Study of Ultrawide-Band Transmission in the Extremely High Frequency (EHF) Band

Yosef Pinhasi, Asher Yahalom, Oren Harpaz, and Guy Vilner

Abstract—The growing demand for broad-band wireless communication links and the lack of wide frequency bands within the conventional spectrum, causes us to seek bandwidth in the higher microwave and millimeter-wave spectrum at extremely high frequencies (EHF) above 30 GHz. One of the principal challenges in realizing modern wireless communication links in the EHF band is the presence of phenomena occurring during electromagnetic wave propagation through the atmosphere. A space-frequency approach to analyzing wireless communication channels operating in the EHF band is presented. Propagation of the electromagnetic radiation is studied in the frequency domain, enabling consideration of ultrawide-band modulated signals. The theory is employed for the analysis of a communication channel operating at EHF which utilizes pulse amplitude modulated signals. The atmospheric absorptive and dispersive effects on pulse propagation delay, pulse width and distortion are discussed. The theory and model are demonstrated in a study of ultrashort-pulse transmission at 60 GHz.

Index Terms—Atmospheric attention, millimeter wave propagation, ultrashort pulses, ultrawide-band transmission.

I. INTRODUCTION

The growing demand for broad-band wireless communication links and the deficiency of wide frequency bands within the conventional spectrum, require utilization of higher microwave and millimeter-wave spectrum at the extremely high frequencies (EHF) above 30 GHz. In addition to the fact that the EHF band (30–300 GHz) covers a wide range, which is relatively free of spectrum users, it offers many advantages for wireless communication and RADAR systems as follows:

• broad bandwidths for high data rate information transfer;
• high directivity and spatial resolution;
• low transmission power (due to high antenna gain);
• low probability of interference/interception (due to narrow antenna beamwidths);
• small antenna and equipment size;
• no multipath fadings (although fading can be caused by atmospheric conditions).

Among the practical advantages of using the EHF region for satellite communications systems is the ability to employ smaller transmitting and receiving antennas. This allows the use of a smaller satellite and a lighter launch vehicle.

Some of the principal challenges in realizing modern wireless communication links at the EHF band are the effects emerging when the electromagnetic radiation propagates through the atmosphere. Fig. 1 is a schematic illustration of a wireless communication line-of-sight (LOS) link, where the distance between the transmitter and receiver sites is $d$.

At frequencies below the EHF band, the ratio between the received power $P_r$ and the transmitted power $P_t$ in line-of-sight (LOS) radio links, operating at a frequency $f$, is given by the well-known Friis transmission (“link budget”) formula [1], [2]

$$
\frac{P_r}{P_t} = G_t \cdot \left( \frac{c}{4\pi f d} \right)^2 \cdot G_r
$$

where $G_t$ and $G_r$ are the transmitting and receiving antenna gains respectively, and $c \cong 3 \times 10^8$ m/s is the speed of light. The above formula describes the “free-space loss” due to diffraction of the transmitted radiation without considering atmospheric attenuation due to absorption and scattering and ignores multipath effects along the path of propagation.

When millimeter-wave radiation passes through the atmosphere, it suffers from selective molecular absorption [3]–[8]. Several empirical and analytical models were suggested for estimating the millimeter and infrared wave transmission of the atmospheric medium. The transmission characteristics of the atmosphere at the EHF band, as shown in Fig. 3 was calculated with the millimeter propagation model (MPM), developed by Liebe [9], [10]. Curves are drawn for several values of relative-humidity (RH), assuming clear sky and no rain. Inspection of Fig. 3(a) reveals absorption peaks at 22 and 183 GHz, where resonance absorption of water (H$_2$O) occurs, as well as absorption peaks at 60 and 119 GHz, due to absorption resonances of oxygen (O$_2$). Between these frequencies, minimum attenuation is obtained at 35 (Ka band), 94 (W band), 130, and 220 GHz, which are known as atmospheric transmission “windows” [6].

