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Evolution of waves in a horizontal pipe propagating on a surface of a liquid film sheared by gas

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ABSTRACT
Different wavy regimes in stratified air–water pipe flow are determined for a wide range of gas and liquid flow rates in a 10 m long horizontal pipe with a diameter of 24 mm. Three sub-regions of wavy stratified flow are identified: ripples, roll waves, and pre-annular wavy flow. Statistical parameters, such as local mean film thickness and its higher moments (root-mean-square, skewness, excess kurtosis) as well as wave characteristics (mean heights and wave height distributions, lengths, propagation velocities, etc.), are measured and analyzed. It is demonstrated that ripples are essentially linear waves and their propagation velocities are described reasonably well by linear wave theory. High amplitude roll and pre-annular waves are substantially nonlinear, and their propagation velocities differ significantly from that of ripples. Transition to roll waves causes a sharp increase in higher statistical moments. Evolution of wave and statistical parameters characterizing each sub-region of stratified gas–liquid pipe flow is studied. Simplified models describing roll waves are presented; the model predictions are verified by experiments.

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I. INTRODUCTION

Stratified gas–liquid horizontal pipe flow is dominated by gravity that causes the liquid to flow in a continuous bottom layer. At low gas velocities, the gas–liquid interface remains smooth. An increase in the gas flow rate, while keeping the liquid flow rate constant, results in excitation of interfacial waves whose characteristic properties vary significantly with the gas flow rate. Mass, heat, and momentum transfer are strongly affected by the various wave patterns at the interface.

Jeffreys1 invoked a sheltering hypothesis to explain the generation of waves by wind. Taitel and Dukler3 adopted this approach to model wave excitation by gas flow in round pipes. Additional modification was suggested by Andrisos and Hanratty. Alternative theoretical approaches for inception of waves on an initially smooth liquid surface due to airflow over unbounded water were developed by Miles, and in subsequent studies, some of those studies were supported by measurements. It should be stressed that, in most cases, waves propagating over deep water were considered. Except for Phillips, the flow in those models was assumed to be unidirectional. Larger amplitude waves are developed with an increase in wind velocity. In deep water, those wind waves may be seen as the resonant stage predicted by Phillips and identified in experiments by Zavadsky and Shemer. Most studies on the characterization of waves created by gas flow over the liquid surface have been conducted in rectangular channels with large aspect ratio (Refs. 16–20 and additional references therein). In wind-wave channels, the wave amplitudes and lengths grow continuously with the distance from the inlet while their frequency decreases.

Relatively few experimental studies of interfacial waves in stratified gas–liquid flow were conducted in closed environments, such as pipes or wide rectangular ducts characterized by a shallow water layer. The wave evolution in those conduits is notably different from that observed when the air flow is effectively unbounded and flows over thick liquid layers. Unlike wind-waves observed in high channels, the transition in pipes from small-amplitude to large-amplitude waves, which are often termed roll waves, is usually associated with Kelvin–Helmholtz instability. Roll waves that can be seen as shock wave-like disturbances are commonly observed not only in gas–liquid pipe flows, but also in downward inclined open-channel flows, such as aqueducts and spillways. A model for such waves was developed by Dressler that compared their results to Brock’s experiments. Roll waves are also formed over thin films in horizontal gas–liquid stratified flow in
pipes or close narrow conduits. Roll waves constitute a transient phenomenon; mechanistic models that attempt to describe steady-state mean values were described in Refs. 31–35, while transient models and numerical simulations of unsteady roll waves were presented in Refs. 35–40.

Andritsos and Hanratty and Tzotzi et al. distinguished between three sub-regimes within the stratified wavy flow: small amplitude waves, roll waves, and atomization that is associated with droplet detachment from the film. Andritsos performed statistical analysis of the film thickness records. In this study, the small amplitude waves were identified as periodic and maintained their identity for several wave periods while large amplitude waves are more random. Lioumbas et al. studied the transition from smooth to wavy stratified flow for slightly inclined 24 mm pipe. Conductance probe records of the liquid film thickness variation with time were related to the laser Doppler anemometer-derived measurements of the velocity field in the liquid. Birvalski et al. simultaneously applied particle image velocimetry to determine the flow field in the liquid and gas phases while using imagery to record the temporal variation of the instantaneous gas–liquid interface shape. An attempt was made to relate the flow fields in both phases and the wavy structure.

Ayati et al. carried out experiments in a 10 cm diameter pipe and analyzed statistical characteristics of interfacial waves in stratified gas–liquid flow in a horizontal configuration. A Gaussian model was implemented to differentiate between linear and nonlinear wave regimes. For nonlinear waves, stagnation in wave growth due to possible micro breaking of waves was observed.

In the present study, data on the gas–liquid interfacial structure were acquired in horizontal stratified flow. The measurements cover the entire domain of stratified flow from very low liquid flow rates up to the transition to intermittent slug flow and from low gas flow rates toward transition to the annular flow regime. Various statistical parameters were extracted from the accumulated records for wave regimes observed in the flow. The evolution of those quantities along the pipe was analyzed.

II. EXPERIMENTAL FACILITY AND PROCEDURE

Experiments were performed in a 10 m long transparent horizontal pipe with an inner diameter of 0.024 m supported by a steel frame. The outline of the experimental facility is presented schematically in Fig. 1. The pipe consists of sections about 2 m long connected by easily loosened and fastened flanges. Each section is aligned horizontally using a digital level (±0.1°). Air and water are used as working fluids. The air is supplied from a high-pressure main and passes through a filter and pressure reducer set at 1 bar; the outlet of the pipe is open to atmosphere. Water is circulated in the pipe using a frequency controlled centrifugal pump (P). Both air and water flow rates are measured by a set of flow meters. Water and air enter the inlet section flowing on the bottom and upper sides of a horizontal splitting plate, respectively, thus forming stratified flow.

Capacitance-type sensors (CP) consisting of 0.3 mm anodized tantalum wires spanning the vertical diameter of the pipe are used for measuring instantaneous film thickness. The accuracy of measuring the instantaneous film thickness by the capacitance probes is estimated at ±0.03 mm. Six sensors were arranged in pairs with an axial spacing of 0.15 m in each pair, with the first sensor in the pair located at distances L = 116, 407, and 698 cm from the inlet. Each pair of sensors was introduced into a separate pipe section. A six-channel conditional unit (CU) provided analogue output to the NI A/D card. For each set of gas and liquid flow rates, the records of all sensors were continuously sampled for 900 s at the rate of 300 Hz/channel. The static calibration of the sensors is carried out before each realization by filling different prescribed amounts of water into the pipe section. As seen in Fig. 2, the calibration curves follow a linear trend. The maximum deviation of individual measurements from the calibration straight line is about 4%, which is in agreement with the estimated error in the mean depth.

Depending on the air and water flow rates, four distinct wavy regimes were identified by visual observations. The records of the film thickness variation δ(t) in each of those regimes have characteristic signal shapes presented in Fig. 3. The small amplitude ripples in Fig. 3(b) are distributed seemingly uniformly along the record, while the
significantly higher roll waves are separated by relatively long plateaus with superposed small waves. High waves also characterize the pre-annular regime; however, unlike in the roll wave regime, those waves are not separated by plateaus.

