Cyclic Coded Integer-Forcing Equalization

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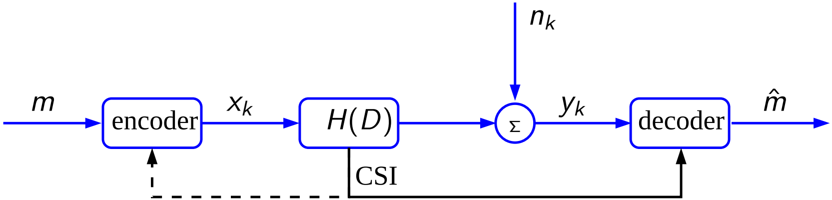
The Gaussian ISI channel



$$y_k = x_k + \sum_{m \neq 0} h_m x_{k-m} + n_k$$
$$= x_k + \mathrm{ISI}_k + n_k$$

- n_k is AWGN with unit power.
- Mutual Info: $I(S_x(\cdot)) = \frac{1}{2\pi}\int_\omega \log\left(1 + S_x(e^{j\omega})|H(e^{j\omega})|^2\right) d\omega$
- Assume (for simplicity) white input: $S_x(e^{j\omega}) = const = \sigma_x^2$
- CSI@Rx **only**

Reliable communication over the ISI channel

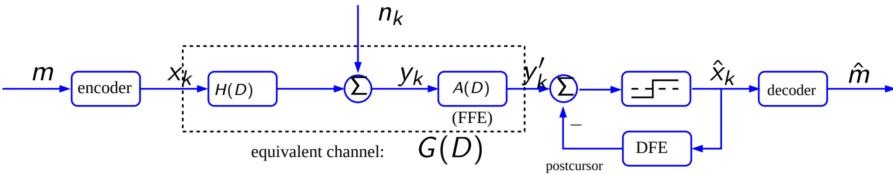
We are interested in schemes where decoding is **decoupled** from equalization.
-Turbo equalization not considered.

- Multi-carrier (frequency domain) - OFDM/DMT
  - Transforms the ISI channel into parallel AWGN subchannels - simplifies equalization
  - Coding over a channel with varying SNR may incur an unbounded gap-to-capacity
  - PAPR
  - Non-applicable to channels with finite alphabet (magnetic etc.)

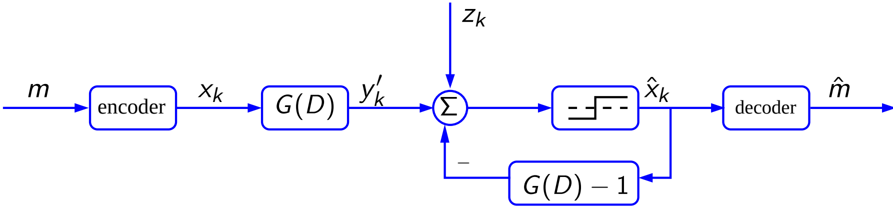
Reliable communication over the ISI channel

- Single carrier (time domain)
  - (Tomlinson-Harashima Precoding (THP) requires complete CSI@Tx - inapplicable...)
  - Linear equalizers: ZF-LE, MMSE-LE
  - Decision-feedback equalization (DFE)

Closer look at DFE



or equivalently:



"Optimality" of DFE

- MMSE-DFE is known to be "optimal" assuming correct detection of past symbols (CDEF)

$$\frac{1}{2}\log\left(1 + SNR_{DFE-MMSE-U}\right) = C$$

- But how can one get **error-free decisions**?
- Must replace slicer with decoder
- Possible solution: Guess-Varanasi interleaving
- We pursue different solution: **Move decoder before feedback loop**

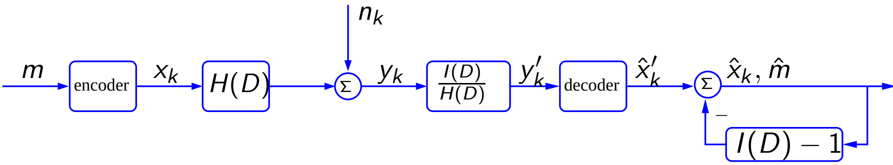
Preview of Suggested Approach

- Equalize the channel to integer-valued impulse response
- Add Zero-Padding/Cyclic prefix (as in OFDM) so that

$$\text{Linear Convolution} \rightarrow \text{Cyclic Convolution}$$

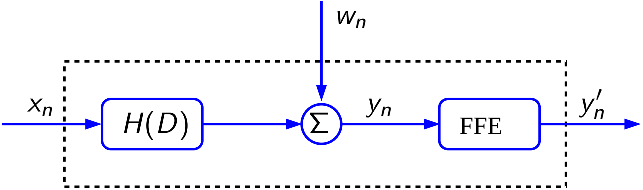
- Use linear cyclic code
- $\Rightarrow$ closed under integer-valued cyclic convolution
- Decode convolved codeword which is also a codeword
- Apply DFE

