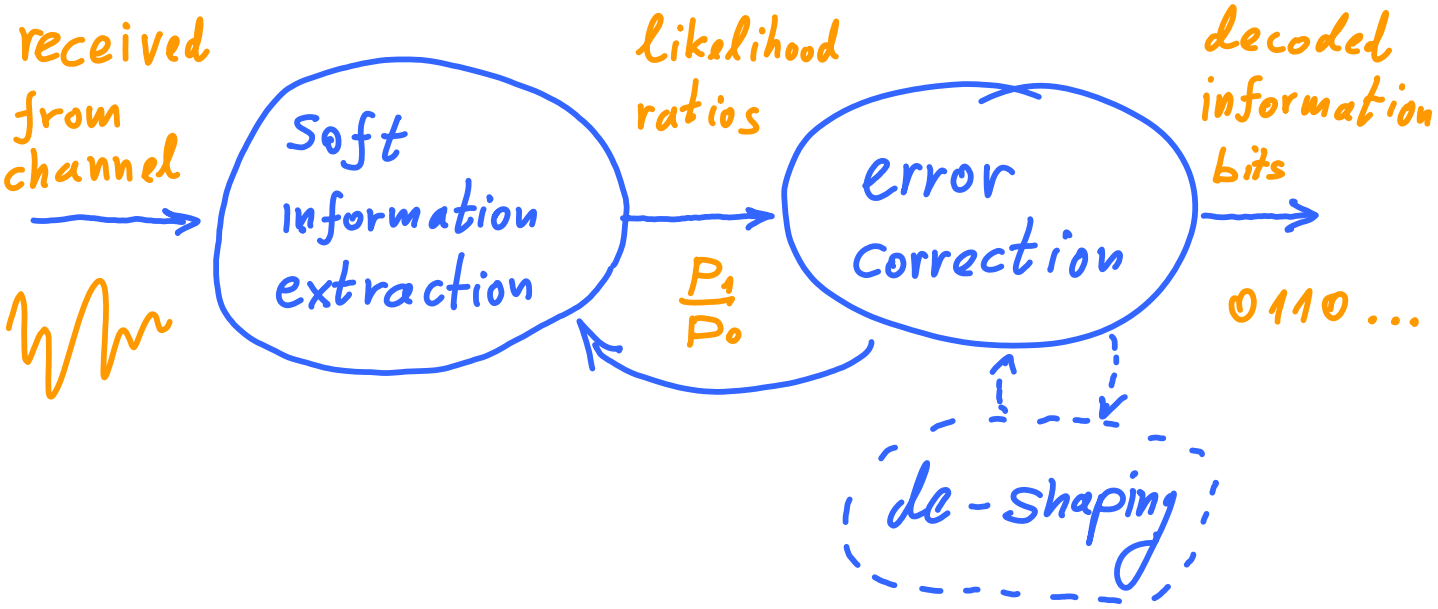
info bits $\longrightarrow$ $\Lambda_{coding} / \Lambda_{shaping}$

Block diagram of a digital modulator





3. Practical receiver / decoder structure:



Block diagram of a digital de-modulator

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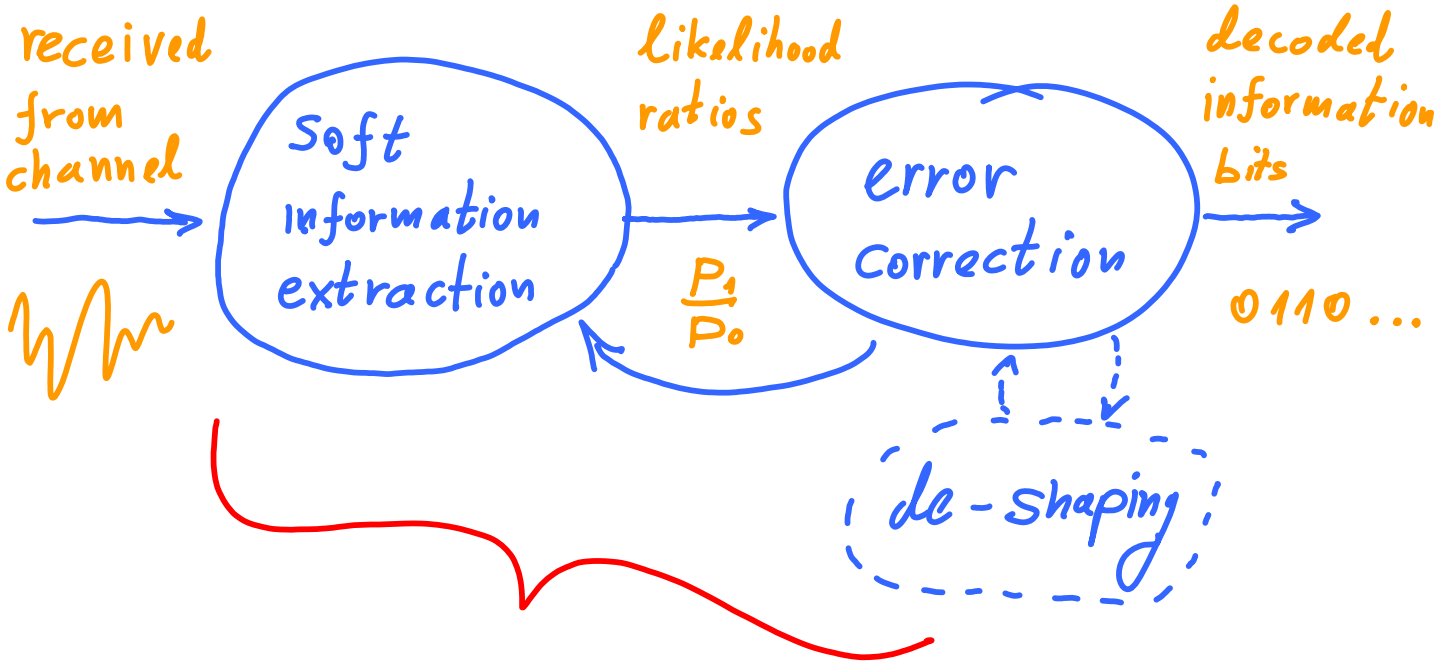
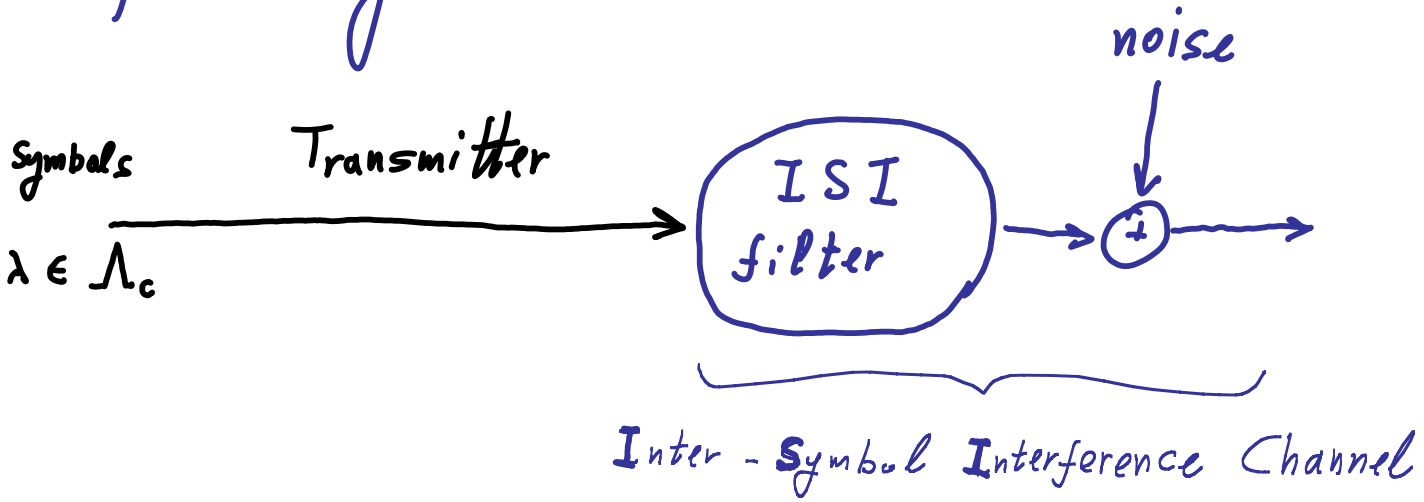