The mean film thickness \( \bar{\delta} \) is defined as the first moment of the instantaneous film thickness,

\[
\bar{\delta} = \frac{1}{N} \sum \delta_i,
\]

where \( N \) is the total length of the data array sampled for a given sensor. Note that one has to take into account possible disturbance induced by the first probe in each pair that may have a minor effect on the downstream sensor; hence, to estimate the mean depth more accurately, the values are averaged over two adjacent probes.

The gas and liquid flow rates in the pipe are characterized by the corresponding superficial velocities \( U_{LS} = Q_L/A \) and \( U_{GS} = Q_G/A \), where \( Q_L \) and \( Q_G \) are the volumetric flow rates of liquid and gas and \( A = \pi R^2 \) is the pipe cross section. Assuming a flat mean water surface, for given water and air superficial velocities, the mean film thickness allows us to determine the average cross-sectional velocity of each fluid as \( U_L = U_{LS} A_{L} / A \) and \( U_G = U_{GS} A_{G} / A \), where \( A_L = (A/2\pi) [\phi - \sin(\phi)] \), with the angle \( \phi = 2\cos^{-1}(1 - \delta/R) \); \( A_G = A - A_L \) is the cross-sectional area covered by gas, \( S_L = 2R[\cos^{-1}(1 - \delta/R)] \) and \( S_G = 2R[\pi - \cos^{-1}(1 - \delta/R)] \) are the wetted perimeter by liquid and gas, respectively, and \( S_i = 2R\sin[\cos^{-1}(1 - \delta/R)] \) is the interfacial perimeter, Fig. 4.

The second central moment corresponds to the root-mean-square (RMS) value of the deviation of the instantaneous film thickness from the mean value,

\[
\text{RMS} (\delta_i - \bar{\delta}) = \left( \frac{1}{N} \sum (\delta_i - \bar{\delta})^2 \right)^{1/2},
\]

where \( N \) is the number of sampled data points \((N = 270000 \text{ for the whole record})\).

The statistics of the surface elevation can be further characterized by higher moments, skewness \( (\lambda_3) \) and kurtosis \( (\lambda_4) \), defined as

\[
\lambda_3 = \frac{1}{\left( \frac{1}{N} \sum (\delta_i - \bar{\delta})^2 \right)^{3/2}} \frac{1}{N} \sum (\delta_i - \bar{\delta})^3,
\]

\[
\lambda_4 = \frac{1}{\left( \frac{1}{N} \sum (\delta_i - \bar{\delta})^2 \right)^{2}} \frac{1}{N} \sum (\delta_i - \bar{\delta})^4.
\]
The skewness is a measure of the vertical asymmetry of the signal, while the excess kurtosis \( \lambda_4 - 3 \) characterizes the spread of the surface elevation distribution relative to the Gaussian shape; for the Gaussian distribution, \( \lambda_4 = 3 \).

The instantaneous surface elevations measured by each pair of adjacent probes may be used to estimate the propagation velocity of waves. This is done by computing the cross correlation coefficient as a function of the time-lag between the signals,

\[
S(\tau) = \frac{\sum_{i=1}^{N/2} \sum_{j=1}^{i} (\delta_1(i\Delta t) - \overline{\delta_1})(\delta_2(j\Delta t) - \overline{\delta_2})}{\text{RMS}(\delta_1 - \overline{\delta_1}) \cdot \text{RMS}(\delta_2 - \overline{\delta_2})},
\]

where \( \delta_1 \) and \( \delta_2 \) are the instantaneous surface elevations measured at the first and second probe in each pair, respectively. The time series is divided into 45 segments, each 20 s long (\( N = 6000 \)). Note that the correlation is computed using Eq. (4) rather than by applying FFT. Examples of the cross correlation coefficient dependence on the time delay \( \tau = (j - i)\Delta t \) at two locations along the pipe, averaged over all segments in the record, are given in Fig. 5.

At both locations, the values of \( S \) decay fast with time, so that the signals become effectively uncorrelated at \( \tau > 1.5 \) s. The maxima in the curves show that the signals close to the inlet are weakly correlated; at a more remote location, the development of flow along the pipe leads to a higher maximum value of the cross correlation coefficient \( S \) and to a more pronounced peak.

An effort is made to identify individual waves in the record. To this end, reference depths should be defined to apply the zero-crossing procedure. Following Fershtman et al.,\(^5\) two different reference values are used for different wave regimes. For nearly linear ripples, the average mean film thickness \( \overline{\delta} \) serves as a reference, see Fig. 6(a). For larger waves, it is advantageous to use as the reference value the average between the mean film thickness \( \overline{\delta} \) and substrate depth \( \overline{\delta}_{\text{sub}} \), defined as

\[
\overline{\delta}_{\text{sub}} = \frac{1}{N} \sum \delta_{\text{mi}},
\]

where the summation is over all local minima that are below the mean film thickness \( \overline{\delta} \), \( \delta_{\text{mi}} \). Examples for identification in roll and pre-annular waves are presented in Figs. 6(b) and 6(c). The reference line \( \overline{\delta}_{\text{ref}} = (\overline{\delta} + \overline{\delta}_{\text{sub}})/2 \) is shown as a dashed line in Figs. 6(b) and 6(c). Note that due to different length and time scales in those wave regimes, different scaling is applied in those panels.

The selection of \( \overline{\delta}_{\text{ref}} \) allows us to identify individual waves between consecutive upward crossings of the reference line. For ripples, crests are defined as the maxima of the surface elevation exceeding the reference [the red dots in Fig. 6(a)], following previous troughs that are defined as the minimum film thickness [the blue dots in Fig. 6(a)] between two successive zero-crossing points. The wave height

**FIG. 4.** Geometric parameters within the pipe cross section.

**FIG. 5.** Examples of cross correlation curves \( S(\tau) \) at two locations along the pipe: black: \( U_{\text{LS}} = 0.02; U_{\text{GS}} = 3 \), red: \( U_{\text{LS}} = 0.04; U_{\text{GS}} = 2 \), blue: \( U_{\text{LS}} = 0.05; U_{\text{GS}} = 2 \).

**FIG. 6.** Instantaneous surface elevation \( \delta \) for (a) small-amplitude ripples, (b) roll waves, and (c) pre-annular waves; solid line: mean film thickness; dashed line: reference film thickness; red dots: crests and blue dots: troughs.
The height of roll and pre-annular waves is obtained as crest heights relative to the substrate. Those waves have significantly larger amplitudes with $h_{wave} = O(1)$ and thus are substantially nonlinear.

Typical frequency amplitude spectra of different types of waves are plotted in Fig. 7. The power spectra are calculated over 10 s long segments, resulting in a frequency resolution of 0.1 Hz. The mean amplitude spectra are obtained from the averaged over 90 independent segments power spectra; they represent the amplitudes of each harmonic. The spectra in Fig. 7 are plotted for a single superficial liquid velocity $U_{LS} = 0.08$ m/s and various superficial gas velocities corresponding to ripples ($U_{GS} = 2$ m/s), roll waves ($U_{GS} = 7$ m/s), and pre-annular waves ($U_{GS} = 14$ m/s). Note that the spectra of ripples and pre-annular waves do not have well-defined peaks, while for roll waves the peak frequency is more pronounced. Since the exact definition of the peak frequency from the spectra is not always possible, the dominant frequency $f_{dom}$ applicable for all wave regimes is introduced as the total number of waves in the time series divided by the total time duration.