The transmission characteristics are determined by weather conditions such as temperature, pressure and humidity. The absorption is proportional to air density, and thus reduces with height. Attenuation due to fog, haze, clouds, rain and snow is one of the dominant causes of fading in wireless communication links operating in the EHF band [11], [12]. Raindrops and dust scatter millimeter wave radiation, resulting in amplitude fluctuations and phase randomness in the received signal. This further degrades the availability and performance of the communication links. Sufficient fade margins are essential for a reliable system.

The inhomogeneous transmission in a band of frequencies causes absorptive and dispersive effects in the amplitude and in phase of wide-band signals transmitted in the EHF band.
The frequency response of the atmosphere plays a significant role as the data rate of a wireless digital radio channel is increased. The resulting amplitude and phase distortion leads to inter-symbol interference, and thus to an increase in the bit error rate (BER). These effects should be taken into account in the design of broad-band communication systems, including careful consideration of appropriate modulation, equalization and multiplexing techniques.

Several theoretical papers dealt with the problem of distortion occurring when a short pulse is propagating in absorptive and dispersive media, including gases and plasmas [13]–[19]. They also studied the delay and pulse shape evolution along the path of propagation. Gibbins [18] extended earlier investigations [14], [16] and examined distortions of short Gaussian pulses, modulating millimeter waves and propagating in the atmosphere. An approximation of the wave propagation factor was used to derive analytical expressions for the pulse shape. Conditions for pulse broadening and compression were identified.

Approximate formulations are restricted to an initial Gaussian pulse and result in errors in the calculation of its shape evolution during propagation. These errors increase as the initial pulse duration is shortened. Moreover, the validity of the analytical expression is limited, and does not permit consideration of ultrashort pulses, which violate the conditions for existence of a solution. In this paper, we develop a general space-frequency approach for studying wireless communication channels operating in the EHF band. The time dependent field $E(t)$ represents an electromagnetic wave propagating in a medium. The Fourier transform of the field is

$$E(f) = \int_{-\infty}^{+\infty} E(t) \cdot e^{-j2\pi ft} dt. \tag{2}$$

Propagation of electromagnetic waves in a medium can be viewed as transformation through a system (see Fig. 2).

In the far field, transmission of a wave, radiated from a localized (point) isotropic source and propagating in a (homogeneous) medium is characterized in the frequency domain by the transfer function, derived in Appendix A

$$H(f) \equiv \frac{E_{out}(f)}{E_{in}(f)} = \frac{d_0}{d_0 + d} \cdot e^{-j\phi(f)\cdot d}. \tag{3}$$
Here, $k(f) = 2\pi f \sqrt{\mu \varepsilon}$ is a frequency dependent propagation factor, where $\varepsilon$ and $\mu$ are the permittivity and the permeability of the medium, respectively. The transfer function $H(f)$ describes the frequency response of the medium. Its inverse Fourier transformation corresponds to the temporal impulse response $h(t)$. In a dielectric medium the permeability is equal to that of the vacuum $\mu = \mu_0$ and the permittivity is given by $\varepsilon(f) = \varepsilon_r(f) \cdot \varepsilon_0$. If the medium introduces losses and dispersion, the relative dielectric constant $\varepsilon_r(f)$ is a complex, frequency dependent function, for which its real and imaginary parts satisfy the Kramers-Kronig relations [24], [25]. The resulting index of refraction $n(f) = \sqrt{\varepsilon_r(f)}$ can be presented by

$$n(f) = 1 + N(f)$$  (4)

where $N(f) = N_r(f) - j \cdot N_i(f)$ is the complex refractivity of the molecules composing the air [11]. The propagation factor can be written in terms of the index of refraction

$$k(f) = \frac{2\pi f}{c} \cdot n(f).$$  (5)

Substituting expressions (4) and (5) in (3), assuming horizontal propagation in an homogeneous medium, results in the transfer function

$$H(f) = \frac{d_0}{d_0 + d} \cdot e^{-[\alpha(f) + j\beta(f)] \cdot d}$$  (6)

where $\alpha(f) = -\text{Im}\{k(f)\} = (2\pi f/c) \cdot N_r(f)$ is the attenuation coefficient and $\beta(f) = \text{Re}\{k(f)\} = (2\pi f/c) \cdot [1 + N_r(f)]$ is the wavenumber of the propagating wave, shown in Fig. 3.

