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SCATTERING AND IMMITTANCE TYPE TABULAR STABILITY TESTS FOR 2-D DISCRETE-TIME SYSTEMS AND THEIR SIMPLIFICATION BY TELEPOLATION

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ABSTRACT

Two recently proposed tabular procedures for testing stability of two-dimensional (2-D) discrete linear (shift invariant, LSI) system polynomials are compared and simplified. One is an immittance-type test that associates a tested 2-D polynomial of degree (n_1, n_2) with a sequence of n_2 or n_1 centro-symmetric matrices of linearly increasing row sizes and decreasing column sizes. The second is a scattering-type tabular test that builds a sequence of similarly sized matrices without structural symmetry. The two tabular tests are of $O(n^6)$ complexity (say $n = n_1 = n_2$) where the symmetries of the matrices makes the immittance test more efficient. Simplification of the tabular tests is achieved by *telepolation* - *tele*scoping the last polynomial of the table using inter*polation*. The telepolation approach reveals that the testing stability of a 2-D system can be carried out by a finite collection of designated 1-D stability tests. It also produces testing procedures of apparently unprecedentedly low $O(n^4)$ complexity. The resulting immittance form test can be carried out by less than $4n^4$ real arithmetic operations and it is again more efficient than the corresponding scattering version.

1. INTRODUCTION

Stability is an important issue in the design and analysis of multidimensional systems. Developing methods to determining stability of multidimensional (n-D) systems has been an active research area in the last three decades. This paper describe two recent tabular stability tests for two-dimensional (2-D) discrete-system polynomials and their further simplification by interpolation. Specifically, it deals with procedures to solve the following problem.

Problem statement. Given a bivariate (two-dimensional,2-D) polynomial

$$D(z_1, z_2) = [1, z_1, \ldots, z_1^{n_1}]D[1, z_2, \ldots, z_2^{n_2}]^t \qquad (1)$$

of degree (n_1, n_2), where $D = (d_{i,k})$ is a real coefficient matrix, determine whether it does not vanish in the closed exterior of the unit bi-circle, viz.,

$$D(z_1, z_2) \neq 0 \,, \forall (z_1, z_2) \in \bar{V} \times \bar{V}. \qquad (2)$$

where $T = \{z : |z| = 1\}$, $V = \{z : |z| > 1\}$, and $\bar{V} = V \cup T$. A $D(z_1, z_2)$ that satisfies (2) will be called *stable*. (Attention is drawn to that several other notations for defining stability in the literature exist and may require a reversion of the coefficient matrix to conform with the present convention cf. [1].) Similarly, a 1-D (real or complex) polynomial $p(z)$ such that

$$p(z) = [1, z, \ldots, z^n][p_0, \ldots, p_n]^t \neq 0 \qquad \forall z \in \bar{V} \qquad (3)$$

will too be called stable. The testing of the condition (2) is the key problem in testing stability of 2-D discrete systems. The similarity in the definition of 1-D and 2-D stable polynomials is somewhat misleading in that it hides several difficulties and differences between stability of 1-D and 2-D systems. Comprehensive background on multidimensional linear discrete-time systems with emphasis on the stability is available in [2] [3].

This paper considers two recent tabular tests that solve the stated problem in only $O(n^6)$ (say, $n_1 = n_2 = n$) complexity and shows alternative procedures that reduces for each of them the overall complexity to $O(n^4)$. The first tabular test is a scattering-type 2-D stability test proposed by Hu and Jury [4]. The second tabular test is the immittance-type tabular test in [5]. The terms *immittance* and *scattering* are used to distinguish algorithms that stem from the zero location test formulation of Bistritz in [6][7] from the Schur-Cohn test and several other related classical algorithms (see [8] and other references therein). The immittance formulation replaces two-term recursions in the classical algorithms by three-term recursions and obtain algorithms of improved efficiency that exploit intrinsic symmetry in the problems. In the current context, the scattering tabular test involves a two-term recursion of 2-D polynomials (or of matrices) with no particular structure whereas the immittance tabular test uses a three-term recursion to propagate 2-D polynomials (or matrices) with a special symmetry (the coefficient matrices are centro-symmetric). A count of operations for the 2-D stability test reveals that they are both $O(n^6)$ complexity ($n = n_1 = n_2$) - a definite advantage over previous 2-D tabular stability tests that until very recently used to be of severe exponential complexity. Examination of the accompanying stability conditions reveals that, in each case, the table is constructed merely to reach its last entry - presented by a symmetric polynomial of degree $2n_1n_2$ that has then to be tested for no zeros on the unit-circle. This last task can be carried out in $O(n^4)$ ($n_1 = n_2 = n$) operations. Nevertheless an overall $O(n^6)$ complexity is dictated by the higher cost of the table's construction. The paper shows that it is possible to circumvent the table's construction and replace it by a finite number of low degree 1-D stability tests from which the last entry of the table can be recovered by an efficient interpolation formula. We call this approach *telepolation*, standing for *tele*scoping (the last entry of the sequence) by inter*polation*. The resulting procedures have a very (apparently unprecedentedly) low overall $O(n^4)$ complexity.