Integer-Forcing Equalization



- J. Zhan, B. Nazer, U. Erez, M. Gastpar ISIT 2010: Integer-Forcing Equalization proposed
- $\Rightarrow FFE(D) = \frac{I(D)}{H(D)}$ such that $I(D) = FFE(D)H(D)$ is a monic polynomial with integer coefficients
  - More general than Zero-Forcing where $I(D) = 1$
  - Less general than DFE since coefficients have to be integers
  - FFE part is reminiscent of partial response equalization by lattice reduction (R. Fischer & C. Siegl 2005)

DFE- IF Equalization



$$Y'(D) = \underbrace{FFE(D)H(D)}_{I(D)} X(D) + FFE(D)W(D)$$

$$y'_n = x_n + \sum_{k=1}^{L} i_{n-k} x_k + z_n$$

- For DFE-IF choose $I(D)$ so as to maximize

$$\text{SNR}_{DFE-IF} = \frac{\sigma_x^2}{\sigma_z^2} = \frac{\sigma_x^2}{\frac{1}{2\pi}\int_{-1/\pi}^{1/\pi} \frac{|I(e^{j\omega})|^2}{|H(e^{j\omega})|^2} d\omega}$$

Cyclic Codes

- Let $\mathcal{C} = \{\mathbf{x}_k\}_{k=1}^{2^{NR}}$ be a linear code over $\mathbb{Z}_q$
- $\mathcal{C}$ is cyclic if any cyclic shift of codeword is also a codeword
- $\Rightarrow$ Cyclic linear code is closed under integer-valued cyclic convolution with operations performed over $\mathbb{Z}_q$.

$$\mathbf{x} \in \mathcal{C} \;\Rightarrow\; \mathbf{x}' = [\mathbf{x} \otimes \mathbf{i}] \in \mathcal{C}$$

- Examples of cyclic codes:
  - "Most" algebraic codes - for example BCH
  - Classes of LDPC codes: type-I EG, type-I PG (Kou, Lin, Fossorier 2001), codes by Shibuya and Sakaniwa (2003)

Finding I(D)

- We would like to maximize

$$\text{SNR}_{DFE-IF} = \frac{\sigma_x^2}{\sigma_z^2} = \frac{\sigma_x^2}{\frac{1}{2\pi}\int_{-1/\pi}^{1/\pi} \frac{|I(e^{j\omega})|^2}{|H(e^{j\omega})|^2} d\omega}$$

- σ_z^2 can be written in matrix form

$$\sigma_z^2 = \mathbf{i} \begin{bmatrix} k_0 & k_{-1} & k_{-2}\dots & k_{-L} \\ k_1 & k_0 & k_{-1}\dots & k_{-(L-1)} \\ \vdots & \vdots & \vdots \ddots & \\ k_L & k_{L-1} & k_{L-2}\dots & k_0 \end{bmatrix} \mathbf{i}^{\mathbf{T}} = \mathbf{i}\tilde{\mathrm{K}}\mathbf{i}^{\mathbf{T}}$$

where $k_m = \frac{1}{2\pi}\int_{-\pi}^{\pi} \frac{1}{|H(e^{j\omega})|^2} e^{-jmw} d\omega$.

- A shortest lattice vector problem
- Can use LLL as an approximate solution

Performance

- How much do we lose w.r.t. "ideal" DFE by integer forcing?

Theorem

The noise enhancement caused by the IF equalizer is upper bounded by

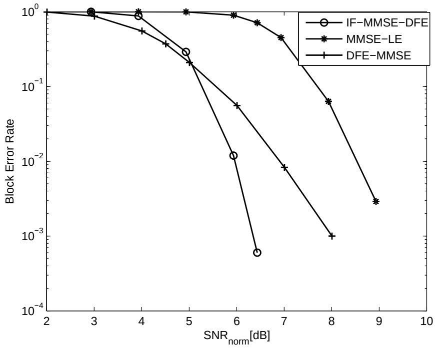
$$\sigma_z^2 \leq \sigma_{\mathrm{ZF-DFE}}^2 \cdot \min_{n \geq p+1} \left[n \frac{2\,(1.4\pi n)^{\frac{1}{n}}}{\pi e} \left(\frac{|z_0 z_1 \ldots z_{p-1}|^{2p}}{\prod_{\mu,\nu} |z_\mu^* z_\nu - 1|} \right)^{\frac{1}{n}} \right]$$

where $z_0, z_1, \ldots, z_{p-1}$ are the maximum-phase zeros of $H(D)H^*(D^{-*})$, and $p+1$ is the channel's length.

- Bound is based on Minkowski bound for shortest lattice vector
- Not tight in general

Simulation results

- 8-PAM constellation is used in a TCM-like manner
- cyclic LDPC n=255, k=175 ($R = 2.6862 \frac{\text{bits}}{\text{cahnnel use}}$) code for IF-DFE and MMSE-LE
- uncoded transmission for MMSE-DFE
- Channel is
  $1 + 0.894D + 0.814D^2 + 0.239D^3 - 0.070D^4 + 0.036D^5 - 0.022D^6$



Summary, Extensions and Open Questions

- Integer-Forcing equalization allows channel decoding before applying the DFE loop.
  Gains:
  - No error propagation
  - Channel coding is much more effective

  Penalties:
  - DFE coefficients must be integers
  - Code must be cyclic
- Method is effective for channels of moderate lengths, and high SNR
- Extension to MMSE exists
- Explore specific channel models for which IF is advantageous