The power transfer function along the propagation path is given by

$$|H(f)|^2 = \left(\frac{d_0}{d_0 + d}\right)^2 \cdot e^{-2\alpha(f) \cdot d}$$  (7)

and is shown in Fig. 4 for frequencies of 35 and 94 GHz, where minimum attenuation (“transmission window”) is obtained, as well for 60 and 119 GHz where maximum absorption by oxygen molecules occurs.

**III. TRANSMISSION OF BROAD-BAND MODULATED SIGNAL**

Assume that a carrier wave at $f_0$ is modulated by a wide-band signal $A_{in}(t)$

$$E_{in}(t) = \text{Re}\{A_{in}(t) \cdot e^{j2\pi f_0 t}\}$$  (8)

as shown in Fig. 1. Here, $A_{in}(t) = I_{in}(t) - jQ_{in}(t)$ is a complex envelope, representing the base-band signal, where $I_{in}(t) = \text{Re}\{A_{in}(t)\}$ and $Q_{in}(t) = -\text{Im}\{A_{in}(t)\}$ are the in-phase and the quadrature information waveforms, respectively. The Fourier transform of the transmitted field is

$$E_{in}(f) = \frac{1}{2} A_{in}(f - f_0) + \frac{1}{2} A_{in}^*(f + f_0)$$  (9)
where $A_{\text{in}}(f)$ is the Fourier transform of the complex envelope $A_{\text{in}}(t)$. After propagating along a path with a distance $d$, the field is

$$E_{\text{out}}(f) = \frac{1}{2} A_{\text{in}}(f - f_0) \cdot H(f) + \frac{1}{2} A_{\text{in}}^*[-(f + f_0)] \cdot H^*(-f).$$

Since the transfer function $H(f)$ is the Fourier transform of a real function $H(f) = H^*(-f)$. The inverse Fourier transformation of (10), results in a received field given in the time domain by

$$E_{\text{out}}(t) = \text{Re}\{A_{\text{in}}(t) \cdot e^{j2\pi f_0 t}\}$$

where the complex envelope of the signal obtained at the receiver cite is given by

$$A_{\text{out}}(t) = \int_{-\infty}^{\infty} A_{\text{in}}(f) \cdot H(f + f_0) \cdot e^{+j2\pi ft} df.$$  \hspace{1cm} (12)

The formalism developed above is utilized in the following for analytical derivation and numerical calculations of the demodulated signal at the receiver site. The flow chart in Fig. 5 summarizes the procedure carried out for solving (12). A baseband digital signal $A_{\text{in}}(t)$, fed to the modulator at the transmitter site modulates a carrier at $f_0$. The MPM model [10]–[12] is called for to calculate the complex refractivity of the atmosphere, as required in the transfer function (6). Finally, the demodulated complex signal $A_{\text{out}}(t)$ and its related quadrature components $I_{\text{out}}(t)$ and $Q_{\text{out}}(t)$ obtained at the receiver output are found, employing an algorithm of fast Fourier transformation (FFT) for solving (12).

IV. ULTRAWIDE-BAND PULSE MODULATION

Assume that the transmitted waveform is a carrier modulated by a Gaussian envelope

$$A_{\text{in}}(t) = e^{-\frac{t^2}{2\sigma^2_{\text{in}}}} \hspace{1cm} (13)$$

characterized by a standard deviation $\sigma_{\text{in}}$. Fourier transformation of the pulse results in a Gaussian line-shape in the frequency domain

$$A_{\text{in}}(f) = \sqrt{2\pi} \cdot \sigma_{\text{in}} \cdot e^{-\frac{1}{2} (2\pi \sigma_{\text{in}} f)^2} \hspace{1cm} (14)$$

shown in Fig. 6. The corresponding standard deviation frequency bandwidth is $\sigma_f = 1/(2\pi\sigma_{\text{in}})$. The full-width half-maximum (FWHM) is the $-3$ dB bandwidth and is equal to $B = 2 \cdot \sqrt{\ln(2)} \cdot \sigma_f \approx 0.265 \cdot \sigma_{\text{in}}$.

