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A SURVEY OF NEW IMMITTANCE-TYPE STABILITY TESTS FOR TWO-DIMENSIONAL DIGITAL FILTERS

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ABSTRACT

The paper brings a brief report on three new algebraic tests to determine whether a two variable polynomial has all its zeros inside the unit bi-circle (is 'stable'). A three-term recursion algorithm associates the tested two-variable polynomial with a sequence of matrices (the 2-D 'table') that possess a symmetry which allows to compute only half of the entries for each matrix. The recursions incorporate a deconvolution/division mechanism that removes recursively redundant common polynomial factors and prevents an exponential grow of the raw dimension of the matrices. A minimal set of conditions necessary and sufficient for stability for a polynomial with variables of degrees (n_1, n_2) requires one or two 1-D tests of degree n_1 and the testing of whether a last 1-D polynomial has zeros in the real interval $[-1, 1]$ or on $|z| = 1$. The degree of this last polynomial is only $2n_1n_2$.

1. INTRODUCTION

A two-dimensional (2-D, two-variable) polynomial

$$D(z_1, z_2) = \sum_{i=0}^{n_1}\sum_{k=0}^{n_2} d_{i,k} z_1^i z_2^k$$

is said to be stable if

$$D(z_1, z_2) \neq 0, \quad \text{for} \quad (z_1, z_2) \in \bar{V} \times \bar{V}$$

where

$$T = \{z : |z| = 1\},\ U = \{z : |z| < 1\},\ V = \{z : |z| > 1\},$$

are used to denote the unit circle, its interior, and its exterior, respectively, and the bar denotes closure, e.g. $\bar{V} = V \cup T$.

A stable 2-D polynomial is the key for the stability of 2-D linear shift-invariant recursive filters and systems. 2-D stability tests are methods to determine whether a given $D(z_1, z_2)$ is stable.

This paper summarizes main results from recent algebraic 2-D stability tests [1, 2, 3, 4] that stem from 1-D stability tests in [5] (for real) and in [6] [7] (for complex polynomials). The referenced 1-D tests propagate symmetric polynomials by three-term recursions with a single multiplier yielding computationally more efficient algorithms than alternative 1-D stability tests in the classical Schur-Cohn class that involves polynomial sequences with no special symmetry. These recursions were also found to offer computational improvement for signal processing algorithm related to linear prediction and were called in that context *immittance* domain algorithms in distinction from corresponding classical *scattering* Levinson, Schur and Schur-Cohn algorithms [8], [9].

Each of the three reported 2-D stability tests consists of an algorithm that builds for the tested polynomial a sequences of matrices $\{E_m\}$ (the 2-D stability 'table') and a set of associated necessary and sufficient conditions for stability. The sequence of matrices $\{E_m\}$ are obtained by three-term recursions of matrices or of two-variable polynomials having these matrices as their coefficients. The polynomial notation will be used for shortness. Each matrix E_m possesses a certain symmetry by which it suffice to compute only half of the entries of each matrix. Exploiting these and derivative symmetries may reduce the computation cost by roughly a factor of two. For each case only the most refined form of table and smallest set of stability condition are cited. The referenced conference proceeding provide a larger set of stability conditions (more conditions implied by stability that need not to be tested to conclude it) while forthcoming publications in journal will provide full details on the derivation and proofs for the theorems.

Typically, a 2-D stability test for $D(z_1, z_2)$ uses one of several simplified stability condition [11]. This includes the next Huang-Strintzis simplification [10] [12] that will be used here (with $a = 1$).

Lemma 1. *$D(z_1, z_2)$ is stable if and only if*

(i) $D(z, a) \neq 0$ for all $z \in \bar{V}$ and some $a \in \bar{V}$

(ii) $D(s, z) \neq 0$ for all $(s, z) \in T \times \bar{V}$.

In view of this simplification the task of an algebraic 2-D stability test is to provide an easy to program and computationally efficient algorithm to test second condition in the Lemma. For the current 2-D tests, the choice $a = 1$ in the first condition blends instrumentally with the special role of $z = 1$ in the underlying 1-D stability tests and contributes to the simplicity of the stability conditions associated with the 2-D tables.