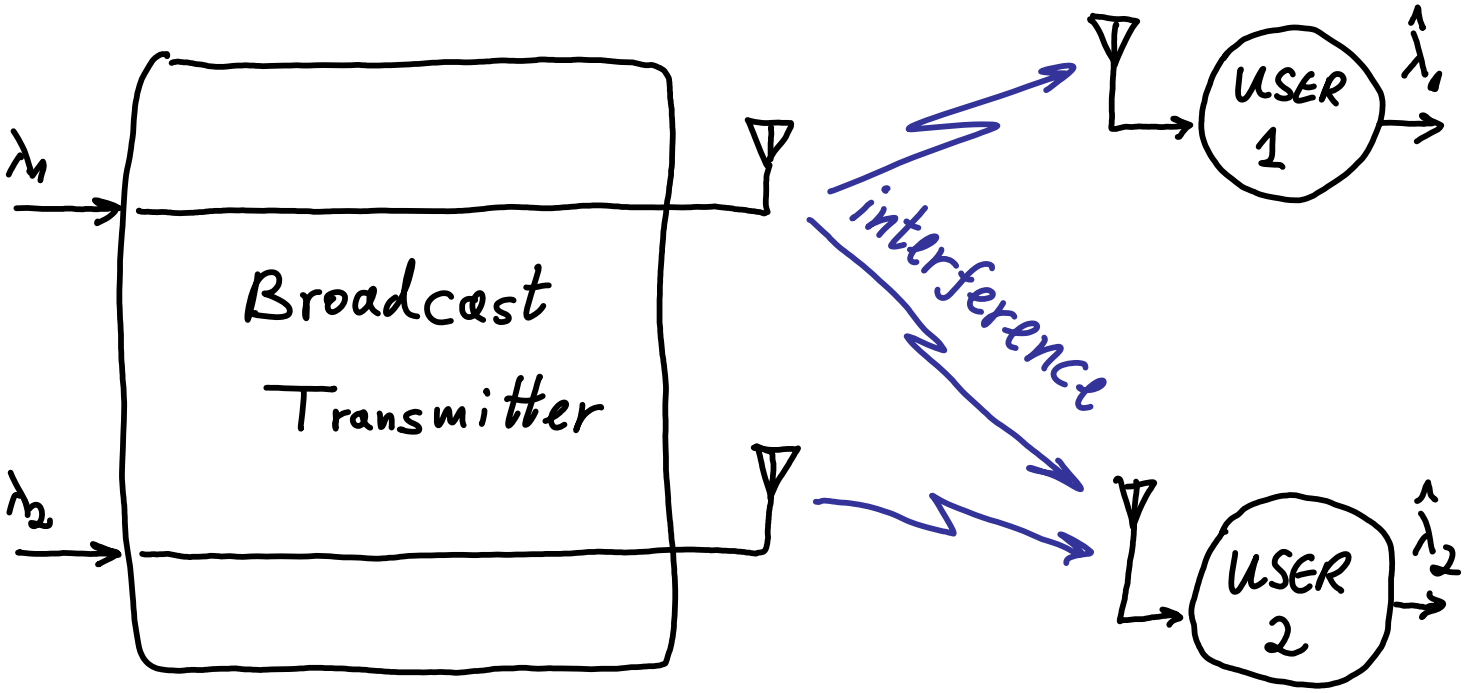
lattice decoding:

