interference microscope, are also shown in Table 1. The two zero-dispersion wavelength values agree well. The small differences between the calculated and measured wavelengths may be due to the core-diameter fluctuation along the fibre length. The zero-dispersion wavelength of the 20 km long fibre calculated from the fibre parameters was 1·48 μm.

Pulse broadening was measured with the 20 km long fibre. Fig. 1 shows the input and output pulse waveform. The full width $\Delta\tau$ at half maximum of the optical pulse before and after propagation, $\Delta\tau_{in}$ and $\Delta\tau_{out}$, were 380 ps and 400 ps, respectively. The laser output spectrum consisted of three longitudinal modes with 4·8 nm width at half-maximum intensity. From these measurements, pulse dispersion was found from the formula $(\Delta\tau_{out}^2 - \Delta\tau_{in}^2)^{1/2}$ to be 26 ps/nm, which was found to agree well with the estimated value from each single-mode fibre.

Conclusion: Low-loss dispersion-free single-mode fibres at 1·5 μm were fabricated by choosing the waveguide parameters so as to cancel the material dispersion. The total loss of the 20 km long fibre was 24·2 dB and 21·0 dB, at 1·50 and 1·52 μm, respectively. The measured dispersion of the fibre was 26 ps/nm. The loss was slightly higher than estimated,[13] owing to waveguide imperfection, which will be reduced by improving the fabrication technique.

This experiment suggests the feasibility and desirability of large-capacity and long-distance single-mode-fibre transmission in the 1·5 μm wavelength region.

Acknowledgment: The authors wish to thank N. Inagaki and T. Kimura for useful discussions. They are also indebted to Y. Horikoshi for supplying the laser, and S. Machida and J. Yamada for the measurement.

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0013-5194/80/050188-02$1.50/0

COMMENT

MIXED TIME/FREQUENCY DOMAIN APPROACH TO MODEL REDUCTION

Indexing terms: Modelling, Systems

Given a high-order system described by the transfer function $H(s)$ and the impulse response $h(t)$, a method has been introduced in Reference A that obtains a reduced pth order model such that the first terms in the power-series expansions about zero of its transfer function $G(s)$ and its impulse response $g(t)$ match the first k and $2p - k$ terms of the corresponding expansions of the system:

$$H(s) = \sum_{i=0}^{\infty} H_i s^i \tag{1}$$

$$h(t) = \sum_{i=0}^{\infty} h_i t^i \tag{2}$$

respectively.

This method has been termed by the authors 'mixed time/frequency approach' owing to the fact that the exprs. 1 and 2 are written in the frequency- and the time-domain formulation, respectively.

Taking the Laplace transform of expr. 2 term by term, it is readily obtained that

$$H(s) = \sum_{i=0}^{\infty} M_i s^{-(i+1)}$$

$$M_i = i!\, h_i \qquad i = 0, 1, \ldots \tag{3}$$

It follows, therefore, that the suggested method is equivalent to methods of mixed Padé approximations at both $s = 0$ and $s = \infty$, which have been suggested in References B–D. The constants M_i in expr. 3, which are termed Markov parameters, are easily obtained by expanding the rational function $H(s)$ about $s = \infty$ using long division, and the reduced model can be derived entirely from frequency-domain considerations.

The proposed method may also be interpreted as a pure time-domain approach, considering expr. 2 and noting that H_i in expr. 1 are related to the time moments T_i of the impulse response

$$T_i = \int_0^{\infty} t^i h(t)\, dt = i!\, (-1)^i H_i, \qquad i = 0, 1, \ldots \tag{4}$$

with the following further 'time-domain' properties:

(i) There is no steady-state error between the outputs of a model that matches k time moments and the system for inputs that can be written in the form

$$u(t) = \sum_{i=0}^{k-1} \alpha_i t^i \tag{5}$$

with any α_i.

(ii) H_i and M_i (and thus T_i and h_i) can be obtained also from the state-space representation (A, b, c) of the system by

$$H_i = cA^{-(i+1)}b \qquad M_i = cA^i b \qquad i = 0, 1, \ldots \tag{6}$$

Every method that obtains reduced models by matching k terms of expr. 1 or expr. 4 and $2p - k$ terms of exprs. 2 or 4 yields the same pth-order model and has four different interpretations (a pure s domain, a pure time domain and two complementary mixed time/s domain). Among these possible combinations, the terms that can be derived and matched most directly are the coefficients of the expansion of the transfer function about $s = 0$ and $s = \infty$.

