Pulse dispersion in hollow optical waveguides

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Abstract. A study of laser (near- and mid-infrared) pulse dispersion in hollow waveguides is presented. We developed an analytical model to describe the pulse dispersion in hollow waveguides and compared our theoretical calculations with measurements done by us and also by two other groups. The pulse dispersion was experimentally measured for a short Q-switched Er:YAG laser in the nanosecond range and for femtosecond Ti:sapphire laser pulses transmitted by hollow optical waveguides. For analytical calculation of the pulse dispersion in these waveguides, a refined ray tracing program was developed. This approach took into account roughness of the internal reflecting and refracting inner layers. A comparison analysis between the measurements and calculations conducted at identical parameters demonstrates good correlation between theoretical and experimental results. © 2005 Society of Photo-Optical Instrumentation Engineers. [DOI: 10.1117/1.2042967]

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1 Introduction

The basic theory of optical waveguides was first reported by Marcatili and Schmeltzer in 1964.1 About 15 years later, metal waveguides for IR were demonstrated by Garmire et al.2 Since then, extensive research efforts have been focused on developing and refining them.

Hollow waveguides are made of tubes that are internally coated by thin metal and dielectric films. The choice of the tube and the thin-layer materials depends on the application and coating technique. Figure 1 shows the cross section of a typical hollow waveguide.

These waveguides are able to transmit high-power laser radiation. Their flexibility enables transmission of energy to a remote location or closed cavity over curved paths. A host of data exist on the characterization of these waveguides, including bending attenuation losses, beam shape analysis, spectral range, peak energy transmission capabilities, and pulse transmission from femtosecond to continuous wave (cw).3–5

In this paper, an alternative study of short-laser-pulse transmission in hollow optical waveguides and of pulse broadening is presented. The analysis is directed toward better understanding the use of these waveguides for laser-tissue interaction and for optical and thermal data transmission and collection.

A theoretical analysis based on the ray tracing model6 for calculating the pulse dispersion in hollow optical waveguides, as well as experimental results on pulse dispersion that correspond to these calculations, is described. Although it is customary to describe the pulse dispersion using the mode approach (solving Maxwell’s equations for the appropriate waveguide and analyzing the propagation constants of each mode as it passes through the waveguide7), this model is simpler, is intuitive, and takes into account the multimode nature of the laser beam and different waveguide phenomena like bending and surface roughness, which are difficult to take into account using other approaches.

Fig. 1 Cross section of a hollow waveguide. Black: tube (fused silica, plastic, etc.); gray: metal layer (Ag, Au, Ni, etc.); light gray: dielectric layer (AgI, ZnS, ZnSe, etc.).
2 Theory

Different light components (modes or wavelengths) travel through the waveguide at different velocities and by different optical paths. Therefore, laser pulses that travel along the fiber may become longer by a certain amount of time, denoted as $\Delta \tau$. This phenomenon is well known as pulse dispersion.8

Hollow waveguides are multimode fibers. Thus, the dominant dispersion mechanism in them is modal or intermodal dispersion (different modes travel along different optical paths because of the different incidence and reflectance angles at the reflecting and refracting layers).

2.1 Pulse Dispersion in Straight, Smooth Multimode Fibers

Different modes travel along different optical paths through the fiber, and the result is a modal dispersion that can be evaluated using Fig. 2.8

The shortest path of a traveling mode in Fig. 1 corresponds to ray 0, which travels directly to the destination along the fiber axis. The travel time for that mode is

$$\tau_{\text{fast}} = \frac{L n_1}{c},$$

where $L$ is the fiber length, and $n_1$ is the refractive index of the core.

The longest path of a traveling mode corresponds to ray 2, which travels with the maximum angle $\theta_{\text{max}}$ with respect to the fiber axis. [This angle is limited by the numerical aperture (NA) of the fiber. In a hollow waveguide there is no real NA, but a limiting input angle, which corresponds to a full angle of 25 deg or an NA of 0.22. A larger NA is possible, but will cause the attenuation to increase.] The travel time for this mode is

$$\tau_{\text{slow}} = \frac{L n_1}{c \cos \theta_{\text{max}}}.\quad (2)$$

The modal dispersion is obtained by subtracting Eq. (1) from Eq. (2):

$$\Delta \tau = \frac{L n_1}{c} \left( \frac{1}{\cos \theta_{\text{max}}} - 1 \right). \quad (3)$$

As can be seen from Eq. (3), the pulse dispersion depends on the fiber’s material properties, its length, and the incidence angle of the input ray.