Λ_c / Λ_s + noise $\rightarrow$ Λ_c $\rightarrow$ info bits

Block diagram of a digital de-modulator

4. Precoding (equalization, Multi-User MIMO):





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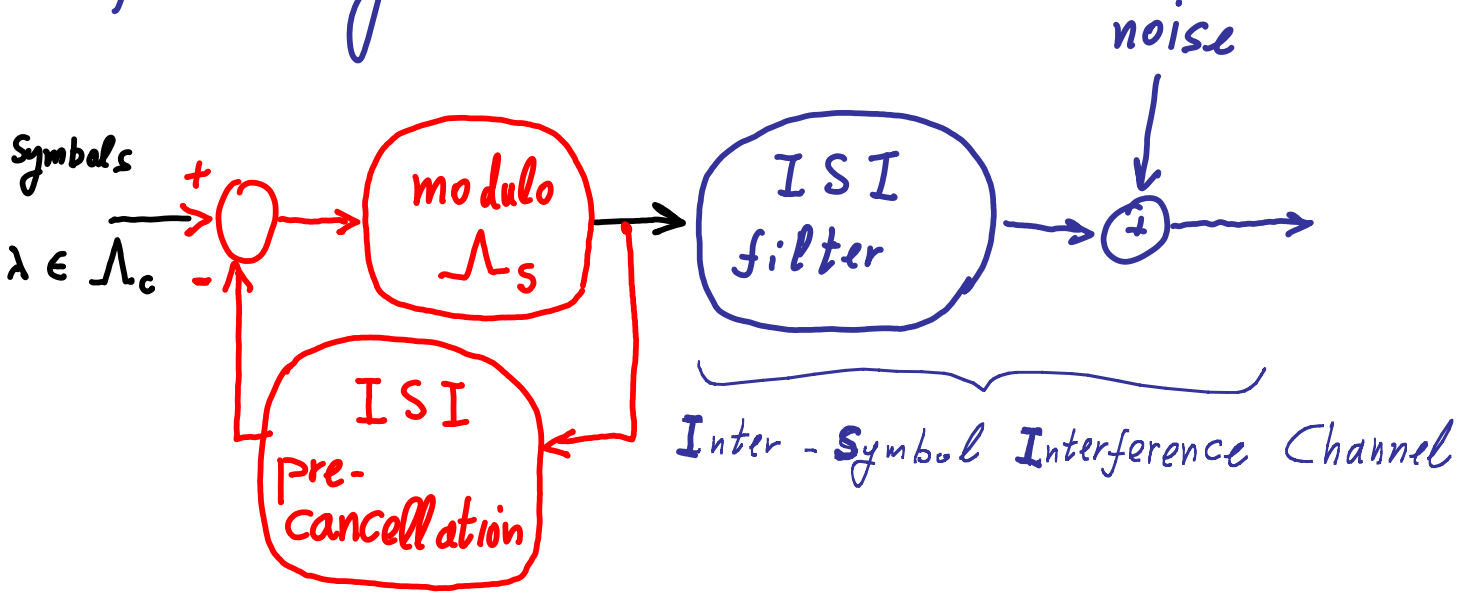
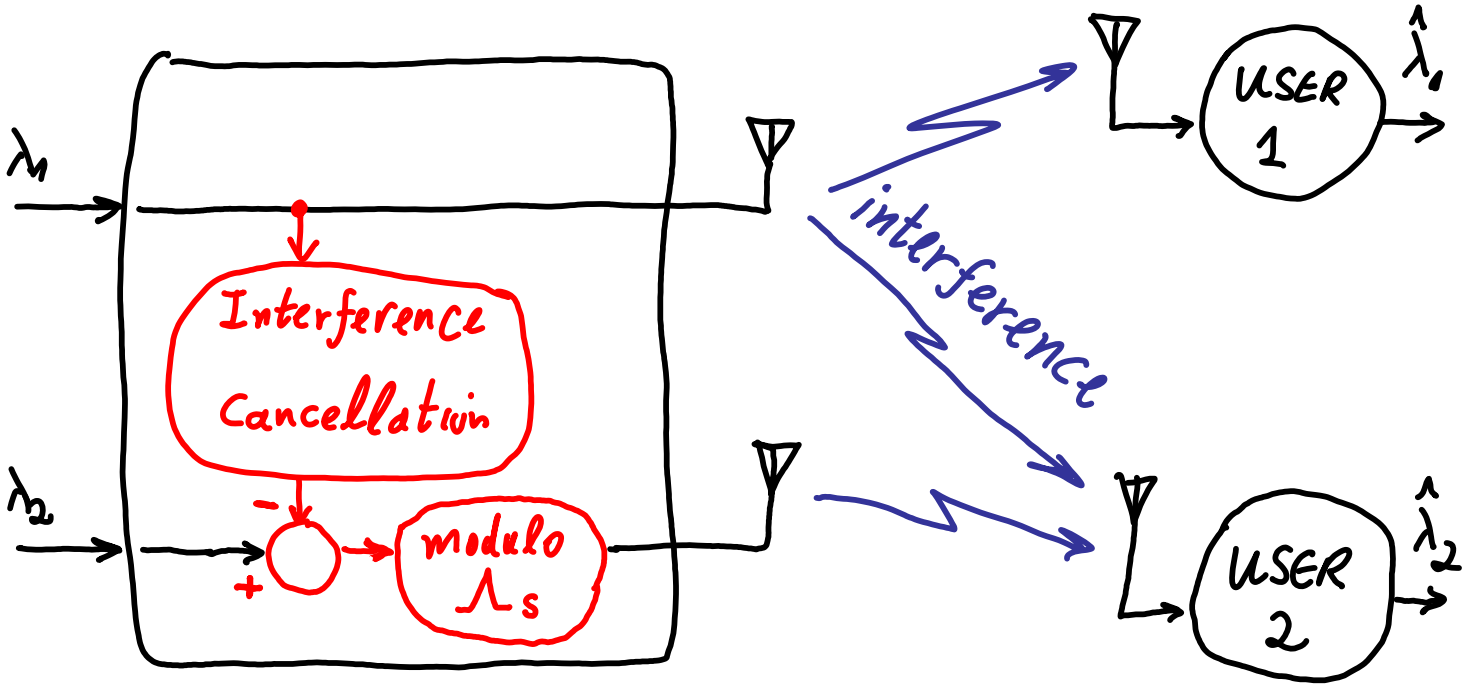
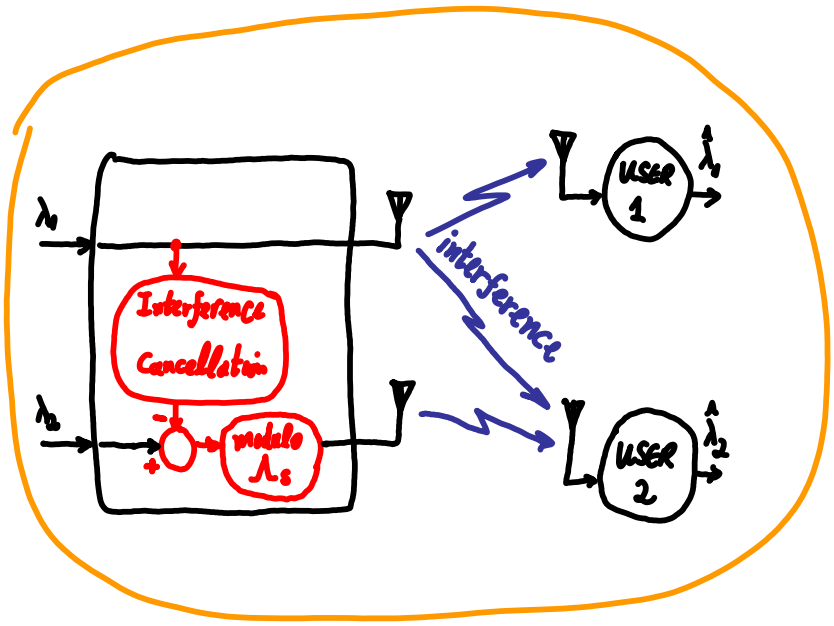






$\Longleftrightarrow$ MIMO Broadcast by Dirty-paper coding [Caire-Shamai 2002]

$\Rightarrow$ lattice binning

$\Rightarrow$ alternative language for multi-user information theory !