**III. RESULTS**

Figure 8 represents the boundaries of stratified gas–liquid flow in a horizontal pipe and the sub-regions within this flow pattern. The stratified regime is bounded by intermittent flow at high liquid flow rates and by annular flow at high gas flow rates. The stratified–non-stratified transition boundaries in this map are determined by neutral rates and by annular flow at high gas flow rates. The stratified–non-stratified regime is bounded by intermittent flow at high liquid flow duration.

Variation of characteristic wave parameters with the gas flow rate $U_{GS}$ measured at the last measuring station ($L = 698$ cm) is now studied for various liquid flow rates $U_{LS}$. The dependence of the mean film thickness $\overline{d}$ on flow conditions is presented in Fig. 9. As expected, the values of $\overline{d}$ decrease with $U_{GS}$ and increase with $U_{LS}$. Following the transition from roll waves to pre-annular wavy flow, the rate of change in $\overline{d}$ decreases notably due to the upward spreading of the liquid film on the pipe wall in agreement with the experiments of Andritsos.

The variation of the characteristic wave height represented by RMS values of the difference between the instantaneous and the mean film thickness, $\delta - \overline{d}$, is shown in Fig. 10 as a function of $U_{GS}$. The abrupt increase in the RMS values corresponds to transition from ripples to roll waves. The characteristic amplitude of roll waves is somewhat higher for higher $U_{LS}$ and increases with $U_{GS}$. Transition to pre-annular wavy flow results in a slight decrease in the RMS ($\delta - \overline{d}$) with $U_{GS}$ due to the upward film spreading along the pipe periphery.

It is instructive to examine the relation between the mean film thickness $\overline{d}$ and the characteristic wave amplitude, RMS ($\delta - \overline{d}$), in Fig. 11. In this plot, these quantities for all three axial positions are plotted for each sub-region defined in Fig. 8 for all gas and liquid flow rates. It can be clearly seen that for each mean film thickness, the characteristic...
wave amplitude increases with transition between the four sub-regions, from smooth to pre-annular wavy flow. In each region, the wave amplitude is growing with \( \delta \). However, the transition from smooth surface to ripples is not well defined; in the stratified smooth flow, the wave amplitudes remain below 0.2 mm, while in ripples, the amplitudes attain 0.5 mm. The ratio between RMS \((\delta - \bar{\delta})/\bar{\delta}\) is below 0.035 for the regime denoted as smooth surface; for the ripples, RMS \((\delta - \bar{\delta})/\bar{\delta}\) < 0.064. This ratio increases significantly to about 0.3 for roll waves and to about 0.5 for pre-annular waves, thus indicating the transition to nonlinear wave regimes. Note that at low liquid and high gas velocities, the time series resembled roll waves. In fact, for those records \( h_{\text{wave}}/\bar{\delta} \ll 1 \), thus the waves are essentially linear; hence, the corresponding data points were not considered in further analysis.

Higher moments that characterize the wave shape, the third order skewness \((\lambda_3)\), and the fourth order excess kurtosis \((\lambda_4 - 3)\) are presented in Fig. 12 as a function of \( U_{GS} \). The skewness of ripples is quite small and mostly negative, indicating at some possible minor asymmetry of the interface relative to the mean value, with more pronounced troughs as compared to crests. The excess kurtosis of the ripples is also quite small, on the order of magnitude observed in the experiments with ripples in a pipe of larger diameter by Ayati and Carneiro. Transition to roll waves corresponds to a sharp increase in both \( \lambda_3 \) and \((\lambda_4 - 3)\), as marked by the dashed arrows. The high values of the skewness coefficient demonstrate that roll waves crests are typically much higher than the wave trough depths, while high excess kurtosis values indicate a significant deviation of the roll wave height distribution from the Gaussian. Transition to pre-annular wavy flow also results in a notable change in the higher moments, with both \( \lambda_3 \) and \((\lambda_4 - 3)\) decreasing significantly from their values for roll waves. Within this regime, they do not vary significantly with flow rates of both fluids. Similar values of \((\lambda_4 - 3)\) were reported by Andritsos.

The wave propagation velocities were calculated as \( \epsilon = \Delta x/\tau \), \( \Delta x \) being the distance between the probes in each pair of sensors, while the time delay \( \tau \) corresponds to the maximum of the cross correlation coefficient dependence on time, see Fig. 5. The dominant frequencies \( f_{\text{dom}} \) were estimated by counting individual waves as detailed Sec. II. The wave propagation velocities \( \epsilon \) and the dominant frequencies measured in various wave regions are plotted in Figs. 13(a) and 13(b), respectively. As in previous figures, an abrupt change in \( \epsilon \) occurs at transition from ripples to larger waves. This sharp velocity change suggests that the physical nature of ripples that are essentially linear differs from that of the larger nonlinear waves (roll and pre-annular). As shown in sequel, the dominant frequency of ripples does not add additional information regarding the transition and therefore not presented in Fig. 13(b). The transition from roll waves to pre-annular waves is accompanied by substantial wave frequency increase.

Exceedance distribution functions of wave height \( h_{\text{wave}} \) for two liquid superficial velocities \( U_{LS} \) are presented in Fig. 14 for ripples and pre-annular waves, and in Fig. 15(a) for roll waves.

The measured exceedance distributions are customarily compared with Rayleigh distribution, that is applicable for linear narrowband Gaussian waves in deep water,

\[
F(h_{\text{wave}}) = \exp \left( -\frac{h_{\text{wave}}^2}{8 \text{RMS}(\delta - \bar{\delta})^2} \right)
\]

(6)

In view of weak dispersion of waves observed in the present experiments, application of the Weibull distribution,

\[
F(h_{\text{wave}}) = \exp \left( -\left(\frac{h_{\text{wave}}}{\lambda}\right)^k \right)
\]

(7)

that is often used in studies of sea waves in shallow environment is examined. Here \( \lambda = \text{RMS} (h_{\text{wave}}) \) and \( k \) has a tunable value. The exceedance distributions of ripples in Fig. 14(a) show that the probability of relatively high waves is significantly smaller than that predicted by the Rayleigh distribution, in general agreement with the results obtained in a pipe with a larger diameter by Ayati and Carneiro. It can be approximated reasonably well by Weibull distribution with \( k = 2.7 \). For pre-annular waves, the best fit to the Weibull distribution yields \( k = 1.9 \), close to the power in the Rayleigh distribution; it also resembles wave height distribution measured in a vertical annular flow.