Clearly, each 2-D stability test may be applied to either $D(z_1, z_2) = \mathbf{z_1}^t D \mathbf{z_2}$ itself (the case that is assumed throughout currently) or to $\mathbf{z_1}^t D^t \mathbf{z_2}$ where $\mathbf{z} := [1, z, \ldots, z^i, \ldots]^t$ (with preference to be determined by whether $n_1 > n_2$ or $n_1 < n_2$, resp.).

2. EXTENDED REAL TEST

This first 2-D stability test stems from a division-free form of the test for real polynomials in [5] and was first presented in [1]. Define for the polynomial $D(z_1, z_2)$ the auxiliary polynomial

$$W(s^{-1}, s, z) = D(s^{-1}, z) D(s, z) = \tilde{\mathbf{s}}^t W \mathbf{z} \quad ,$$

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where the notation $\tilde{\mathbf{s}} := [s^{-m},\ldots,s^{-1},1,s,\ldots,s^{m}]^t$ is introduced. This operation creates a matrix W of size $(2n_1+1)\times(2n_2+1)$. The columns q_i of W are symmetric vectors, i.e. $Jw_i = w_i$ where J is the reversion matrix (with 1's on the antidiagonal and zeros elsewhere). By mapping the coefficient polynomials from T to the finite interval $[-1,1]$ via the transformation

$$x = \frac{s+s^{-1}}{2} \quad s \in T \quad , \quad x \in [-1,1]$$

it is possible to return to the original row size. We shall denote the resulting 2-D polynomial by

$$R(x,z) = W(s^{-1},s,z)_{|x=\frac{1}{2}(s+s^{-1})}$$

The above substitution is simply implemented by exploiting trigonometric relations that follows from regarding s and x as $s = e^{j\theta}$ and $x = cos\theta$ ($j=\sqrt{-1}$) and is also equivalent to replacing a series expansion in Chebyshev polynomials $T_m(x) = \frac{1}{2}[s^m + s^{-m}]$ by expansion in a power series.

Note that testing condition (ii) in Lemma 1 is equivalent to testing $R(x,z)$ for the condition

$$R(x,z) \neq 0 \quad \forall x \in [-1,1] \quad \text{and} \quad \forall z \in \bar{V}$$

Algorithm 1: Real 2-D Table.

Use $R(x,z)$ to assign to the tested polynomial a sequence of polynomials $n+1$ ($n := 2n_2$, is the column size of R) 2-D polynomials

$$E_m^{(r)}(x,z) = \sum_{k=0}^{n-m} e_{[m]\ k}^{(r)}(x) z^k\ , \quad m = 0,\ldots,n$$

Each coefficient matrix $E_m^{(r)}$ exhibits the symmetry $E_m^{(r)}J = E_m^{(r)}$, i.e., $e_{[m]\ k}^{(r)} = e_{[m]\ n-m-k}^{(r)}$ $k = 0,\ldots,n-m$

Initiation. $E_0^{(r)}(x,z) = R(x,z) + z^n R(x,z^{-1})$

$$E_1^{(r)}(x,z) = \frac{R(x,z) - z^n R(x,z^{-1})}{z-1}$$

Recursion. For $m = 0,1,\ldots,n-2$ compute:

$$zE_{m+2}^{(r)}(x,z) =$$

$$= \frac{e_{[m]\ 0}^{(r)}(x)(z+1)E_{m+1}^{(r)}(x,z) - e_{[m+1]\ 0}^{(r)}(x)E_m^{(r)}(x,z)}{\eta_{m-1}(x)}$$

where $\eta_m(x) = e_{[m]\ 0}^{(r)}(x)$ for $m \geq 1$, $\eta_m(x) = 1$ for $m < 1$.

Theorem 1. (Stability conditions for Algorithm 1) *$D(z_1,z_2)$ is stable if, and only if, the next three conditions (i) (ii) and (iii) or (iii') hold.*

(i) $D(z,1) \neq 0$ *for all* $z \in \bar{V}$
(ii) $D(1,z) \neq 0$ *for all* $z \in \bar{V}$
(iii) $\epsilon_n^{(r)}(x) \neq 0$ *for all* $x \in [-1,1]$
(iii') $\epsilon_n^{(r)}(x) > 0$ *for all* $x \in [-1,1]$
where $\epsilon_n^{(r)}(x) := E_n^{(r)}(x,1) = e_{[n]\ 0}^{(r)}(x)$.