2. TABULAR 2-D STABILITY TESTS

The notation in this paper follows that in [1] and related works. It associates a 2-D polynomial as in (1) with the matrix D of its coefficients, by $D(z_1, z_2) = \mathbf{z_1}^t D\mathbf{z_2}$. The vector $\mathbf{z} := [1, z, \ldots, z^i, \ldots]^t$ is of length depending on context. This notations is also used to associate vectors p with a 1-D polynomials $p(s) = \mathbf{s}^t p$. We also denote $D^\sharp := JD^\star J$ for a matrix and $e^\sharp_k := Je^\star_k$ for a vector, where J is the reversion matrix and $\star$ denotes complex conjugate. A vector may also be associated with a 'balanced polynomial' $p(\tilde{s}) := \tilde{\mathbf{s}}^t p = s^{-m/2}p(s)$ where m is the degree of $p(s)$ and $\tilde{\mathbf{s}} = [s^{-m/2}, \ldots, 1, \ldots, s^{m/2}]^t$. Balanced polynomials may also be associated with the columns of matrices, for example $D(\tilde{s}, z)$ stands for $\tilde{\mathbf{s}}^t D\mathbf{z}$ where the length of $\tilde{\mathbf{s}}$ is compatible with the row size of the matrix. A polynomial such that $p^\sharp(z) =: \mathbf{s}^t p^\sharp = p(z)$, and a vector such that $p^\sharp = p$ are called symmetric. Similarly a 2-D polynomial such that $D^\sharp(z_1, z_2) := \mathbf{z_1}^t D^\sharp \mathbf{z_2} = D(z_1, z_2)$ and a matrix such that $D^\sharp = D$ are called centro-symmetric.

2.1. An Immittance tabular 2-D stability test

The next 2-D tabular test is based on the original form of the author's 1-D stability test [6, 7].

Algorithm 1, [9][5]. Construct for $D(z_1, z_2)$ the sequence of polynomials. $\{E_m(\tilde{s}, z) = \sum_{k=0}^{n-m} e_{[m]\ k}(\tilde{s})z^k, m = 0, 1, \ldots, n\ (= n_2)$ using the following recursion.

$$M(\tilde{s}, z) = D(s^{-1}, 1)D(s, z)$$

$$E_0(\tilde{s}, z) = M(\tilde{s}, z) + M^\sharp(\tilde{s}, z)$$

$$E_1(\tilde{s}, z) = \frac{M(\tilde{s}, z) - M^\sharp(\tilde{s}, z)}{z - 1}$$

$$q_0(\tilde{s}) = E_0(\tilde{s}, 1)$$

For $m = 1, \ldots, n - 1$ obtain $E_{m+1}(\tilde{s}, z)$:

$$g_m(\tilde{s}) = e_{[m-1]\ 0}(\tilde{s})e^\sharp_{[m]\ 0}(\tilde{s})$$

$$q_m(\tilde{s}) = e_{[m]\ 0}(\tilde{s})e^\sharp_{[m]\ 0}(\tilde{s})$$

$$zE_{m+1}(\tilde{s}, z) = \frac{g_m(\tilde{s})E_m(\tilde{s}, z) + g^\sharp_m(\tilde{s})zE_m(\tilde{s}, z) - q_m(\tilde{s})E_{m-1}(\tilde{s}, z)}{q_{m-1}(\tilde{s})}$$