The shape of the exceedance of roll waves plotted in Fig. 15(a) deviates significantly from both Rayleigh and Weibull distributions. The measured distribution can be seen as consisting of two...
qualitatively different parts that are denoted by I and II in this figure. The boundary between these two parts is at the wave height \( h_{\text{wave}} \approx 3 \text{ mm} \). It thus seems reasonable to plot the exceedance distributions for each part separately. The distributions that only account for small waves in the ensemble that mainly characterize the plateau, see Fig. 6(c), and have heights \( h_{\text{wave}} < 3 \text{ mm} \) indeed resemble those plotted for ripples in Fig. 14(a). The best fit yields \( k = 2.1 \) in the Weibull distribution that is slightly above that of the Rayleigh distribution. Accordingly, the tail of the distribution in Fig. 15(b) is above that corresponding to the Rayleigh distribution. This can be attributed to the fact that relatively few waves in the ensemble with heights exceeding about \( h_{\text{wave}} = 1.5 \text{ mm} \) can hardly be seen as ripples, see also Fig. 14(a) for \( U_{\text{LS}} = 0.06 \text{ m/s} \). The exceedance of the larger waves that characterize roll wave regime in turn resembles that of pre-annular waves with \( k = 3.2 \).

Note that the initial distinction between different wave regimes was made based on visual observations and by examining the shape of the surface elevation dependence on time in Fig. 6. Figures 10–13 demonstrate that this distinction is strongly supported by the sharp variation of the statistical wave parameters that occur at the transition between the wave regimes.

In the following, we take a closer look at each wave regime separately.

**A. Small amplitude ripples**

The mean film thickness \( \bar{d} \) and the characteristic wave amplitude RMS \( (\bar{d} - \bar{d}) \) measured along the pipe and averaged for the two sensors in each pair are presented in Fig. 16 for various gas and liquid flow rates. A certain increase in the mean film thickness with the distance is observed for all values of \( U_{\text{LS}} \) and \( U_{\text{GS}} \). The characteristic wave amplitudes RMS \( (\bar{d} - \bar{d}) \) in Fig. 16(b) increase slightly with \( L \) and show a growing trend with wind and liquid velocity. The
FIG. 13. (a) Wave velocity $c$ and (b) dominant wave frequency as a function of $U_{GS}$, $L = 698$ cm. The symbols represent different wave regimes, see the legend of Fig. 8; black: $U_{LS} = 0.02$, red: $U_{LS} = 0.04$, blue: $U_{LS} = 0.06$, magenta: $U_{LS} = 0.08$ m/s.

FIG. 14. Exceedance distribution of the wave height $h_{wave}$ at $L = 698$ cm: (a) $U_{GS} = 2$ (ripples), (b) $U_{GS} = 14$ (pre-annular waves); blue: $U_{LS} = 0.06$, magenta: $U_{LS} = 0.08$ m/s.
non-monotonic behavior for few cases at low $U_{GS}$ may stem from experimental error. However, the trend of these statistical parameters is estimated based on the similar overall behavior at different flow conditions.

Identification of individual waves as presented in Fig. 6(a) allows determining the probability density function (pdf) for the wave height distribution. The typical wave height probability density distributions with a bin size of 0.05 mm are plotted in Fig. 17 for three sets of $U_{LS}$ and $U_{GS}$. The distributions are characterized by considerable scatter; hence, Savitzky–Golay filter is applied. The filter is based on local-least squares polynomial approximation and post-filtering; it retains the normalizing condition $\int pdf(h)dh = 1$. The most probable wave height can be identified more accurately by applying to the smoothed distributions parabolic fit in close vicinity ($\pm 0.15$ mm) to each maximum; in analogy with frequency, it is denoted as $h_{dom}$.

The variation of $h_{dom}$ along the pipe for various gas and liquid flow rates presented in Fig. 18 demonstrate that, for $U_{GS} = 2$ m/s, an increase in the liquid flow rate results in higher ripples, while for stronger winds, the characteristic ripple wave heights vary in the range of 0.2 to about 0.6 mm. Similar to the behavior of the characteristic wave amplitude RMS $(\delta - \overline{\delta})$, there is an upward trend with $L$ in the most probable wave heights. However, the heights of the most probable ripples under all flow conditions remain mostly below 0.6 mm, quite small relative to the film thickness.

The amplitude frequency spectra of the instantaneous film thickness $d(t)$ measured at $L = 698$ cm are plotted in Fig. 19(a) for conditions of Fig. 17. The results show that waves at low frequencies below about 4 Hz dominate the spectrum, but no well-defined peak can be identified at those frequencies. A secondary peak corresponding to wave amplitudes can be observed at frequencies around about 8 Hz. Those waves, however, have energies smaller than those around the dominant frequency by more than two orders of magnitude. The lack of peak frequency in the spectra of ripples suggests defining the dominant frequency by counting the waves and averaging over ensembles of 10 s ($N = 3000$) as described in Sec. II. The resulting $f_{dom}$ plotted as dashed lines in Fig. 19(a) roughly correspond to the weighted average of the spectral amplitudes in the low-frequency range. For $U_{LS} = 0.05$ and $U_{GS} = 2$, and $U_{LS} = 0.03$ and $U_{GS} = 3$ m/s, $f_{dom}$ estimated through counting waves and weighted average yields frequencies 1.91; 2.01; and 1.1; 1.3 Hz, respectively. For $U_{LS} = 0.02$ and $U_{GS} = 5$ m/s with a less prominent secondary peak, the corresponding quantities are 0.8
and 1.2 Hz. The dominant frequencies tend to decrease along the pipe, Fig. 19(b).

The ripple propagation velocities estimated from the time delays \( \tau \) corresponding to the maxima of the cross correlation coefficients \( S(\tau) \) as presented in Fig. 5 are plotted in Fig. 20. The accuracy of the velocities \( \frac{d}{C_0} \) is estimated from the scatter of correlation curves obtained separately for numerous time series segments as discussed in Sec. II. Note that only the cases where the maximum in \( S(\tau) \) is reasonably well-defined are presented in this figure. The results of Fig. 20 show a slightly increasing trend in the variation of the wave propagation velocity \( c \) along the pipe for each set of \( U_{LS} \) and \( U_{GS} \); however, all values are within a range \( 35 \leq c \leq 60 \text{ cm/s} \).

Since \( \frac{(\delta - \bar{\delta})}{\bar{\delta}} \ll 1 \), these velocities can be compared with predictions based on linear theory of surface waves. The characteristic mean depths represented by \( \bar{\delta} \) in Fig. 16(a) and the range of the measured wave frequencies presented in Fig. 19 suggest that both the finite depth and capillary effects should be accounted for. The dispersion relation that relates the angular frequency \( \omega = 2\pi f \) with the wave number \( k = 2\pi / \lambda \) for gravity-capillary waves propagating over the surface of stagnant water is

\[
\omega^2 = \left( \frac{\sigma}{\rho} \right) \tanh \left( \frac{k \delta}{L} \right),
\]

where \( \lambda \) is the wavelength and \( \sigma = 73 \text{ cm}^2/\text{s}^2 \) is the surface tension coefficient divided by water density. Since the bottom of the pipe cross

![Figure 16](image1.png)

![Figure 17](image2.png)

![Figure 18](image3.png)