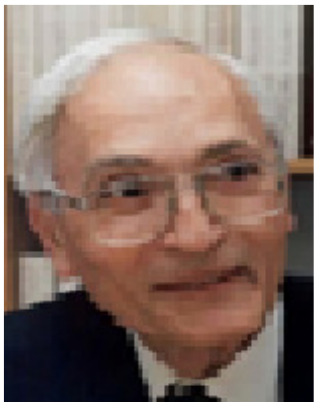
Many Cons, what about pros?

- Beauty of Lattices
- History
- Circumstances
- Teaching (basis for a course)
- Anecdotes, Future ...

How did it all start ?

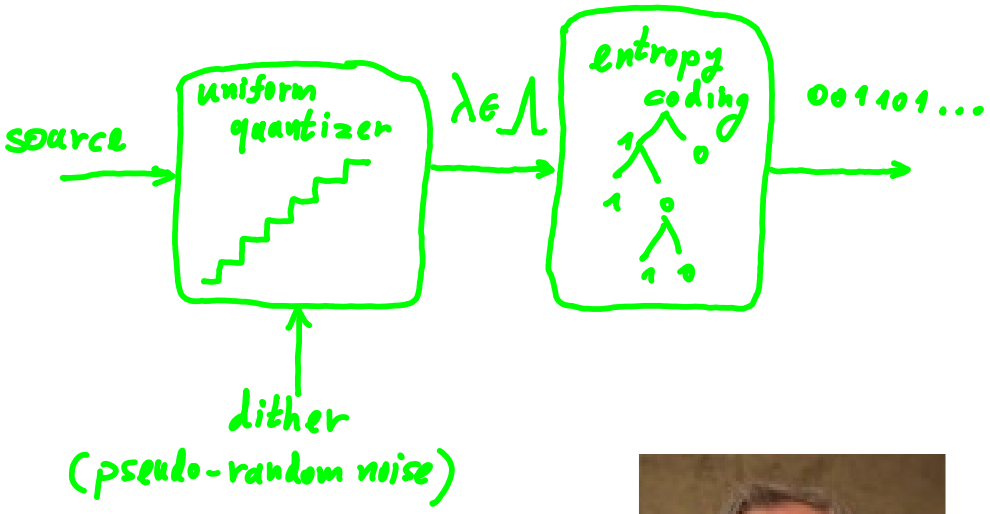
Meir Feder

data compression class 1989...



Jacob Ziv

universal (dithered) quantization [1985]



Gersho's conjecture [1979]

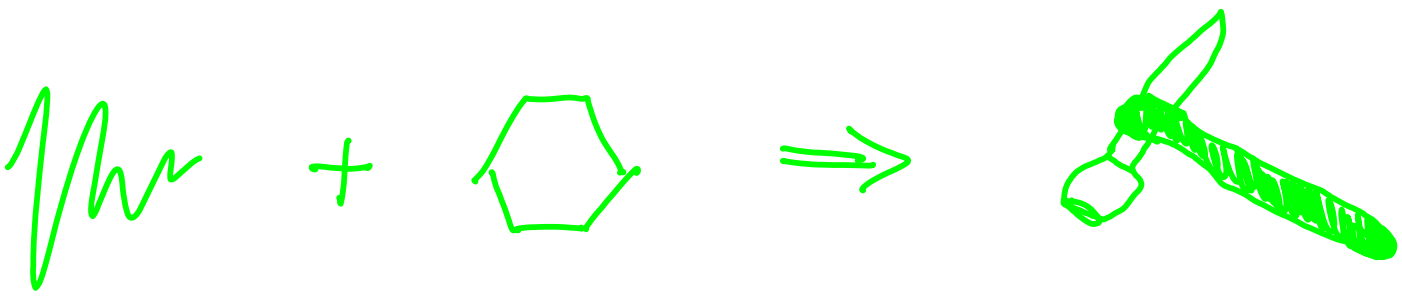
Allen Gersho

$\Rightarrow$ MSc [~ Jan 1991]

Moral 1 :

Dither (randomization) + Lattice

$\Rightarrow$ a promissing analatic tool !



How did it all start ?

Meir Feder

Yair Beery

~ 1991

seminar on the...

first book about lattice codes

John Conway

Neil Sloane

Gregory poltyrev

~ 1992

challange : dithered modulation ?!