In order to calculate the pulse shape $A_{\text{out}}(t)$ after propagation along a horizontal path in the atmospheric medium, we substitute the Fourier transform (14) into expression (12). Analytical result can be found if the complex propagation factor (5) is
approximated in the vicinity of the carrier frequency $f_0$, by a second-order Taylor expansion [14]–[16]

$$k(f) \equiv k_0 + k' \cdot (f - f_0) + \frac{1}{2} k'' \cdot (f - f_0)^2$$  \hspace{1cm} (15)$$

where $k_0 \equiv k(f_0), k' \equiv (dk/df)|_{f_0}, k'' \equiv (d^2k/(df)^2)|_{f_0}$. The second-order approximation given in (15) can be used if the standard deviation $\sigma_{in}$ of the Gaussian pulse satisfies

$$\sum_{n=0}^{2} \frac{1}{n!} \cdot k^{(n)} \cdot \sigma_{in}^{-n} \gg \sum_{n=3}^{\infty} \frac{1}{n!} \cdot k^{(n)} \cdot \sigma_{in}^{-n}$$

resulting in a complex envelope

$$A_{out}(t) = \frac{d_0}{d_0 + d} \cdot \sigma_{in} \cdot e^{\left(\frac{(\alpha''d)^2}{(2\pi)^2}\right)} \cdot e^{-j\alpha_0 \cdot d}$$  \hspace{1cm} (16)$$

where

$$\sigma^2 = \sigma_{in}^2 + \frac{j \cdot k''}{(2\pi)^2} \cdot d.$$  \hspace{1cm} (17)$$

The expression obtained at (16) is valid if

$$\text{Re}\{\alpha^2\} = \frac{1}{\sigma_{in}} + \frac{\alpha''d}{(2\pi)^2} > 0.$$  \hspace{1cm} (18)$$

This condition is always satisfied if $\alpha'' > 0$. However, at frequencies for which the attenuation curve is convex $\alpha'' < 0$ (in the vicinity of the absorption lines) the analytical results are valid only for $\sigma_{in}^2 > -\alpha''d/(2\pi)^2 > 0$. Another interpretation of this result is that for a given initial pulse width $\sigma_{in}$, the distance should not exceed $d < -((2\pi\sigma_{in})^2)/(\alpha'')$ in order for (16) to be valid.

The magnitude (absolute value) of the complex envelope given by (16), has a Gaussian shape

$$|A_{out}(t)| = \frac{d_0}{d_0 + d} \cdot \sigma_{in} \cdot \exp\left[\frac{-\alpha_0 \cdot d + \frac{1}{2} \cdot \frac{\alpha''d^2}{(2\pi\sigma_{in})^2}}{\frac{1}{2\sigma_{out}}}ight] \cdot e^{-\frac{(\alpha'd)^2}{(2\pi\sigma_{out})^2}}.$$  \hspace{1cm} (19)$$
with a temporal delay

\[ t_d = \frac{1}{2\pi} \left[ \beta - \frac{\alpha''d}{(2\pi\sigma^2) + \alpha''d} \right] \cdot d \]  

(19)

and a standard deviation \( \sigma_{\text{out}} \) given by

\[ \sigma^2_{\text{out}} = \sigma^2_{\text{in}} + \frac{\alpha''d}{(2\pi)^2} + \left( \frac{\beta d}{(2\pi)^2} \right)^2 \]  

(20)

These results, which can be obtained from the solutions derived in [14] and [16], show that in the framework of the approximation (15), a Gaussian magnitude \( |A_{\text{out}}(t)| = \sqrt{P_{\text{out}}(t) + Q_{\text{out}}(t)} \) is preserved while propagating in the medium (although its width changes) and thus can be retrieved by the receiver.