![Figure 19](image4.png)
Due to presence of mean water velocity \( U_L \), the radian frequency computed from Eq. (8) should be amended by the corresponding Doppler shift, \( \Delta \omega_D = kU_L \). The expression for the phase velocity \( c = \omega / k \) of linear gravity waves that accounts for capillarity, finite depth, and the Doppler shift by the mean flow is thus\(^5\)

\[
c = \frac{f \lambda}{k} = \sqrt{\frac{g}{k} \left(1 + \frac{(\sigma k^2)}{g}\right) \tanh(k\delta_L)} + U_L, \tag{9}
\]

where the first term on the right-hand side represents the wave propagation velocity on the surface of stagnant water and the second term is the advection by mean flow. The bulk liquid velocity \( U_L \) presented in Fig. 21 for different liquid flow rates \( U_{LS} \) as a function of the effective depth \( d_L \) varies in the range \( 0.1 < U_L < 0.25 \) m/s. In view of thin films encountered in the present experiments, the wave propagation velocity was also calculated assuming that waves are sufficiently long, so that the capillarity effects can be neglected, and the water is sufficiently shallow, so that \( k\delta_L \ll 1 \). Under those conditions, the dispersion relation (9) can be simplified significantly to

\[
c = \sqrt{\frac{g\delta_L}{k}} + U_L. \tag{10}
\]

The phase velocities calculated using Eqs. (9) and (10) for the extreme values of \( U_L \) in Fig. 21 are plotted in Fig. 22(a) as a function of ripples’ frequencies for two representative effective mean water depths \( \delta_L \). The results of this figure demonstrate that for the present experimental conditions, contributions to the resulting ripple phase velocity due to the advection by mean flow and to inherent ripple phase velocity are on the same order. Apparently, the decrease in the film thickness and increase in \( U_L \) lead to a larger relative contribution of the mean liquid flow to the resulting \( c \).

To estimate the range of validity of the shallow water approximation in the present experiments, the relation between the ripple frequency \( f \) and the dimensionless water depth \( k\delta_L \) calculated using Eq. (9) is presented in Fig. 22(b) for the values of \( U_L \) in Fig. 22(a). Note that the depth is considered shallow as long as \( k\delta_L < \pi/10 \), while for longer waves with \( k\delta_L > \pi/10 \) that propagate in water of intermediate depth, the assumptions adopted in the simplified dispersion relation (10) cease to be valid.\(^6\) The results of Fig. 22(b) demonstrate that the shallow water limit is only applicable for lower frequencies in the ripple spectra plotted in Fig. 19(a). Indeed, for frequencies below about
3 Hz for $\delta_L = 8$ and below 5 Hz for $\delta_L = 4 \text{ mm}$, Fig. 22(a) shows that both equations yield practically identical results at dominant ripple frequencies, see Fig. 19(a). Thus, for the present experimental conditions, capillarity practically does not affect the propagation velocity of those ripples, and they largely behave as long gravity waves in shallow water advected by the bulk water velocity.

However, at higher frequencies in the spectrum, the phase velocity of the ripples $c$ becomes affected by both the finite water depth and wave capillarity, and they become notably dispersive, see Fig. 22(a). As seen from the dimensionless water depths $k_d$ plotted in Fig. 22(b), the shallow water conditions are not satisfied for higher frequency ripples propagating over a relatively thin liquid film ($\delta_L = 4 \text{ mm}$); for $\delta_L = 8 \text{ mm}$, the finite film thickness is essential at even lower frequencies. The joint effect of capillarity and depth on the ripple phase velocity is studied in Fig. 22(c) for a range of effective film thicknesses relevant for the present study. This plot demonstrates that for $\delta_L \approx 5 \text{ mm}$ and both values of the liquid velocity $U_L$, the ripples can be seen as essentially nondispersive waves. For lower values of $\delta_L$, the phase velocity of the ripples increases with the frequency, while for $\delta_L > 5 \text{ mm}$, it decreases with $f$. The effect of water depths weakens as the water bulk velocity $U_L$ increases; thus, Doppler shift becomes a dominant contribution to $c$. This behavior clarifies the reasons for the qualitatively different dependence of $c$ on frequency for the two values of $\delta_L$ in Fig. 22(a).

Comparison of the measured ripple velocities with those calculated using Eq. (9) for the corresponding flow conditions is carried out in Fig. 23. Reasonable agreement is observed; however, the experimentally measured values are somewhat higher than the theoretically predicted results. Note that Liberzon and Shemer\(^{17}\) and Zavadsky and Shemer\(^{15}\) observed that the deep-water phase velocity measured in wind-wave tank experiments with no mean water flow is higher by 10%–15% than that predicted by the linear dispersion relation. They attributed this discrepancy to the effect of wind shear. It is reasonable to assume that in a closed pipe environment the wind shear is even more pronounced and causes higher ripple propagation velocities as compared to predictions based on Eq. (9).

The characteristic wavelengths can be estimated from the measured dominant wave frequencies (Fig. 19) and the corresponding phase velocities. The wavelength variation along the pipe for the conditions of Fig. 20 is presented in Fig. 24(a). The typical ripples at the dominant frequency are few decimeters long, in reasonable agreement with the measurements of Lioumbas et al.\(^{16}\) Ripples are thus longer by orders of magnitude than their heights, $h_{\text{wave}}/\lambda \ll 10^{-2}$. This small steepness of the ripples is consistent with the linear approximation adopted in their analysis. The wavelengths $\lambda$ tend to increase along the pipe although in some cases a different trend is obtained. To examine the reasons for the opposite trends in the variation of $\lambda$ along the pipe in Fig. 24(a), it is instructive to plot the wave spectra and the corresponding dominant frequencies for two sets of gas and liquid flow rates that exhibit qualitatively different dependence $\lambda(L)$. Note that for better visibility, the spectra in Fig. 24(b) are plotted in semilog coordinates. For $U_{LS} = 0.03$ and $U_{GS} = 3 \text{ m/s}$, the wave amplitude at lower frequencies in the spectra in Fig. 24(b) increase notably with the axial location $L$, causing the downshifting of the dominant frequency. For a higher liquid flow rate corresponding to $U_{LS} = 0.05$ but...
weaker wind, $U_{GS} = 2$ m/s, higher frequencies become more dominant at a more remote location; thus, the dominant frequency increases. Since the phase velocity in both cases does not change significantly along the pipe, the notable variation in the dominant frequency in opposite directions leads to the observed effect on $\omega(L)$. The wavelengths $\lambda_2$ estimated using the linear dispersion relation corresponding to the frequency range around the secondary peak ($f_2$) in the spectra of Fig. 19(a) are about few cm and are shown separately in Fig. 24(c).

B. Roll waves regime

1. Experimental results

As seen in Fig. 3(c), roll waves are random, widely spaced, and occur intermittently. They are characterized by amplitudes notably larger than those of ripples, see Figs. 10 and 11, and have steep fronts and gradually sloping backs [Figs. 3(c) and 6(b)]. The strong asymmetry of roll wave results in high values of the skewness coefficient $\lambda_3$, Fig. 12(a), and of the excess kurtosis, $\lambda_4 - 3$, Fig. 12(b). The exceedance distributions plotted in Fig. 15 suggest considering high roll waves separately while discarding small waves ($h_{wave} < 3$ mm) in the plateaus [Fig. 3(c)]. This requires defining their heights $h_{wave}$ as the crest height over the reference film thickness, $\left(\delta + \delta_{ref}\right)/2$, see Fig. 6(b). This definition thus differs from that adopted for ripples. 51

Examples of the smoothed probability density functions of the measured individual roll wave heights are plotted in Fig. 25. As in Fig. 17, the most probable wave height $h_{dom}$ is estimated from the parabolic fit. For a given $U_{GS}$, the value of $h_{dom}$ increases with $U_{GS}$.