The first variable in the 2-D polynomials is distinguished by s and will usually be interpreted most as taking values $s \in T$. It is possible to replace everywhere in the algorithm the balanced variable $\tilde{s}$ by s and have normal 2-D and 1-D polynomials e.g., $E(s, z) = \mathbf{s}^t E_m\mathbf{z}$, $q_m(s) = \mathbf{s}^t q_m$. The choice $E(\tilde{s}, z) = \tilde{\mathbf{s}}^t E_m\mathbf{z}$ makes more transparent in the forthcoming linking of the tabular algorithms with 1-D polynomial recursions that reversion of rows may be presented by complex conjugation (for $s \in T$). It is also possible to drop variables altogether and regard the algorithm as operating on vectors and matrices where multiplication / division between a vector and a matrix mean convolution/deconvolution between the vector and each column of the matrix. The latter view may be the most transparent for programming these tabular tests.

The $q_m(s)$ represents a factor common to all the coefficients in the numerator that is eliminated by dividing it out. This elimination reduces drastically (from exponential to linear growth) the row sizes of the E_m and improves the efficiency of the algorithm. The degree of $E_m(s, z)$ is $(2mn_1, n_2 - m)$ for $m \geq 1$, and the matrices E_m are all centro-symmetric, $E^\sharp_m = E_m$. It therefore suffices to calculate only half of their entries. The more effective approach (less computation and better accuracy) is to compute the upper half of the rows of each E_m (rather than half of the columns) [5].

Theorem 1, [5]. *Assume Algorithm 1 is applied to $D(z_1, z_2)$. $D(z_1, z_2)$ is stable if, and only if, the following three conditions hold.*

(i) $D(z, 1) \neq 0$ *for all* $z \in \bar{V}$

(ii) $D(1, z) \neq 0$ *for all* $z \in \bar{V}$

(iii) $\epsilon(s) := \mathbf{s}^t E_n \neq 0$ *for all* $s \in T$

2.2. A scattering tabular 2-D stability test

The next tabular stability test has been obtained by Hu and Jury [4]. It originates from the modified Jury test for 1-D stability [10] and it is brought here in the version we derived for it in [11].

Algorithm 2. Assign to $D(z_1, z_2) = [d_0(z_1), \ldots, d_n(z_1)]\mathbf{z_2}$, $n := n_2$, a sequence of polynomials $\{C_m(s, z) = \sum_{k=0}^{m} c_{[m]k}(s)z^k$, $m = n - 1, \ldots, 0\}$, , using the following recursions.

$$zC_{n-1}(s, z) = d^\sharp_n(s)D(s, z) - d_0(s)D^\sharp(s, z),\ q_{n-1}(s) = 1$$

For $m = n - 1, \ldots, 1$ do:

$$zC_{m-1}(s, z) = \frac{c_{[m]m}(s)C_m(s, z) - c_{[m]0}(s)C^\sharp_m(s, z)}{q_m(s)}$$

$$q_{m-1}(s) = c_{[m]m}(s)$$

The division by the $q_m(s)$ is again exact. Namely, $q_m(s)$ divides without remainder the numerator 2-D polynomial. The polynomials $C_m(s, z)$, $m = n_2 - 1, \ldots, 0$, are of degrees $(2(n_2 - m)n_1, m)$. Again, the algorithm may be equally presented with variable s replaced by the balanced variable $\tilde{s}$, or as recursion of arrays with convolution and deconvolution replacing multiplication and division of polynomials.

Theorem 2, [11]. *Assume Algorithm 2 is applied to $D(z_1, z_2)$. $D(z_1, z_2)$ is stable if, and only if, the following three conditions hold.*

(i) $D(z, 1) \neq 0\ \forall z \in \bar{V}$

(ii) $D(1, z) \neq 0\ \forall z \in \bar{V}$

(iii) $\epsilon(s) := C_0(s, z) \neq 0 \quad \forall s \in T$

2.3. Evaluation

It is apparent that both tabular tests serve to derive a target polynomial $\epsilon(s)$ that has to be examined for having no zeros on T. The polynomial $\epsilon(s) = \mathbf{s}^t\epsilon$ is a symmetric real polynomial of even degree $2M$, $M := n_1n_2$. It follows that $\epsilon(\tilde{s}) := s^{-M}\epsilon(s)$ is real for $s \in T$. In fact, the conditions (iii) in Theorems 1 and 2 can be replaced by the positivity condition

$$\epsilon(\tilde{s}) > 0 \quad \forall s \in T$$

because in the context of these theorems conditions (ii) holds only if at $s = 1$ $\epsilon(\tilde{s}) > 0$.