Since the pulse cannot propagate above a velocity of the speed of light the time delay, given by (19) is always \( t_d > (d/c) \). This fact points out that the medium response should fulfill \( \beta - (\alpha''/\alpha''') > (2\pi/c) \) at any frequency. For a short distance (or a wide pulse), the time delay can be approximated by \( t_d \approx (\beta/2\pi)c \). This becomes the exact solution at attenuation peaks, where \( \alpha'' = 0 \). When \( \alpha'' < 0 \), the denominator of (19) may become arbitrary small, (however should be kept positive in order to satisfy the validity condition (17)). In that case the time delay is approximately \( t_d \approx -\frac{1}{(2\pi)}(\alpha''\beta/(2\pi\sigma^2) + \alpha''d) \) (where must be \( \alpha''\beta < 0 \)), resulting in and arbitrary long delay in the pulse arrival at a distance \( d \).

Examination of (20) reveals that when \( \alpha'' \geq 0 \) the pulse always widens along the path of propagation. However, for \( \alpha'' < 0 \) (e.g., in the vicinity of absorption frequencies), the pulse may become narrower while propagating in the atmospheric medium. Pulse compression occurs when \( \sigma^2_{\text{out}}/(d) = -(1)/(2\pi)^2 \alpha''(\alpha'' + \beta'^2) \). In both cases minimum pulse width is obtained for \( \sigma^2_{\text{in}} = (d/(2\pi)^2(|\beta| - \alpha'') \) resulting in \( \sigma^2_{\text{out}} = 2(\beta d)/(2\pi)^2 \).

For long propagation distances, the time delay approaches \( t_d \rightarrow (1/2\pi)(\alpha''\beta - \alpha'') \cdot d \) and the standard deviation \( \sigma^2_{\text{out}} \rightarrow (1/(2\pi)^2)\alpha''(\alpha'' + \beta'^2) \cdot d \).

V. NUMERICAL RESULTS—PROPAGATION AT 60 GHz

An interesting application of the described approach is the analysis of transmission of ultrashort-pulses in the 60 GHz band. The high atmospheric attenuation and dispersive effects, caused by the absorption of the oxygen molecules at this frequency region [23]–[25], are expressed as evident distortions on the pulse delay, duration and shape. The complex propagation factor \( jk(f) = \alpha(f) + j\beta(f) \) in the 60 GHz band is shown in Fig. 7 as a solid line. The dashed lines describe curves of polynomials, which resulted from second-order Taylor expansions at 60.5 GHz, where the absorption peaks (\( \alpha' = 0 \)) and at 56.5 and 63 GHz, where \( \alpha'' = 0 \).

Table I summarizes the values of the complex propagation factor at these three frequencies, including their first and second derivatives. These numbers are used in the analytical
investigation of the delay (19) and pulse width (20) of a pulse modulated signal transmitted through the atmosphere.

The procedure, following the flow chart of Fig. 5, was employed in a numerical program aimed at the simulation of pulse transmission in the atmosphere. In our study, the modulating base-band signal \( A_{in}(t) \) was taken to be Gaussian, as given in (13). The initial standard deviation \( \sigma_{in} \) of pulse duration was varied, examining its effect on the resultant pulse envelope \( A_{out}(t) \) along the path of propagation. It calculates the \( n \)th order “moments” of the temporal magnitude of the envelope \( |A_{out}(t)| \)

\[
\bar{m} = \frac{\int_{-\infty}^{\infty} t^n |A_{out}(t)| \, dt}{\int_{-\infty}^{\infty} |A_{out}(t)| \, dt}, \quad (21)
\]

The first-order \((n = 1)\) moment represents the delay of the pulse \( T \cong t_d \). Examination of the approximated analytical expression (19) derived in the preceding section, discloses that the first-order derivative \( \alpha' \) of the attenuation coefficient plays a role in determining the pulse delay. Its major effect on increasing of the group delay is observed in Fig. 8(a) in the vicinity of the frequencies 56.5 and 63 GHz, where the most negative values of the product \( \alpha'/\beta'' \) are obtained. Minimum delay is obtained at 60.5 GHz where \( \alpha' = 0 \) as expected by (19).