3. EXTENDED COMPLEX TEST

The current test stems from a division-free form of the test for complex polynomials in [6] and was first presented in [2]. Define for the polynomial $D(z_1,z_2)$ the auxiliary polynomial

$$M(\tilde{s},z) := D(s^{-1},1)D(s,z) = \tilde{\mathbf{s}}^t M\mathbf{z}$$

Note that currently, condition (ii) of Lemma 1 is equivalent to

$$M(\tilde{s},z) \neq 0 \ \text{for}\ s \in T, z \in \bar{V}$$

In the forthcoming Superscript $\sharp$ to denote (conjugate) reversion, defined for a matrix and a vector, respectively, by $P^\sharp = JPJp^\sharp = Jp$ where J is the previously mentioned reversion matrix of appropriate size.

Algorithm 2: Complex 2-D Table.

Use $M(\tilde{s},z)$ to assign to the tested polynomial a sequence of $n+1$ ($n = n_2$) polynomials

$$E_m^{(c)}(\tilde{s},z) = \sum_{0}^{n-m} e_k^{(c)}(\tilde{s})z^k\ , \quad m = 0,\ldots,n$$

Each coefficient matrix $E_m^{(c)}$ exhibits the symmetry $JE_m^{(c)}J = E_m^{(c)}$, i.e., $e_{[m]\ k}^{(c)} = Je_{[m]\ n-m-k}^{(c)}$, $k = 0,\ldots,n-m$

Initiation. $E_0^{(c)}(\tilde{s},z) = M(\tilde{s},z) + M^\sharp(\tilde{s},z)$

$$E_1^{(c)}(\tilde{s},z) = \frac{M(\tilde{s},z) + M^\sharp(\tilde{s},z)}{z-1}$$

Recursion. For $m = 0,1,\ldots,n-2$ compute:

$$g_{m+1}^{(c)}(\tilde{s}) = e_{[m]\ 0}^{(c)}(\tilde{s})e_{[m+1]\ 0}^{(c)\ \sharp}(\tilde{s})$$

$$q_{m+1}^{(c)}(\tilde{s}) = e_{[m+1]\ 0}^{(c)}(\tilde{s})e_{[m+1]\ 0}^{(c)\ \sharp}(\tilde{s})$$

and $q_0^{(c)}(\tilde{s}) := E_0^{(c)}(\tilde{s},1)$.

$$zE_{m+2}^{(c)}(\tilde{s},z) =$$

$$= \frac{g_{m+1}^{(c)}(\tilde{s})E_{m+1}^{(c)}(\tilde{s},z) + g_{m+1}^{(c)\ \sharp}(\tilde{s})zE_{m+1}^{(c)}(\tilde{s},z) - q_{m+1}^{(c)}(\tilde{s})E_m^{(c)}(\tilde{s},z)}{q_m^{(c)}(\tilde{s})}$$

Theorem 2. (Stability conditions for Algorithm 2) *$D(z_1,z_2)$ is stable if, and only if, the next three conditions (i) (ii) and (iii) or (iii') hold.*

(i) $D(z,1) \neq 0$ *for all* $z \in \bar{V}$
(ii) $D(1,z) \neq 0$ *for all* $z \in \bar{V}$
(iii) $\epsilon_n^{(c)}(\tilde{s}) \neq 0$ *for all* $s \in T$
(iii') $\epsilon_n^{(c)}(\tilde{s}) > 0$ *for all* $s \in T$
where $\epsilon_n^{(c)}(\tilde{s}) = E_n^{(c)}(\tilde{s},1) = e_{[n]\ 0}^{(c)}(\tilde{s})$.

4. EXTENDED MODIFIED COMPLEX TEST

The last 2-D test for the current survey stems from a division-free form of the modified test for complex polynomials in [7] and was first presented in [3].