The variation along the pipe of dominant roll wave heights that are defined on the basis of identification of individual waves is plotted in Fig. 26(a) for two values of $U_{LS}$ and four values of $U_{GS}$. It is compared in Fig. 26(b) with the corresponding RMS values of $\left(\delta - \bar{\delta}\right)$. Both $h_{dom}$ and RMS $\left(\delta - \bar{\delta}\right)$ increase with the distance from the inlet, following the trend similar to that observed for the ripples, Figs. 16(b) and 18. However, unlike the ripples where $h_{dom}$ and RMS $\left(\delta - \bar{\delta}\right)$ are on the same order of magnitude, the dominant heights of roll waves in Fig. 26(a) are notably higher than RMS $\left(\delta - \bar{\delta}\right)$ in Fig. 26(b). This can be attributed to the relatively short duration of the roll waves that are separated by extensive plateaus with no significant waves, see Fig. 3(c).

The variation along the pipe of the higher spectral moments, $\lambda_3$ and $(\lambda_4 - 3)$, is plotted in Figs. 27(a) and 27(b). For given $U_{LS}$ and $U_{GS}$, both moments increase with $L$. An increase in the liquid flow rate at a constant $U_{GS}$ and in $U_{GS}$ at constant $U_{LS}$ causes results in lower values of both skewness $\lambda_3$ and excess kurtosis $(\lambda_4 - 3)$ coefficients.

An example of roll wave amplitude frequency spectra plotted in Fig. 28(a) demonstrates the existence of a well-defined peak; the spectral peak frequency is in good agreement with the dominant frequency obtained by counting individual waves. Summary of the dominant frequencies for various gas and liquid flow rates is presented in Fig. 28(b); for each $U_{LS}$ and $U_{GS}$, the values of $f_{dom}$ decrease along the pipe. However, at higher $U_{GS}$, the $f_{dom}$ decreases significantly from first to second station followed by small change from the second to the third station, while at lower $U_{GS}$, the overall decrease in $f_{dom}$ is quite small.

For roll wave regimes, the signals at two sensors spaced by 15 cm are fairly well correlated, and the temporal variation of cross correlation coefficients $S(\tau)$ plotted in Fig. 29(a) demonstrates that their maximum values increase somewhat with the distance $L$ along the pipe, while the time delay at which the maximum of $S(\tau)$ is attained decreases with $L$. Accordingly, the mean propagation velocities of roll waves $c$ increase along the pipe, see Fig. 29(b). Higher values of $c$ are obtained for a given gas flow rate $U_{GS}$ at higher liquid flow rate $U_{LS}$ for a fixed $U_{GS}$, the roll wave propagation velocity decreases with $U_{GS}$, see also Fig. 13(a).

The variation of additional statistical parameters of roll waves along the pipe is presented in Fig. 30 for gas flow rate $U_{GS} = 6$ m/s. The panels in the upper row correspond to $U_{LS} = 0.08$ and in the bottom row to $U_{LS} = 0.04$ m/s. The mean propagation velocities of roll waves’ $c$ as presented in Fig. 29(b) and the time intervals between consecutive wave crests detected by the two sensors in the pair can be used to estimate the length of roll wave units, $L_w$ as the distance between consecutive crests. The probability density functions of $L_w$ are presented in the left column. In both panels, the most probable values of $L_w$ increase along the pipe while the distributions become wider. Higher liquid flow rate corresponds to shorter roll waves. The increase along the pipe of $L_w$ is accompanied by a notable increase in the roll wave crest heights $\delta_{c}$, see the middle column in Fig. 30. The peaks of roll waves identified in the time series, see Fig. 6(b), are used to estimate the crest heights $\delta_{c}$ relative to pipe’s bottom. The time-lag $\Delta t$ between the crests in the time series of two sensors corresponding to the same roll wave is used to estimate the local wave propagation velocity $c_{wave}$. The pdf of $c_{wave}$ along the pipe is presented in the third column of Fig. 30. The increase along the pipe of the mean wave propagation velocity $c$ estimated from cross correlation analysis and of the
FIG. 26. (a) The variation of dominant wave height (\(h_{\text{dom}}\)) and (b) RMS (\(\delta - \delta\)) along the pipe for \(4 \leq U_{\text{GS}} \leq 7\) m/s.

FIG. 27. The variation along the pipe for \(4 \leq U_{\text{GS}} \leq 7\) m/s: (a) skewness (\(k_3\)); (b) excess kurtosis (\(k_4/\delta\)).

FIG. 28. (a) The frequency spectrum at \(L = 698\) cm; the dashed line: \(f_{\text{peak}}\) and solid line: \(f_{\text{dom}}\); (b) variation of dominant frequency (\(f_{\text{dom}}\)) along the pipe for \(4 \leq U_{\text{GS}} \leq 7\) m/s.

FIG. 29. (a) Cross-correlation function at two values of \(L\); black: \(U_{LS} = 0.08\); \(U_{GS} = 5\), red: \(U_{LS} = 0.08\); \(U_{GS} = 7\) m/s; (b) variation of measured mean wave velocity, \(c\), along the pipe for \(4 \leq U_{\text{GS}} \leq 7\) m/s.
local wave velocities $c_{\text{wave}}$, as well as growth in $\delta_{cr}$, are consistent with formation of larger accelerating waves. This behavior, together with the decrease in the dominant frequency $f_{\text{dom}}$, see Fig. 28, suggests that roll waves undergo coalescence.

The relation between the crest height $d_{cr}$ of the individual roll wave and its propagation velocity $c_{\text{wave}}$ is examined in Fig. 31. The considerable scatter in the data for each set of $U_{LS}$ and $U_{GS}$ manifests the random nature of those waves. Nevertheless, all three plates of Fig. 31 demonstrate clear trend in the relation between $d_{cr}$ and $c_{\text{wave}}$.

2. Model for roll waves’ propagation velocity

Although roll waves are quite long and their characteristic steepness $\delta_{cr}/L_u = O(10^{-2})$ is thus very low, their crest heights are on the order of the mean film depth. The linear approximation adopted in the analysis of ripples is thus inapplicable for those waves. A different approach is therefore applied here.

Characteristic roll wave profile in a frame of reference moving with its propagation velocity $c_{\text{wave}}$ that is taken here to be constant is shown in Fig. 32. In this frame of reference, the roll wave is assumed to retain a stationary shape, and the gas flows to the right with the velocity $u_{G} - c$, while the velocity of the liquid that moves to the left is $c - u_{L}$. The maximum film thickness at the roll wave peak $\delta_2$ is notably higher than the background substrate thickness $\delta_1$.

Roll waves exist when the Froude number in the moving frame of reference defined as $Fr^2 = (u_{L} - c)^2/gh_{L}$, see Fig. 4, exceeds unity in the substrate, resulting in supercritical flow there. The abrupt transition from the supercritical to subcritical flow in the vicinity of the roll wave peak resembles a hydraulic jump. For roll wave conditions in the present experiments, at $L = 698$ cm, the Froude number is in the range $2 < Fr < 6$.