~1993

covering $\Longleftrightarrow$ quantization

$G(\Lambda_n) \longrightarrow \frac{1}{2\pi e}$

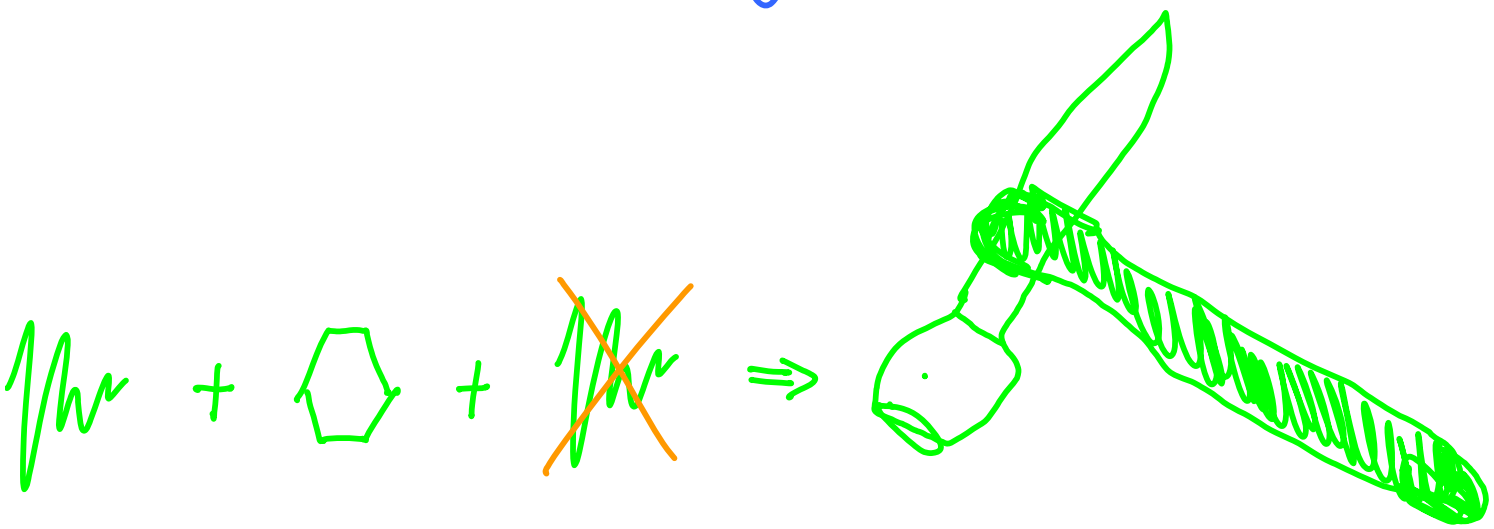
Simon Litsyn

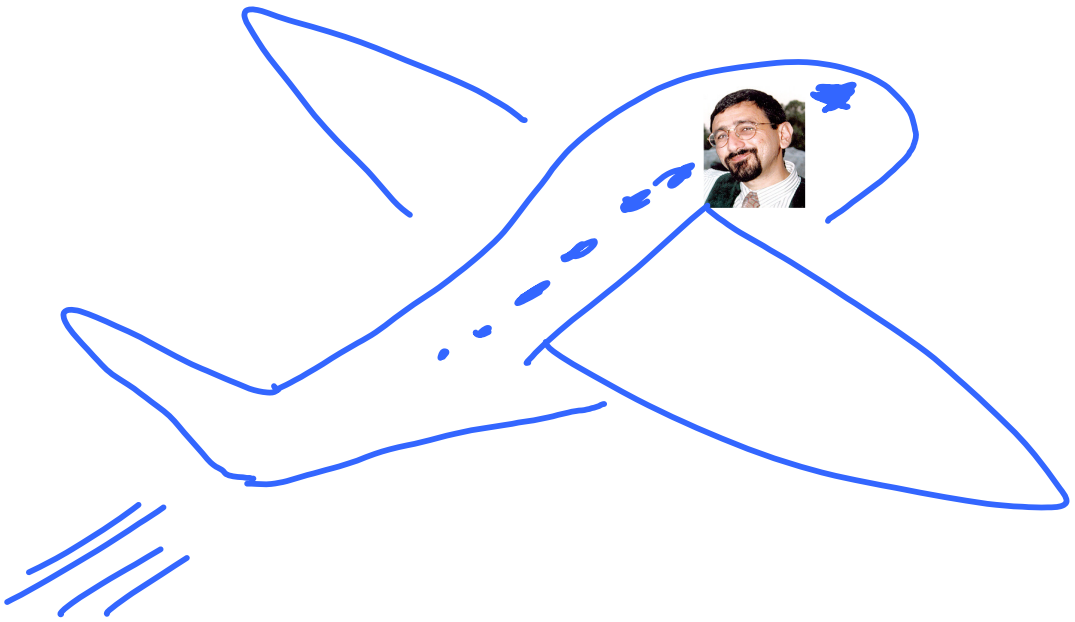
$\Rightarrow$ PHD [1993 , 1994]

Moral 2:

Dither + Lattice + Wiener estimation (MMSE)

$\Rightarrow$ a very promissing analatic tool!!





July 1994 - going to post doc ...

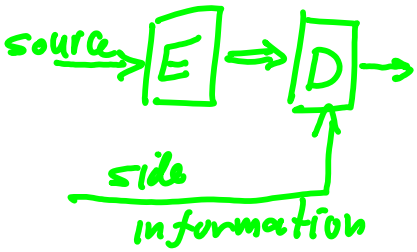
Toby Berger

Mathematical Theory of Data Compression [1971]
& Multi-terminal source coding [1978 ÷]

Wyner-Ziv Problem

Aaron Wyner

Jacob Ziv

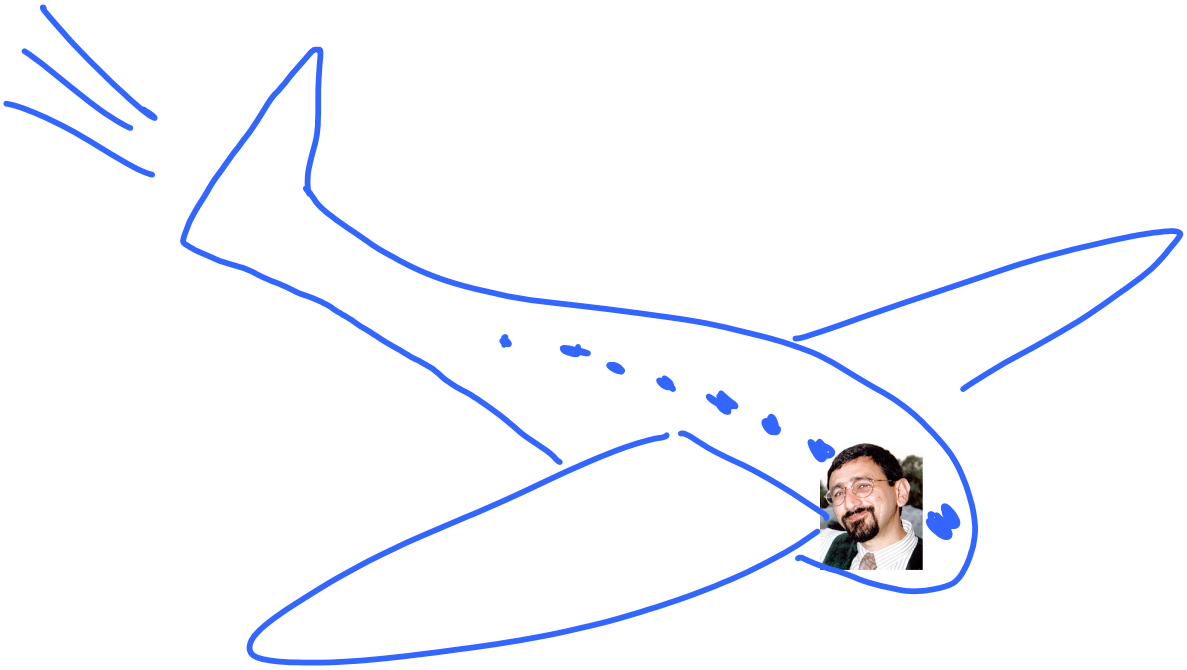


nested binary / lattice Wyner-Ziv coding

Shlomo Shamai

Sergio Verdu

⇒ post doc



July 1996 - back to TAU ...