Algorithm 3: Modified Complex 2-D table

Assign to the tested polynomial a sequence of a sequence of $n+2$ $(n = n_2)$ polynomials

$$E_m^{(\mu)}(\bar{s}, z) = \sum_0^{n-m} e_k^{(\mu)}(\bar{s}) z^k, \quad m = -1, 0, \ldots, n$$

Each coefficient matrix $E_m^{(\mu)}$ exhibits the symmetry $JE_m^{(\mu)}J = E_m^{(\mu)}$, i.e. $e_{[m]\ k}^{(\mu)} = Je_{[m]\ n-m-k}^{(\mu)}$, $k = 0, \ldots, n-m$

Initiation. $E_{-1}^{(\mu)}(\bar{s}, z) = (z-1)(D(\bar{s}, z) - D^\sharp(\bar{s}, z))$

$$E_0^{(\mu)}(\bar{s}, z) = D(\bar{s}, z) + D^\sharp(\bar{s}, z)$$

Recursion. For $m = 0, 1, \ldots, n-1$ compute:

$$g_m^{(\mu)}(\bar{s}) = e_{[m-1]\ 0}^{(\mu)}(\bar{s}) e_{[m]\ 0}^{(\mu)\ \sharp}(\bar{s})$$

$$q_m^{(\mu)}(\bar{s}) = e_{[m]\ 0}^{(\mu)}(\bar{s}) e_{[m]\ 0}^{(\mu)\ \sharp}(\bar{s})$$

and let $q_{-1}^{(\mu)}(\bar{s}) := 1$.

$$zE_{m+1}^{(\mu)}(\bar{s}, z) =$$

$$= \frac{g_m^{(\mu)}(\bar{s}) E_m^{(\mu)}(\bar{s}, z) + g_m^{(\mu)\sharp}(\bar{s}) z E_m^{(\mu)}(\bar{s}, z) - q_m^{(\mu)}(\bar{s}) E_{m-1}^{(\mu)}(\bar{s}, z)}{q_{m-1}^{(\mu)}(\bar{s})}$$

Theorem 3. (Stability conditions for Algorithm 3) *$D(z_1, z_2)$ is stable if, and only if, the next three conditions (i) (ii) and (iii) or (iii') hold.*

(i) $D(z, 1) \neq 0$ *for all* $z \in \bar{V}$

(ii) $D(1, z) \neq 0$ *for all* $z \in \bar{V}$

(iii) $\hat{\epsilon}_n^{(\mu)}(\bar{s}) \neq 0$ *for all* $s \in T$

(iii') $\hat{\epsilon}_n^{(\mu)}(\bar{s}) > 0$ *for all* $s \in T$

where $\hat{\epsilon}_n^{(\mu)}(\bar{s}) = \frac{\epsilon_n^{(\mu)}(\bar{s})}{\epsilon_0^{(\mu)}(\bar{s})}$ *and* $\epsilon_m^{(\mu)}(\bar{s}) = E_m^{(\mu)}(\bar{s}, 1)$.

Note that $\epsilon_n^{(\mu)}(\bar{s}) = e_{[n]\ 0}^{(\mu)}(\bar{s})$.

5. NUMERICAL EXAMPLE

For illustration, consider the polynomial used as an example in several papers following [10].

$$D(z_1, z_2) = \begin{bmatrix} 1 & z_1^1 & z_1^2 \end{bmatrix} \begin{bmatrix} 0 & 0 & 0.2500 \\ 0 & 0.2500 & 0.5000 \\ 0.2500 & 0.5000 & 1.0000 \end{bmatrix} \begin{bmatrix} 1 \\ z_2^1 \\ z_2^2 \end{bmatrix}$$

For conditions (i) and (ii), $D(z,1) = D(1,z) =$ [0.2500 0.7500 1.7500]z are easily determined to be stable. Entries reflecting structural symmetry will be shown inside parentheses.

5.1. Extended Real 2-D Test

$$R = \begin{bmatrix} 0.0625 & 0.2500 & 0.6875 & 1.0000 & 0.8125 \\ 0.0000 & 0.1250 & 0.5000 & 1.1250 & 1.2500 \\ 0.0000 & 0.0000 & 0.2500 & 0.5000 & 1.0000 \end{bmatrix}$$

$$E_0^{(r)} = \begin{bmatrix} 0.8750 & 1.2500 & 1.3750 & (1.2500) & (0.8750) \\ 1.2500 & 1.2500 & 1.0000 & (1.2500) & (1.2500) \\ 1.0000 & 0.5000 & 0.5000 & (0.5000) & (1.0000) \end{bmatrix}$$