Consider the control volume defined by cross sections 1 and 2. Continuity equations in gas and liquid phases are

$$A_G \frac{u_{G1}}{C_0} = A_G \frac{u_{G2}}{C_0}$$

$$A_L \frac{c_{\text{wave}}}{C_0} = A_L \frac{c_{\text{wave}}}{C_0}$$

where $A$ denotes the corresponding cross-sectional areas, see Fig. 4.

The momentum balance for the liquid phase is

$$\sum F_x = (F_1 + F_2) - P_2 = \rho_L Q(U_{L2} - U_{L1}).$$

The horizontal forces acting on the control volume consist of hydrostatic and air pressure forces. The pressure drop $\Delta P$ in the gas between cross sections 2 and 1 can be attributed to gas deceleration due to rapid expansion,

$$\Delta P = \frac{P_{G2} - P_{G1}}{\rho_g} = k \frac{1}{2} U_{G2}^2 = k \frac{1}{2} (u_{G2} - c)^2.$$ 

The local head loss coefficient $k$ is estimated for a sudden expansion of airflow from $A_{G2}$ to $A_{G1}$ as

$$k = \left(1 - \frac{A_{G2}}{A_{G1}}\right)^2.$$ 

The hydrostatic pressure forces are calculated as the product of the corresponding liquid cross-sectional area by the liquid depth at its centroid $\delta_1$, which is given by...
where $u = 2 \cos^{-1}(1 - \delta/R)$, Eq. Fig. 4. Thus,

$$F_1 = (p_{G1} + \rho_A \delta_{A1}) A_{L1},$$

$$F_2 = (p_{G2} + \rho_A \delta_{A2}) A_{L2}, \quad F_0 = p_{G1}(A_{L2} - A_{L1}).$$

From Eqs. (11)–(14), the following relation between the liquid cross-sectional areas and the roll wave propagation velocity can be written:

$$\delta = \frac{4R \sin^2 \left( \frac{u}{3(1 - \sin(\varphi))} \right) - \sqrt{R - \delta}}{\sqrt{R - \delta}},$$

$$g \left( \frac{\delta_{A1}}{A_{L1}} - \frac{\delta_{A2}}{A_{L2}} \right) + \left( c - \frac{u_{L1}}{A_{L1}} \right)^2 \left( 1 - \frac{A_{L1}}{A_{L2}} \right) \frac{A_{L1}}{A_{L2}} \frac{A_{L1}}{A_{L2}}$$

$$- \frac{1}{2} \frac{\rho_g}{\rho_l} \kappa \frac{A_{G1}}{A_{G2}}^2 \left( u_{L1} - c \right)^2 = 0.$$  

The wave propagation velocity $c$ is calculated from the implicit Eq. (18) using the value of $k \approx 0.36$ that corresponds to areas estimated from the film thickness measurements. Note that a constant value for $k$ and Eq. (15) give similar results. The calculated and the measured mean velocities $c$ compare well at various values of $U_{LS}$ and $U_{GS}$; see Fig. 33.

### 3. Estimates of substrate depth $\delta_{L1}$ and crest height $\delta_{L2}$

An alternative approach based on one-dimensional two fluid model, and the available estimated roll wave propagation velocity $c$ is now applied to calculate the substrate area $A_{L1}$, its depth $\delta_{L1}$, and the liquid area at the crest $A_{L2}$ with the height $\delta_{L2}$. Unlike the local control volume analysis in Sec. III B 2. that only accounts for the normal stress, we now extract those values from the two fluid model equations,

$$\frac{d}{dx} (\rho_l A_l U_{L1}) = 0, \quad \frac{d}{dx} (\rho_g A_g U_{G1}) = 0,$$

$$\frac{d}{dx} (\rho_l A_l U_{L2}^2) = -A_l \frac{d}{dx} (\rho_l A_l \delta) - \rho_l A_l g \frac{d\delta}{dx} - \tau_l S_l + \tau_l S_l,$$

$$\frac{d}{dx} (\rho_g A_g U_{G2}^2) = -A_g \frac{d}{dx} (\rho_g A_g \delta) - \rho_g A_g g \frac{d\delta}{dx} - \tau_g S_g - \tau_g S_g.$$
where $U_l = u_l - c$ and $U_G = u_G - c$, and the shear stresses are given in terms of actual velocity,

\[ \tau_L = f_L \rho_L |u_1| |u_L| / 2, \]

\[ \tau_G = f_G \rho_G |u_1| |u_G| / 2, \]

\[ \tau_i = f_i \rho_i (|u_G - u_l|(|u_G - u_l|)), \]

where $f_L = 0.046Re_0^{0.2}$ represents the friction factor between liquid and wall, $f_G = 0.046Re_0^{0.2}$ between gas and wall, and $f_i = f_G$ is the friction factor at the gas–liquid interface.

Eliminating pressure gradient from Eqs. (20) and (21) and using the mass balance as in Eq. (19) yields the following differential equation:

\[ \frac{d\delta}{dx} = \frac{\tau_L S_L}{A_L} - \frac{\tau_G S_G}{A_G} - \frac{\tau_i S_i}{1 + \frac{A_L - A_G}{A_G}} = \frac{M_1}{M_2}. \]

Equating the numerator $M_1$ in Eq. (25) to zero for given values of $U_G$ and $U_L$ and using the calculated value of $c$ from Eq. (18) yields two solutions for the film thickness $\delta$. The lower value corresponds to the equilibrium film thickness far away from the roll wave and is indeed quite close to the measured substrate thickness $\delta_{sub}$, see Fig. 34. The higher solution corresponds to the maximum in $\delta(x)$ and compares well with the measured mean crest height $\delta_{cr}$ as also is evident in the same panel.

Integrating Eq. (25) starting from the calculated value of $\delta_{cr}$ yields variation of the film thickness in roll waves in a reasonable agreement with measurements and tends to $\delta_{sub}$, see Fig. 35.

C. Pre-Annular Wavy Regime

Temporal variation of surface elevation characterizing pre-annular waves is presented in Fig. 3(d). There are significant similarities between roll and pre-annular waves. The differences between them stem from higher gas flow rates and from spreading of the liquid film over the pipe circumference that lead to reduction of the mean film thickness at the measuring location at the bottom of the pipe. Thus, absolute heights of pre-annular waves are lower as compared to those of roll waves; however, the ratio between wave heights and the substrate thickness is quite similar in both cases, $h_{wave}/\delta = O(1)$, Figs. 6, 9, and 10. Due to increase in $U_{GS}$ in pre-annular waves as compared to roll waves, the dominant frequency also increases significantly, Fig. 13(b). The non-Gaussianity of pre-annular waves is reduced as compared to the essentially by-modal roll waves, see Figs. 14 and 15, as well as in a notable decrease in skewness, Fig. 12(a), and excess kurtosis, Fig. 12(b).

The variation along the pipe of mean film thickness $\delta$ and the RMS ($\delta - \overline{\delta}$) that may represent the characteristic wave amplitude are plotted in Fig. 36 for $0.04 \leq U_{LS} \leq 0.08$ and $8 \leq U_{GS} \leq 12$ m/s. An increase in the mean film thickness along the pipe is observed for all values of $U_{LS}$ and $U_{GS}$ in Fig. 36(a). The characteristic wave amplitudes RMS $\delta - \overline{\delta}$ in Fig. 36(b) also increase slightly with $L$.