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8th January 1980

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0013-5194/80/050189-02$1.50/0

STABILITY OF CERTAIN TIME-VARYING NONLINEAR SYSTEMS

Indexing terms: Control theory, Stability, Systems

A sufficient condition for the feedback stability of the cascade of a nonlinear element and a linear element with positive impulse response is given.

Consider a nonlinear scalar feedback system defined by the equations

$$e = u - y \qquad y = LN(e)$$

where u and y are the input and output of the system, L and N are, respectively, a linear and a nonlinear operator and e is the input to the nonlinearity. We shall assume that the action of L on a signal $x(t)$ passing through it is given by the convolution

$$L[x(t)] = \int_0^t g(t-s, t)x(s)\, ds$$

and N is defined by a function of two variables $n(s, t)$ such that the action of N at time t on a signal $x(t)$ passing through it is given by $n[x(t), t]$. Thus the system equations are

$$e(t) = u(t) - y(t) \tag{1}$$

$$y(t) = \int_0^t g(t-s, t)n[e(s), s]\, ds \tag{2}$$

Our aim is to show that under certain conditions, the most notable of which is the positivity of g and the existence of a lower bound on n, the system is bounded-input/bounded-output stable. A strengthening of our assumptions gives L^1 stability of the system.

Assumptions on n: Suppose that $n(s, t)$ is defined on $R \times R_+$ and satisfies the following conditions:

(i) $sn(s, t) \geq 0$, for all $s \in R$ and $t \geq 0$;

(ii) There exist $M \geq 0$ such that $n(s, t) \geq -M$ for all $s \leq 0$ and $t \geq 0$;

(iii) For every bounded set B of real numbers there exists a constant $c_B > 0$ such that $n(s, t) \leq c_B$ whenever $s \in B$ and $t \geq 0$.

Assumptions on g: $g(s, t)$ is defined on R_+^2; it is a measurable function of s for all t and

(iv) $g(s, t) \geq 0$ for all $(s, t) \in R_+^2$

(v) For some constant C,

$$G(t) = \int_0^t g(s, t)\, ds \leq C$$

for all $t \geq 0$.

We shall further assume that eqns. 1–2 have at least one solution in e and y. Our results are as follows.

Theorem 1: Suppose that conditions (i)–(v) hold. Then whenever u is bounded, so are e and y.

Proof: Let $|u(t)| \leq R$. Put

$$u^+(t) = \max[0, u(t)],$$
$$u^-(t) = \max[0, -u(t)]$$

and define e^+ and e^- similarly. Let

$$S = \{t: e(t) \geq 0\}$$
$$T = \{t: e(t) < 0\}$$
$$S_t = S \cap [0, t]$$
$$T_t = T \cap [0, t]$$

From eqn. 2 we have

$$e(t) = u(t) - \left(\int_{S_t} \int_{T_t}\right)[g(t-s, t)n\{e(s), s\}]\, ds$$

For $s \in S_t$, $e(s) \geq 0$, so that by assumptions (i) and (iv) we have

$$e^+(t) \leq u^+(t) - \int_{T_t} g(t-s, t)n\{e(s), s\}\, ds$$

Thus, by assumption (ii)

$$e^+(t) \leq u^+(t) + MG(t) \leq R + MC \tag{3}$$

Similarly, $e(s) < 0$ on T_t, so that by assumption (i) $n\{e(s), s\} < 0$ and by (iv) we have

$$e^-(t) \leq u^-(t) + \int_{S_t} g(t-s, t)n\{e(s), s\}\, ds \tag{4}$$

From (iii) there exists a constant c depending on $R + MC$ such that $n(e, t) \leq c$ whenever $0 \leq e \leq R + MC$. Thus, by eqns. 3 and 4

$$e^-(t) \leq u^-(t) + cG(t) \leq R + cC \tag{5}$$

The boundedness of e follows from eqns. 3 and 5 and that of y follows from eqn. 1.

A strengthening of assumption (v) gives an L^1 stability result as follows. Suppose that

(vi) $G(t)$ as defined in assumption (v) is measurable and

$$\int_0^\infty G(t)\, dt = K < \infty$$