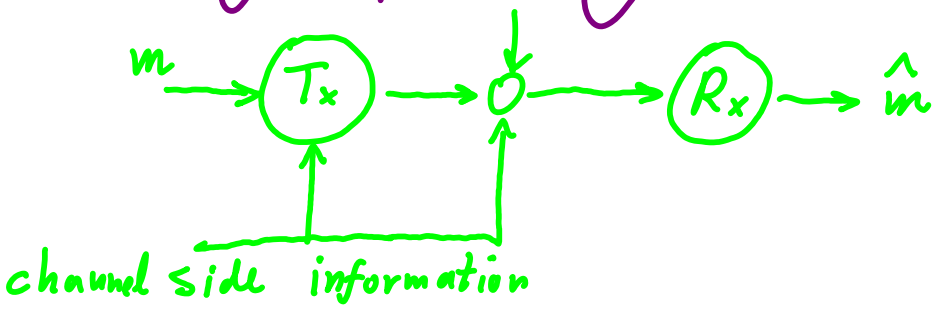
Uri Erez

advanced information theory class 1997:

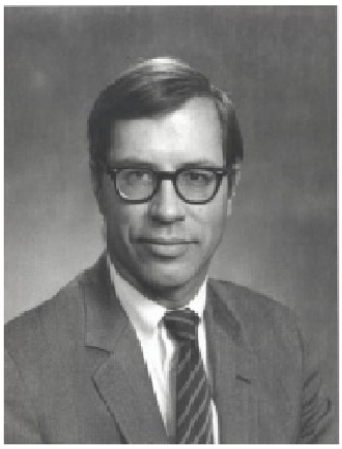
side-information @ transmitter

Shlomo Shamai

dirty-paper coding [Costa 1983]



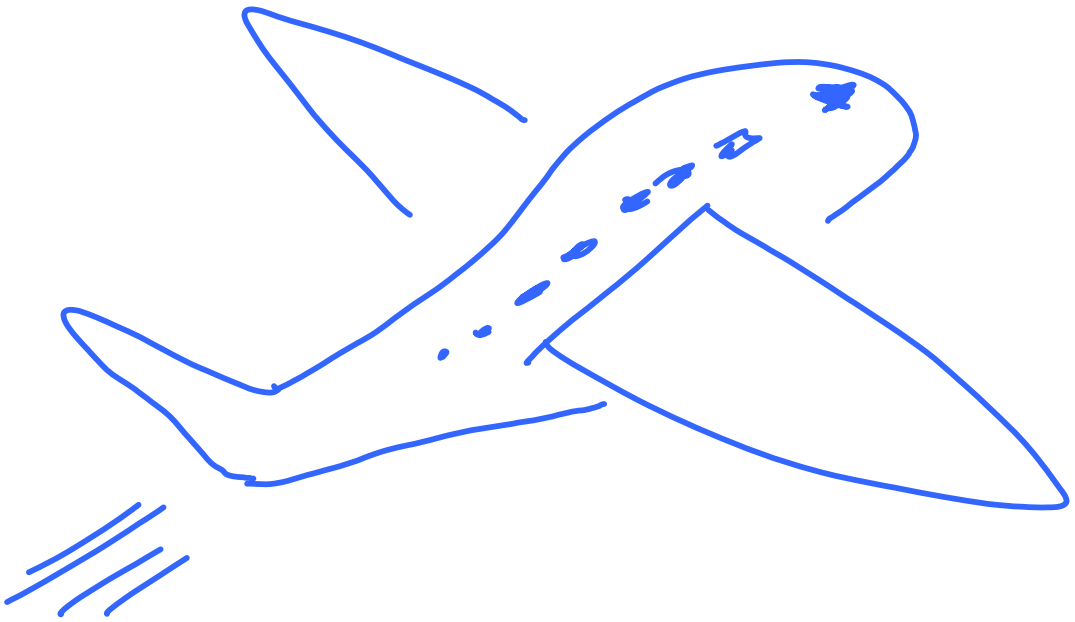
⇒ lattice DPC !