The standard deviation \( \sigma_{out} = \sqrt{\bar{m}^2 - \bar{m}^2} \) represents the typical width of the pulse at the receiver output. Its frequency dependent behavior is shown in Fig. 8(b) for several initial pulse widths \( \sigma_{in} \). In this example, a pulse with initial duration of \( \tau_c = 0.2 \) ns is shown to be shortened \((\sigma_{out}/\sigma_{in} < 1)\) at frequencies where \( \alpha'' < 0 \), satisfies conditions for compression resulting from (20).

Fig. 9 shows a set of pulse envelopes \( |A_{out}(t)| \) obtained from numerical calculations for several distances along the propagation path (solid line). The quadrature components \( I(t) \) and \( Q(t) \) are also drawn. An initial Gaussian shape pulse with \( \sigma_{in} = 0.2 \) ns was assumed. Distortions in the shape increase as shown as the pulse propagates to long distances.

Curves of the group delay and standard deviation of the propagating pulse as a function of its initial width \( \sigma_{in} \) are drawn in Fig. 10 for the three frequencies discussed above. A minimum delay is observed in Fig. 10(a) at a carrier frequency of 60.5 GHz where the absorption peaks \((\alpha' = 0)\), while higher delays are found at 56.5 and 63 GHz, where \( \alpha'/\beta'' \) is negative. Fig. 9(b) shows the range of values of \( \sigma_{in} \), for which conditions for pulse compression occur at 60.5 GHz \((\alpha'' < 0)\).

VI. CONCLUSION

The space-frequency approach presented in this paper for the propagation of electromagnetic waves in dielectric media, enables consideration of the effects of the absorptive and dispersive characteristics of materials composing the media, on the transmission of wide-band signals. Using a complex representation of a frequency dependent refraction index, a transfer function that describes the frequency response of the medium is formulated.

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numerically in the frequency domain revealing spectral effects on pulse propagation delay, width and distortions. Using second-order expansion of the propagation factors leads to the derivation of approximated analytical expressions for the delay and width as a function of distance and carrier frequency. Conditions under which pulse compression or expansion occurs were identified. The analytical solution is found to be limited in describing wide-band communication links where ultrashort pulses are involved. In such links the numerical model use is required.

The effects, predicted by the analytical solution, were compared to the results obtained from a numerical simulation, aimed at the calculation of pulse evolution while propagating in the atmospheric media. By studying propagation of a pulse in the atmosphere, characterized by the millimeter-wave propagation model (MPM), it was shown that even in a medium of atmospheric air some of the effects that we predict are pronounced especially for carrier frequencies in the vicinity of the 60 GHz, where high absorption of oxygen molecules occurs.

**APPENDIX A**

This appendix is aimed at derivation of the transfer function given in (3). In an homogeneous dielectric medium, the electric \( \mathbf{E}(r, t) \) and magnetic \( \mathbf{H}(r, t) \) fields, excited by current \( \mathbf{J}(r, t) \) and charge \( \rho(r, t) \) densities, follow the macroscopic Maxwell equations, consisting of Faraday law

\[
\nabla \times \mathbf{E}(r, t) = -\frac{\partial}{\partial t} \mathbf{B}(r, t) \quad (A.1)
\]

Ampere law

\[
\nabla \times \mathbf{H}(r, t) = \mathbf{J}(r, t) + \frac{\partial}{\partial t} \mathbf{D}(r, t) \quad (A.2)
\]

and Gauss laws

\[
\nabla \cdot \mathbf{D}(r, t) = \rho(r, t) \quad (A.3)
\]

\[
\nabla \cdot \mathbf{B}(r, t) = 0.
\]