The immittance 2-D stability test requires approximately $\frac{5}{6}n_1^2n_2^4$ $+O(n_{1,2}^5)$ flops, [5], where here and on $+O(n_{1,2}^k)$ is used to denote that other additive terms with powers $n_1^{\alpha_1}n_2^{\alpha_2}$ such that $\alpha_1 + \alpha_2 \leq k$ are neglected, . The count of operations for the scattering tabular test is approximately $\frac{4}{3}n_1^2n_2^4 + O(n_{1,2}^5)$, [11]. An exact count of operations for the two tests shows that the cost ratio is higher than the asymptotic factor of 1.6 for all degrees of interest. (For $n = n_1 = n_2$, the ratio factor is greater than 2 for $3 \leq n \leq 10$.)

Theorems 1 and 2 reveal that the only role that the construction of the tables serves is to obtain $\epsilon(s)$, its last entry. The testing of the condition $\epsilon(s) \neq 0 \; \forall s \in T$ can be carried out in $O(n^4)$ ($n = n_1 = n_2$) by any zero location procedure that handles also possible singular cases. (Least count of only $n_1^2 n_2^2$ real multiplications is attainable by using the zero location method in [6]). The higher overall cost of computation is caused by the computation required to construct the 'tables', i.e. the sequences $\{E_m\}$ and $\{C_m\}$.

The remaining of the paper shows that it is possible to maintain an overall $O(n^4)$ complexity by *telepolation* of $\epsilon(s)$. Namely, it is possible to *tele*scope the last entry of these 2-D tables by inter*polation* without their full construction. The next section shows how sample values of $\epsilon(\tilde{s})$ at any desirable $s_o \in T$ can be obtained by certain companion 1-D stability tests. Afterwards, an efficient formula will be brought to recover $\epsilon(\tilde{s})$ from a sufficient set of sample values.

3. COMPANION 1-D STABILITY TESTS

This section brings for each of the above two 2-D polynomial algorithms, a companion 1-D polynomial algorithm that follow its action on $D(\tilde{s}_o, z)$ at a fixed point $s_o \in T$. Each such "projection" of a 2-D polynomials algorithm on a 1-D polynomials algorithm may be used to obtain samples of $\epsilon(\tilde{s})$. Each 1-D polynomial algorithm is turned into a stability test by coupling it with necessary and sufficient conditions for 1-D stability.

3.1. Immittance companion 1-D stability test

Algorithm 1-C, [12]. Assume $p(z)$ (3) with complex coefficients and that $p(1) \neq 0$. Form $\hat{p}(z) = p(1)^\star p(z)$ and construct the next sequence $\{e_m(z), m = 0, 1, \ldots n\}$, of (conjugate) symmetric polynomials $e_m(z) = \sum_{i=0}^{n-m} e_{m,i} z^i$ ($Je_m = e_m^\star$).

$$e_0(z) = \hat{p}(z) + \hat{p}^\sharp(z)$$

$$e_1(z) = \frac{\hat{p}(z) - \hat{p}^\sharp(z)}{(z-1)} \quad , \quad q_0 = 2|p(1)|^2$$

For $m = 1, \ldots, n-1$ obtain $e_{m+1}(z)$:

$$g_m = e_{m-1,0} e_{m,0}^\star \quad , \quad q_m = |e_{m,0}|^2$$

$$z e_{m+1} = \frac{(g_m + g_m^\star z) e_m(z) - q_m e_{m-1}(z)}{q_{m-1}}$$

The requirement in the algorithm that $p(1) \neq 0$ is guaranteed in the forthcoming application.

Algorithm 1-C turns into a stability test for $p(z)$ in conjunction with the next theorem.