David Forney

achieving $\frac{1}{2}\log(1+SNR)$ with lattice (Λ_c/Λ_s) coding

⇒ first PHD student

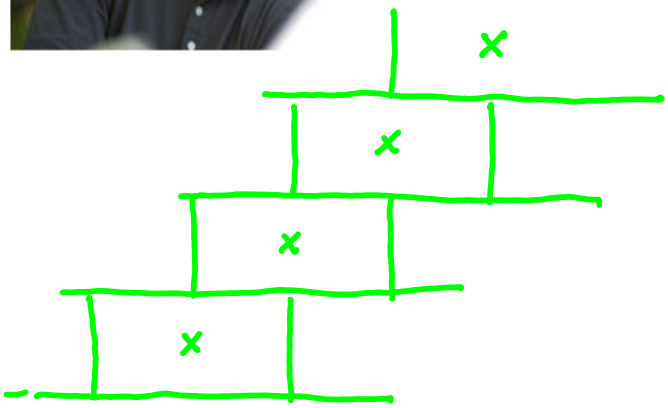


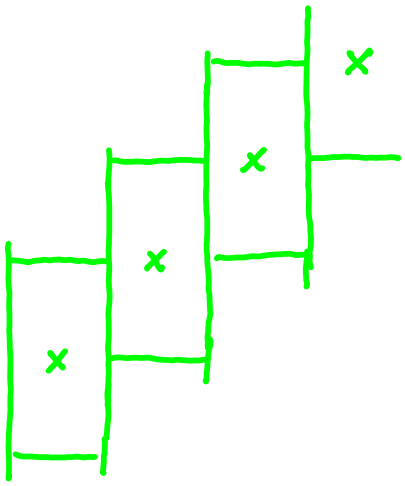
Aug. 2002: going to Sabbatical @ MIT

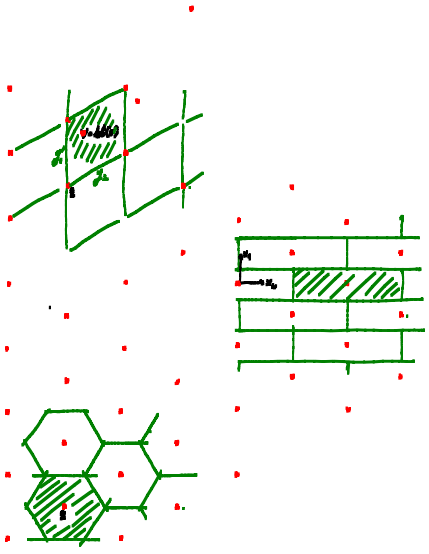
Greg Wornell

Emin Martinian

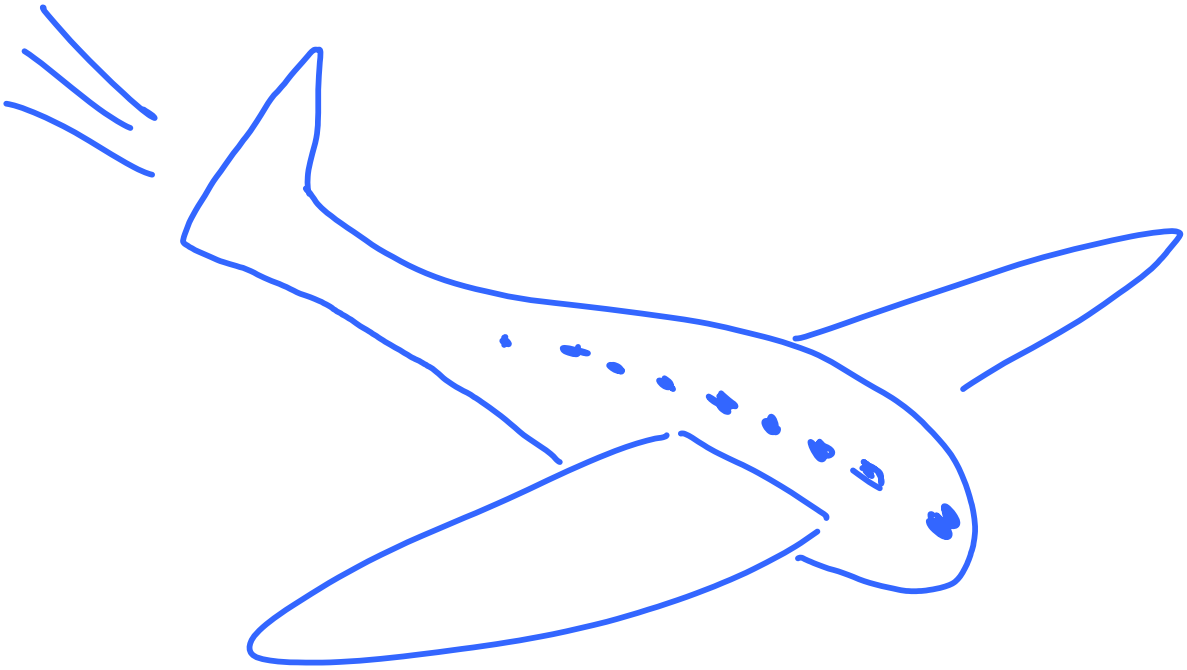
distortion side information using variable partition codes







$\Rightarrow$ professorship



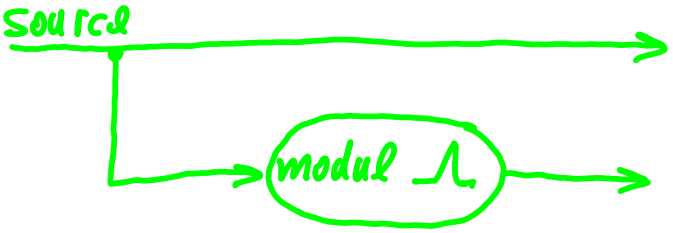
back to TAU ...

Tal Philosof

Yuval Kochman

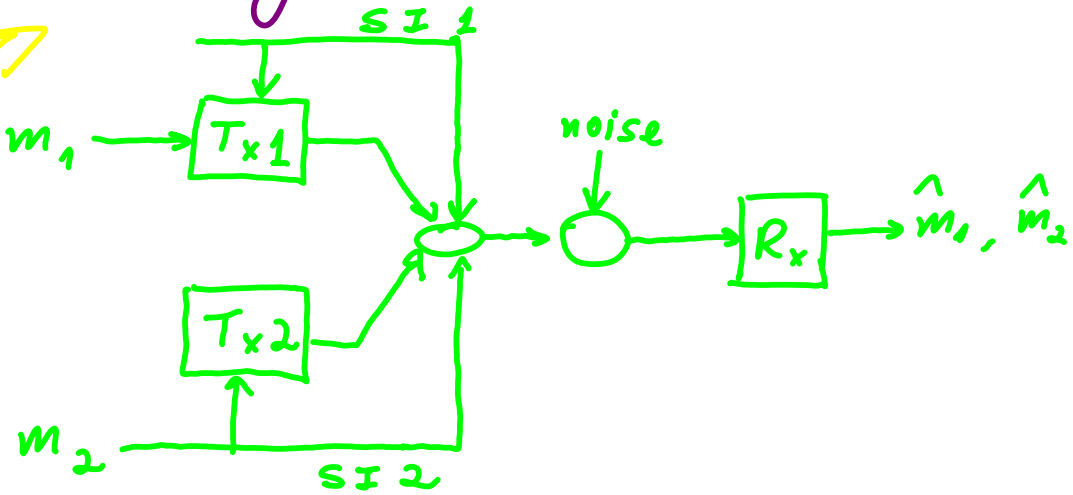
Zvi Reznic

modulo-lattice modulation



Uri Erez

dirty multiple-access channel



Many Cons, what about pros?

- Beauty of lattices
- History
- Circumstances
- Teaching (basis for a course)
- Anecdotes, Future ...

Triggers to start writing...