Here, \( \mathbf{D}(r, t) \) and \( \mathbf{B}(r, t) \) are the electric and magnetic flux densities respectively. In a homogeneous isotropic dielectric medium, their Fourier transforms can be written as

\[
\mathbf{D}(\mathbf{r}, f) = \mathcal{F}\{\mathbf{D}(\mathbf{r}, t)\} = \mathcal{F}\{\varepsilon(f)\mathbf{E}(\mathbf{r}, f)\} \quad \text{and} \quad \mathbf{B}(\mathbf{r}, f) = \mu \mathbf{H}(\mathbf{r}, f),
\]

where \( \varepsilon(f) \) is the frequency dependent permittivity and \( \mu \) is the permeability of the medium. Substituting the last relations in the Fourier transform of the Maxwell equations results in a space-frequency Helmholtz equation

\[
\nabla^2 \mathbf{E}(\mathbf{r}, f) + k^2(\mathbf{f})\mathbf{E}(\mathbf{r}, f) = \frac{1}{\varepsilon} \nabla \rho(\mathbf{r}, f) - j2\pi f \mu \mathbf{J}(\mathbf{r}, f) \quad (A.4)
\]
where \( k(f) = \sqrt{\mu\varepsilon(f)} \cdot 2\pi f \). Defining a driving term \( \Phi(r, f) \)

\[
-4\pi \Phi(r, f) = \frac{1}{\varepsilon} \nabla \phi(r, f) - j 2\pi f \mu \cdot \mathbf{j}(r, f)
\]

we obtain the equation in the form

\[
\nabla^2 \Phi(r, f) + k^2(f) \Phi(r, f) = -4\pi \delta(r - r').
\]

The Green function \( G(r, f) \) is the result of a point-source excitation

\[
\nabla^2 G(r, f) + k^2(f) G(r, f) = -4\pi \delta(r - r'),
\]

This equation can be written in terms of \( R = r - r' \) as

\[
\nabla^2 G(r, f) + k^2(f) G(r, f) = -4\pi \delta(R).
\]

Since the Green function must be spherically symmetric, the (A.9) can be written in the form

\[
\frac{1}{R} \frac{d^2}{dR^2}[RG(r, f)] + k^2(f)[RG(r, f)] = 0
\]

For \( R \neq 0 \) one obtains

\[
\frac{d^2}{dR^2}[RG(r, f)] + k^2(f)[RG(r, f)] = 0
\]

which has the trivial solution

\[
G(r, f) = \frac{1}{R} \left[ A e^{-j(k(f)\cdot R)} + B e^{+j(k(f)\cdot R)} \right].
\]

For \( R \rightarrow 0 \) the equation takes the form

\[
\frac{1}{R} \frac{d^2}{dR^2}[RG(r, f)] = -4\pi \delta(R)
\]

with the solution

\[
G(r, f) = \frac{1}{R}.
\]

Adjusting the two solutions (A.12) and (A.14) yields

\[
G(r, f) = \frac{1}{R} \left[ A e^{-j(k(f)\cdot R)} + (1 - A) e^{+j(k(f)\cdot R)} \right].
\]

For a radiating source we expect only an outgoing wave. Hence, \( A = 1 \) and the solution for the radiated field is given by

\[
\tilde{E}(r, f) = \int \int G(r - r', f) \cdot \Phi(r', f) dr' = \int \int \frac{e^{-j\phi(f)\cdot|r-r'|}}{|r-r'|} \cdot \Phi(r', f) dr'.
\]

In the far field \( |r - r'| = |r| = r \) and the field can be approximated by a spherical wave radiated from a localized source

\[
\tilde{E}(r, f) = \frac{e^{-j\phi(f) r}}{r} \int \int \Phi(r', f) dr'.
\]

If we measure the field \( \tilde{E}_{\text{in}}(f) \) at a far-field distance \( r = d_0 \) from the source and the field \( \tilde{E}_{\text{out}}(f) \) at a distance \( r = d + d_0 \), then dividing any component of the two fields results in

\[
\frac{\tilde{E}_{\text{out}}(f)}{\tilde{E}_{\text{in}}(f)} = \frac{d_0}{d + d_0} e^{-j\phi(f) d}
\]

leading to the transfer function given in (3).

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**REFERENCES**


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