Theorem 1-C, [12]. *Assume Algorithm 1-C is applied to a 1-D polynomial $p(z)$ of degree n.*
(a) *$p(z)$ is stable if, and only if, $e_m(1) > 0$, $m = 0, 1, \ldots, n$ where $\{e_m(z)\}$ are obtained by Algorithm 1-C.*
(b) *If $e_{m,0} = 0$ then $p(z)$ is not stable.*

Furthermore, it can be shown that applying Algorithm 1-C to $p(z) = D(s_o, z)$ represents the action of Algorithm 1 on $D(s, z)$ at $s = s_o$ for any fixed $s_o \in T$. In particular, it follows that if Algorithm 1-C is applied to $p(z) = D(\tilde{s}_o, z)$ then its last entry e_n is equal to the value of $\epsilon(\tilde{s}) = \tilde{\mathbf{s}}^t E_n$ at s_o [12].

3.2. Scattering companion 1-D stability test

Algorithm 2-C. Assign to a polynomial $p(z)$ (3), with complex coefficients, a sequence of polynomials $\{c_m(z), m = n-1, \ldots, 0\}$, where $c_m(z) = \sum_{i=0}^{m} c_{m,i} z^i$ as follows.

$$z c_{n-1}(z) = p_n^\star p(z) - p_0 p^\sharp(z) \; ; \; q_{n-1} = 1$$

For $m = n-1, \ldots, 1$ do:

$$z c_{m-1}(z) = \frac{c_{m,m} c_m(z) - c_{m,0} c_m^\sharp(z)}{q_m} \; ; \; q_{m-1} = c_{m,m}$$

Theorem 2-C [13]. *Assume Algorithm 2-C is applied to $p(z)$. $p(z)$ is stable if, and only if,*

$$c_{m,m} > 0 \quad m = n-1, \ldots, 0$$

For any fixed $s_o \in T$ applying Algorithm 2-C to $p(z) = D(\tilde{s}_o, z)$ (or $p(z) = D(s, z)$) produces the action of Algorithm 2 on $D(\tilde{s}, z)$ at $s = s_o$. Most importantly, $c_{0,0} = \epsilon(\tilde{s}_o)$, [11].

The above stability test is the main form for type-C tests in the classification of the Schur-Cohn-Marden-Jury tests into four classes in [13]. Type-C tests were first obtained by Jury in several occasions including [10] as a modification for the Schur-Cohn-Marden stability tests that has a more direct relation with the principal minors of the Schur-Cohn Bezoutian. A proof of these relations and generalization to zero location is available in [13]. In particular, these relations can be used to prove that $\epsilon(s)$ forms the determinant of the Schur-Cohn Bezoutian of the polynomial $p(z) = D(s, z)$.

4. THE INTERPOLATION PROBLEM

Assume $\epsilon(s) = [\epsilon_0, \ldots, \epsilon_{2M}]$**s** is a symmetric real polynomial. Let

$$\theta = \frac{2\pi}{2M+1} \quad , \quad w = e^{j\theta}$$

where $j = \sqrt{-1}$. Assume values of $\epsilon(\tilde{s})$ are at $M+1$ points on the unit circle as follows

$$b_i = \epsilon(\tilde{s}_i) \quad , \quad s_i = w^{-M+i} \quad , \quad i = 0, 1, \ldots, M \, .$$

Then $\epsilon(s)$ can be determined from them by next expression, [12][11].

$$\epsilon_{M-m} = \frac{b_M + 2\sum_{k=1}^{M} b_{M-k} \cos(mk\theta)}{2M+1} \, , \, m = 0, \ldots, M \, ,$$

$$\epsilon_{M+m} = \epsilon_{M-m} \qquad , \quad m = 1, \ldots, M \, . \tag{4}$$

(Of course, the division by $2M+1$ is not required for the purpose of examining positivity on T of $\epsilon(\tilde{s})$.)

5. TELEPOLATED 2-D STABILITY TEST PROCEDURES

The simplified procedures for testing a 2-D polynomial (1) for the stability condition (2) combines the results brought so far and is outlined (in parallel for brevity) in the next four steps procedure. ('exit' marks a point of termination with conclusion that $D(z_1, z_2)$ is not stable.)