$$E_1^{(r)} = \begin{bmatrix} 0.7500 & 1.5000 & (1.5000) & (0.7500) \\ 1.2500 & 2.2500 & (2.2500) & (1.2500) \\ 1.0000 & 1.5000 & (1.5000) & (1.0000) \end{bmatrix}$$

$$E_2^{(r)} = \begin{bmatrix} 1.0312 & 1.5938 & (1.0312) \\ 3.3750 & 5.2188 & (3.3750) \\ 5.6250 & 8.2500 & (5.6250) \\ 4.7500 & 6.6250 & (4.7500) \\ 2.0000 & 2.5000 & (2.0000) \end{bmatrix}$$

$$E_3^{(r)} = \begin{bmatrix} 0.4219 & (0.4219) \\ 2.3438 & (2.3438) \\ 6.1953 & (6.1953) \\ 9.6250 & (9.6250) \\ 9.3438 & (9.3438) \\ 5.3750 & (5.3750) \\ 1.5000 & (1.5000) \end{bmatrix}$$

The computation of the next matrix involves for the first time a division of the right hand side polynomial by a (non-trivial) factor $\eta_1(x) =$ [0.7500, 1.2500, 1.0000]**x**. The first deconvolution shortens the right hand side matrix (that has $2n_2 + 1 - 4$ columns, i.e one in this example) by n_2 (=2 in this example) rows.

$E_4^{(r)}$ = [0.2637, 1.8867, 6.8486, 15.7266, 24.7578, 27.1406, 20.3281, 9.5625, 2.2500]^t

The polynomial $\mathbf{x}^t E_4^{(r)} \neq 0$ for $x \in [-1, 1]$. This may be verified numerically or algebraically, by conversion from x to s with the aforementioned mapping of [-1,1] to T then using the method in [5] to show that the resulting symmetric polynomial does not vanish on T. Thus, according to Theorem 1, $D(z_1, z_2)$ is stable.

5.2. Extended Complex 2-D Test

$$M = \begin{bmatrix} 0 & 0 & 0.4375 \\ 0 & 0.4375 & 1.0625 \\ 0.4375 & 1.0625 & 2.1875 \\ 0.1875 & 0.4375 & 0.8750 \\ 0.0625 & 0.1250 & 0.2500 \end{bmatrix}$$

$$E_0^{(c)} = \begin{bmatrix} 0.2500 & 1.2500 & (0.5000) \\ 0.8750 & 0.8750 & (1.2500) \\ 2.6250 & 2.1250 & (2.6250) \\ 1.2500 & (0.8750) & (0.8750) \\ 0.0625 & (0.1250) & (0.2500) \end{bmatrix}$$

$$E_1^{(c)} = \begin{bmatrix} 0.2500 & (0.3750) \\ 0.8750 & (1.2500) \\ 1.7500 & (1.7500) \\ 0.8750 & (0.8750) \\ 0.3750 & (0.2500) \end{bmatrix}$$

Set m=0 to compute $E_2^{(c)}$: $g_1^{(c)} = [0.0938 ,0.5469 ,2.1875 ,4.5156 ,6.7031 ,5.1406 ,2.6250 ,0.7500 ,0.1250]^t$, $q_1^{(c)} = [0.0938 ,0.5469 ,1.8594 ,3.6094 ,4.7969 ,3.6094 ,1.8594 ,0.5469 ,0.0938]^t$. Obtain the right hand side numerator polynomial's matrix coefficient that in this case the matrix has one column [0.0547, 0.4336, 2.0840, 6.3281, 14.066, 22.301, 26.447, (22.301), (14.066), (6.3281),(2.0840),(0.4336),(0.0547)]^t. Obtain $q_0^{(c)}$ by summing the columns of $E_0^{(c)}$, $q_0^{(c)}$ = [0.8750, 3.0000, 7.3750, (3.0000), (0.8750)]^t and deconvolve the numerator with $q_0^{(c)}$. The row size is reduced by 4 ($=2n_1$) and the result is $E_2^{(c)}$ = [0.0625, 0.2812, 0.8906, 1.5938, 2.0781, (1.5938), (0.8906), (0.2812), (0.0625)]^t

The corresponding symmetric polynomial $\mathbf{s}^t E_2^{(c)}$ can be tested (e.g. by [5]) to have 4 zeros in V their 4 reciprocals in U and no zeros on T. Thus $D(z_1, z_2)$ is stable by Theorem 2.