The dominant wave heights $h_{dom}$ and dominant frequencies $f_{dom}$ are plotted in Fig. 37 as a function of $L$ for two values of $U_{LS}$. The
dominant wave heights $h_{\text{dom}}$ remain constant along the pipe for both values of $U_{LS}$ and $U_{GS}/C_{20}$. Note that for the relatively short pre-annular waves, the RMS ($\delta - \bar{\delta}$) and $h_{\text{dom}}$ are comparable, in contrast to much longer roll waves (see Fig. 26). The dominant frequency $f_{\text{dom}}$ in all cases remains nearly constant, indicating that no significant coalescence of pre-annular waves occurs at $U_{GS} = 14$, but slightly decreases for $U_{GS}/C_{20} = 12$ m/s.

The variation of the cross correlation coefficients $S(\tau)$ between the adjacent probes at two measurement locations $L$ is plotted for $U_{LS} = 0.08$ and $U_{GS} = 14$ m/s in Fig. 38(a). The time delay $\tau$ corresponding to the maximum allows calculating the propagation velocity $c$ of pre-annular waves; the results presented in Fig. 38(b) show slight increase in $c$ along the pipe.

**IV. CONCLUDING REMARKS**

Visual identification of wavy regimes in horizontal stratified gas–liquid pipe flow supported by analysis of the temporal variation of the instantaneous film thickness is carried out in a 10 m long pipe with a diameter of 24 mm for a wide range of gas and liquid flow rates. Four sub-regions were identified in the stratified flow domain: stratified smooth, ripples, roll waves, and pre-annular waves. For each wave regime, statistical analysis of the film thickness records obtained in gas–liquid stratified pipe flow for a wide range of gas and liquid flow rates is presented. Wave parameters such as heights, frequencies, propagation velocities, characteristic lengths, wave height exceedance distributions, higher statistical moments, etc., were determined for each type of wave. The evolution of those parameters along the pipe is investigated.
An increase in gas velocity for any given value of $U_{GS}$ results in sequence of transitions between the four sub-regions, see Fig. 8. Ripples first appear on the initially smooth water surface; the values of their RMS $\langle \delta - \delta \rangle$ may attain about 0.5 mm. It is demonstrated that ripples are essentially linear and are characterized by small steepness $h_{wave}/\lambda = O(10^{-3})$; the ripple amplitudes are small compared to the mean film thickness, $h_{wave}/\delta = O(10^{-4})$. Higher order statistical moments of those waves are close to zero, while mean film thickness $\delta$ and dominant wave heights increase somewhat along the pipe. The frequency spectra of the surface elevation are characterized by the lack of well-defined peaks. A secondary peak corresponding to waves smaller by an order of magnitude than the energetic low frequency waves is observed around 8 Hz. Wave propagation velocities estimated from temporal variation of the cross correlation coefficient between two adjacent sensors showed no distinct trend along the pipe, remaining within the range of $35 \leq c \leq 60 \text{ cm/s}$. It is found that typical wave-lengths exceed significantly $10 \text{ cm}$ and thus are unaffected by capillarity. The relatively narrow range of waves' propagation velocities is in agreement with their weak dispersion that results from the intermediate dimensionless depth $\delta/c$, as well as from significant contribution of the Doppler shift to $c$ in the presence of water flow.\(^5\) Nevertheless, the calculations somewhat underpredict the measured velocities, as also observed by Liberzon and Shenmer,\(^{17}\) and Zavadsky and Shenmer\(^{15}\) in experiments in a wind-wave tank. The effect of wind shear, which is more prominent in a closed pipe environment as compared to wind-wave facilities, leads to even higher propagation velocities. The relatively weak waves around the secondary spectral peak are much shorter with lengths of few cm that represent gravity-capillary waves; those waves are more affected by dispersion.

Transition from quasi-linear ripples to nonlinear roll waves occurs when water flow in the substrate becomes supercritical ($Fr > 1$). This transition results in an abrupt increase in RMS values of the surface elevation, higher wave heights, and a pronounced jump in the higher statistical moments. The steepness $h_{wave}/\lambda = O(10^{-3})$ of roll waves is similar to that of ripples and is very small, but they are notably higher than ripples as compared to the substrate depth, with $h_{wave}/\delta = O(1)$. Roll waves are thus essentially nonlinear and propagate over shallow water. Both dominant wave height and RMS $\langle \delta - \delta \rangle$ increase along the pipe. However, while those quantities are larger than those for the ripples but retain the same order of magnitude, the wave heights of roll waves exceed notably their RMS $\langle \delta - \delta \rangle$ values due to relatively large spacing between consecutive roll waves that are separated by long plateaus. The roll waves have an essentially bi-modal structure as demonstrated in Figs. 15 and 25(a). This structure of roll waves is consistent with high values of skewness and excess kurtosis coefficients of the film. However, an increase in $U_{GS}$ for constant $U_{GS}$ or increase in $U_{GS}$ for constant $U_{GS}$ causes a certain decrease in both skewness and excess kurtosis coefficients.

The dominant frequency decreases, and wave propagation velocity of roll waves increases along the pipe. The wave height exceedance curves are strongly non-Gaussian and demonstrate the bimodal nature of those waves that can be seen as a combination of low-amplitude ripple-like waves that dominate the plateau regions and much larger widely separated roll waves, see Fig. 15. Additional statistical demonstration of the bimodal structure of roll wave is carried out by presenting their probability density functions (pdf), Fig. 25. The variation of pdf's of roll wave wavelengths, crest heights, and wave propagation velocities along the pipe, as well as the decrease in $f_{dom}$ plotted in Fig. 30, indicates that the waves may undergo coalescence along the pipe, as suggested by Andritsos and Hanratty.\(^4\)

Two approaches were applied to estimate the mean characteristics of nonlinear roll waves. In both analyses, a steady shape of the wave is assumed. The first approach considered the local balance of mass and momentum within the liquid, using the measured film thickness at the characteristic locations within the wave. The action of gas was accounted for by adding an estimated pressure force acting on the control volume. This approach yields the roll wave propagation velocity that is in agreement with measurements for various gas and liquid flow rates. Alternatively, two-fluid model was applied to predict the height of the roll wave crests and the substrate thickness using the mean roll wave propagation velocity as input. This approach allows estimating the longitudinal profile of a representative roll wave that compared well with experiment.

Further increase in gas velocity results in transition to the pre-annular wavy regime. Similar to roll waves, they are essentially nonlinear high amplitude waves propagating over a shallow substrate. The circumferential spreading of the film and high gas flow rates in this sub-region cause a decrease in the substrate thickness as compared to roll waves; thus, the characteristic wave heights are notably lower than those measured for roll waves. The transition to the pre-annular regime also causes a significant drop in both skewness and excess kurtosis, which do not vary notably with gas velocity. The wave height exceedance distribution of pre-annular waves in a horizontal pipe obtained in the present study is quite similar to that measured in vertical annular pipe flow.\(^5\) The wave propagation velocity increases along the pipe and is similar to that observed for roll waves. However, circumferential film spreading and high gas velocities result in a substantial increase in a dominant wave frequency.

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DATA AVAILABILITY

The data that support the findings of this study are available from the corresponding author upon reasonable request.

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