Step 1. Test stability of $D(z, 1)$ is 1-D stable. If not stable - 'exit'.

Step 2. Set $M = n_1 n_2$, $\theta = \frac{2\pi}{2M+1}$, $w = e^{j\theta}$.

For $i = 0, 1, \ldots, M$ do: Set $s_i = w^{-M+i}$.

[**either: immittance form**] Apply to $p_{s_i}(z) = D(s_i, z)$ the 1-D stability test of Algorithm 1-C and Theorem 1-C. If not stable (as soon as a $e_{m,0} = 0$ or a $e_m(1) \leq 0$ is detected)- 'exit'. Otherwise, retain $e_n(> 0)$ as $b_i := e_n$.

[**or: scattering form**] Apply to $p_{s_i}(z) = D(s_i, z)$ the 1-D stability test of Algorithm 2-C and Theorem 2-C. If not stable (as soon as a $c_{m,m} \leq 0$ is detected) - 'exit'. Otherwise, retain the last element $c_{0,0}(> 0)$ as $b_i := c_{0,0}$.

Step 3. Obtain $\epsilon(s)$ from the values b_i $i = 0, \ldots, M$, using (4).

Step 4. Examine the condition "$\epsilon(s) \neq 0 \; \forall s \in T$". $D(z_1, z_2)$ is stable if and only if this condition is true and the current step has been reached without an earlier 'exit'.

Next is a brief account of the approximate complexity of these procedures. It counts multiplication of two complex numbers as four real multiplications and two real additions and a real times complex numbers as two real multiplications. The immittance-type procedure requires $2n_1^2n_2^2$ + $1.5n_1n_2^3$ real multiplications and $3n_1^2n_2^2$ + $n_1n_2^3$ real additions +$O(n_{1,2}^3)$ [12] [15]. It assumes that in step 2 Algorithm 1-C is performed using two multipliers (g_m/q_{m-1} and g_m/q_{m-1}) per recursion step (rather than three) and that step 4 uses the method of least count of operations for the task, the 1-D zero location test in [6]. (A similar reduction in computation is possible by telepolation for also the immittance tabular test in [1], [15].) The complexity of the scattering-type procedure is $3n_1^2n_2^2$ + $3n_1n_2^3$ +$O(n_{1,2}^3)$ real operations and an equal number of real additions. It assumes that in step 2 Algorithm 2-C uses two multipliers per recursion step ($c_{mm}/c_{m-1,m-1}$ and $c_{m0}/c_{m-1,m-1}$) and that the condition in step 4 is tested by Raible's test [14] - the most efficient test in the Schur-Cohn Marden-Jury class of tests [13]. Of course, step 4 may be carried out too by [6] independently of using Algorithm 2-C for step 2. Note that it is always possible, to apply the tests to 2-D polynomial with transposed coefficient matrix ($D \to D^t$). According to the expressions for the count of operations this transposition should precede the procedure when $n_1 < n_2$.

6. CONCLUDING REMARKS

The paper presented procedures to test stability of discrete-time (LSI) two-dimensional systems. The two procedures require very (apparently unprecedentedly) low amount of computation (culminating in the immittance form). The reduced complexity is achieved by telepolation of recent tabular tests. It profits on the already advanced features of these tabular test in terms of efficiency (previously available 2-D tabular tests were of severe exponential complexity) and their simple stability condition (a single positivity test).

The new procedures reveal that testing the stability of a polynomial of degree (n_1, n_2) can be carried out by a collection of (properly designed) n_1n_2+1 1-D stability tests of degree n_1 or n_2 plus one zero location test of a 1-D polynomial of degree $2n_1n_2$. This establishes an interesting feature that was not known before in the theory of multidimensional system. This property also makes the programming of the method very easy. It also minimizes the effort wasted on an unstable polynomial because the procedure is passes a densely paved path of necessary conditions for 1-D stability that are also necessary condition for 2-D stability.

Since the major gap in the mathematics for n-D systems occurs upon moving from 1-D to 2-D systems, the extension of the current approach to stability testing of higher dimensional systems poses no conceptual obstacles. The generalization of the presented tabular tests and their telepolation to 2-D polynomial with a complex valued coefficient matrix is also possible and will be presented in forthcoming papers.

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