5.3. Extended Modified Complex 2-D test

$$E_{-1}^{(\mu)} = \begin{bmatrix} 1.0000 & -0.5000 & (-0.5000) & (0.0000) \\ 0.5000 & -0.5000 & (-0.5000) & (0.5000) \\ 0.0000 & -0.5000 & (-0.5000) & (1.0000) \end{bmatrix}$$

$$E_0^{(\mu)} = \begin{bmatrix} 1.0000 & 0.5000 & (0.5000) \\ 0.5000 & 0.5000 & (0.5000) \\ 0.5000 & (0.5000) & (1.0000) \end{bmatrix}$$

Set $m = 0$ to compute $E_1^{(\mu)}$. $g_0^{(\mu)} = [0.5000 ,0.7500 ,1.2500 ,0.5000 ,0.0000]^t$, $q_0^{(\mu)} = [0.5000 , 0.7500 , 1.5000 , 0.7500 , 0.5000]^t$

$$E_1^{(\mu)} = \begin{bmatrix} 0.5000 & (0.5000) \\ 1.7500 & (1.5000) \\ 4.1250 & (3.7500) \\ 4.3750 & (4.3750) \\ 3.7500 & (4.1250) \\ 1.5000 & (1.7500) \\ 0.5000 & (0.5000) \end{bmatrix}$$

Set $m = 1$ to compute $E_2^{(\mu)}$. $g_1^{(\mu)}$ = [0.5000, 1.7500, 4.7500, 7.0000, 8.1875, 6.0000, 3.4375, ,1.1250, 0.2500], $q_1^{(\mu)}$ = [0.2500 ,1.6250 ,6.5625 ,17.1250 ,33.6250 ,48.9219 ,56.0312 ,(48.9219) ,(33.6250) ,(17.1250) ,(6.5625) ,(1.6250) ,(0.2500)]. Each column in the right hand side numerator matrix is deconvolved by $q_0^{(\mu)}$. There are $n_2 - m$ columns at step m - one column currently that is given by: [0.2500, 1.6875 , 7.3750 , 21.469 , 48.102 , 82.781 , 115.02 , 127.27 , (115.02) , (82.781) , (48.102) , (21.469) , (7.3750) , (1.6875) , (0.2500)]^t. After deconvolution it becomes $E_2^{(\mu)}$ = [0.5000, 2.6250, 9.3125, 20.344, 33.312, 37.969, (33.312), (20.344), (9.3125), (2.6250), (0.5000)]^t

As the final construction step, $\hat{\epsilon}_2^{(\mu)}(\tilde{s}) = E_2^{(\mu)}(\tilde{s})/\epsilon_0^{(\mu)}(s)$ has to be computed where $\epsilon_0^{(\mu)}(\tilde{s}) = E_0^{(\mu)}(\tilde{s}, 1)$ = [2.000 , 1.500 , (2.000)]$\tilde{\mathbf{s}}$.

$\hat{\epsilon}_2^{(\mu)}$ = [0.2500, 1.1250, 3.5625, 6.3750, 8.3125, (6.3750), (3.5625), (1.1250), (0.2500)]^t

The condition $\hat{\epsilon}_2^{(\mu)}(s) = \mathbf{s}^t \hat{\epsilon}_2^{(\mu)} \neq 0 \;\forall s \in T$ may be verified algebraically (e.g. by [5]) or numerically. Therefore $D(z_1, z_2)$ is stable by Theorem 3.

6. CONCLUDING REMARKS

Three efficient algorithms and conditions for testing whether all the zeros of a two-variable polynomial reside inside the unit bi-circle were presented. A computational cost effective implementation should exploit the symmetries of the involved arrays. The second and the third algorithms are of comparable count of operations and for a polynomial of variable degrees (n_1, n_2) they end with test zeros on $|z| = 1$ of a symmetric polynomial of degree $2n_1n_2$. The first algorithm involves a double amount of iterations and ends with a positivity test on [-1,1] for a polynomial of degree $2n_1n_2$ that is mappable to testing a symmetric polynomial of degree $4n_1n_2$ for zeros on $|z| = 1$. Proofs of theorems, more stability conditions, details and improvements will be the subject of forthcoming publications.

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