* Followers & competitors

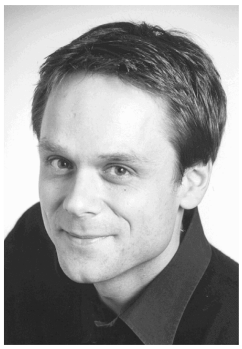
Sandeep Pradhan

binning via syndromes [1999]

Lattice Korner-Marton [2007]

compute & forward [2007]

Bobak Nazer

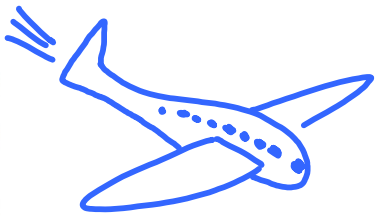


Michael Gastpar

* Full professor

* Next generation

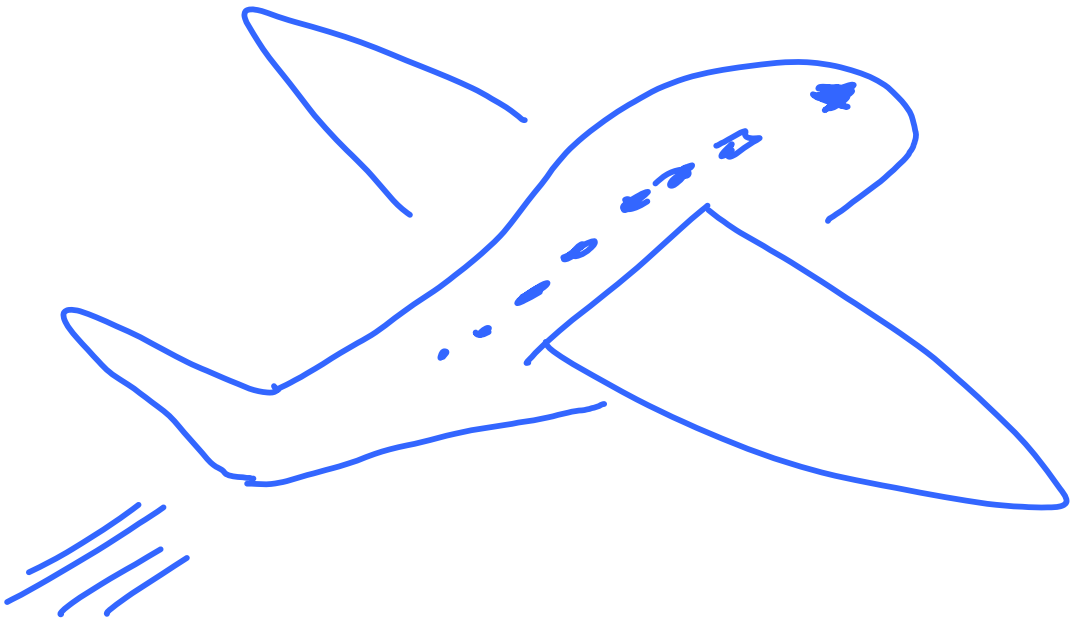
Uri Erez @ TAU



Tal Philosof @ GM

Yuval Kochman @ HUJI

* So what's next ?...



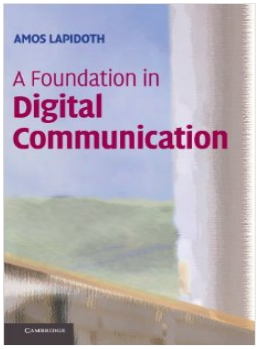
Aug. 2008 - going to Sabbatical @ ETH

Triggers to start writing ...

* Followers & competitors
* Full professor

* Next generation
* So what's next ?

* Colleagues who did it !
  (Sabbatical @ ETH)



Amos Lapidoth

* Colleagues who encouraged
  (visit @ Aalborg University Danmark)

Jan Østergaard

Many Cons, what about pros?

- Beauty of Lattices
- History
- Circumstances
- Teaching (basis for a course)
- Anecdotes, Future ...

Draft 1 : Tutorial @ ITA 2009 (UCSD, Feb. 2009)

==========

Title: Lattices are Everywhere!

"Lattice Codes, Signals and Networks"

Author: Ram Zamir

==========

Abstract:
As bees and crystals (and people selling oranges in the market) know it for many years, lattices provide efficient structures for packing, covering, quantization and channel coding. In the recent years, interesting links were found between lattices and coding schemes for multi-terminal networks. This book covers close to 20 years of my research in the area; of enjoying the beauty of lattice codes, and discovering their power in problems like: dithered quantization, dirty paper coding, Wyner-Ziv video coding, modulo-lattice modulation, distributed interference cancellation, and more.

Table of Contents:
==========

Draft 2 : Proposal to Cambridge University Press (May 2009)

Contents

Chap. 2
Chap. 3
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Chap. 4
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Chap. 8
Chap. 9
Chap. 13
Chap. 10
X
Chap. 11
Chap. 12

Introduction to Lattice Codes

~ first 6 lectures in every course

Advanced Course on Digital Communication

chapters 1 ÷ 4 , and

possible extensions (requires also part of chapter 6):

Course on Data Compression (Vector Quantization)

chapters 1 ÷ 4 , and

Possible extensions:

Advanced Course on Information Theory

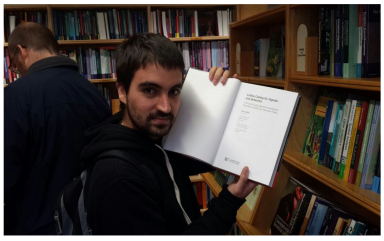
chapters 1 ÷ 4 , and

Many Cons, what about pros?

- Beauty of lattices
- History
- Circumstances
- Teaching (basis for a course)
- Anecdotes, Future ...

Anecdotes

- Who can help me with the figures ?
  (Ilai Bistritz)

- Collaboration in writing chapters 11 and 12
  (Yuval Kochman & Bobak Nazer)


- Story of a book ...
  (Shannon - Slepian - Wyner - Berger - Ephremides)





Future ...

Home CFP Committee Program Registration About

ITW Jerusalem 2015

2015 IEEE Information Theory Workshop

Jerusalem, ISRAEL | April 26 – May 1

CFP

The Information Theory Workshop (ITW) is an international conference on advances broadly related to the transmission, storage, processing and manipulation of information.

pdf version of CFP is here:

Submission is now open through EDAS:

http://edas.info/newPaper.php?c=18275&track=61231

Dates

- Paper submission: ~~October 24, 2014~~ October 31, 2014 (23:59:59pm PDT)
- Acceptance notification: January 10, 2015
- Final paper submission: March 1, 2015 (23:59:59pm PDT)

Dates for invited papers

- Paper registration (title+abstract): December 15, 2014
- Final manuscript: March 1, 2015 (23:59:59pm PDT)

Upload link for invited papers:

http://edas.info/newPaper.php?c=18275&track=61233

Scope

Original contributions in information and coding theory are solicited. The scope of topics is broad, with emphasis on the following themes:

IT and C&

Search

Recent Posts

New submission deadline: Oct. 31, 2014

"Jerusalem Explained" video added

Submission now open through EDAS

Plenary speakers and invited sessions added

Welcome TPC members!

Categories

News

Last but not least ...

Student @ Lab 102

The Zamir Family

Thank You!