2.2 Pulse Dispersion in Real Hollow Waveguides

The preceding calculation is useful only for straight, smooth waveguides. Real hollow waveguides, however, are bent and have a certain amount of surface roughness. Bending and roughness cause the incidence angle to change and thus cause a coupling between different laser modes, which leads to an increase in pulse dispersion.

In order to calculate the real pulse dispersion under these conditions, the ray model9 can be used. According to the ray model,8 one can decompose the laser beam into separate rays and use the laws of geometrical optics to calculate their propagation through the waveguide. This model may be used in this case because $\lambda \ll \text{ID}$, where $\lambda$ is the wavelength of the coupled laser beam and ID is the inner diameter of the waveguide cross section. Multiple incidences on the metal and dielectric layers are assumed to guide the rays, by refraction and reflection. The dielectric layer has been measured experimentally to have a normal distribution of thicknesses.9 The following assumptions were used in order to model the propagation of the laser beam through the hollow fiber:

1. Fresnel’s equations give the reflection coefficient after each time the ray impinges the waveguide’s wall.
2. The rays only propagate frontally and not rotationally (there are no skewed rays). According to Miyagi et al.,10 the contribution of skew rays is of the second order and can be neglected.
3. Two coordinates determine the laser beam cross section and the point where the ray enters the waveguide: $r$ (with Gaussian distribution) and $\theta$ (with uniform distribution) [Fig. 3(a)], where $0 \leq r \leq R$ ($R$ is the laser spot size) and $0 \leq \theta \leq 2\pi$. The angle $\theta$ also determines the plane in which the ray propagates. (b) The entrance angle $\varphi$ has a Gaussian distribution. This angle determines the angle of propagation, $\phi$, by $\phi = 90\,\text{deg} - \varphi$. 

![Fig. 2 Different ray paths in a section of hollow waveguide.](image)

![Fig. 3 The ray parameters: (a) Two coordinates determine the laser beam cross section and the point where the ray enters the waveguide: $r$ (with Gaussian distribution) and $\theta$ (with uniform distribution), where $0 \leq r \leq R$ ($R$ is the laser spot size) and $0 \leq \theta \leq 2\pi$. The angle $\theta$ also determines the plane in which the ray propagates. (b) The entrance angle $\varphi$ has a Gaussian distribution. This angle determines the angle of propagation, $\phi$, by $\phi = 90\,\text{deg} - \varphi$.](image)
distribution. This angle determines the angle of propagation, \( \phi \), by \( \phi = 90 \, \text{deg} - \varphi \).

5. The rays have random polarization, TE or TM.

6. The total energy of the laser beam, \( I \), is the sum of the energies of all rays, and is given by

\[
I = \sum_i I_i(\sigma_r, \sigma_{\varphi}, r, \varphi_i),
\]

where \( \sigma_r \) and \( \sigma_{\varphi} \) are the standard deviations of the Gaussian beam size and angle, respectively.

7. The surface of the dielectric layer is rough.

8. The roughness centers are distributed randomly.

9. The scattering is produced only on the surface of the incident layer and not inside the AgI layer, since the AgI layer is more granular than the Ag layer, and hence its roughness is greater.

10. The scattered energy is taken only in the positive direction; scattered energy in the negative direction is assumed lost. The scattering coefficient \( S \) is given by

\[
S = \frac{\sum_{\varphi = 90 \, \text{deg}}^{\varphi = -90 \, \text{deg}} S(\varphi)}{\sum_{\varphi = 90 \, \text{deg}}^{\varphi = -90 \, \text{deg}} S(\varphi)}.
\]

11. The scattering of a ray-changes the ray’s angle of propagation. The new angle is the average angle of the scattered energy.

12. The laser beam is decomposed to a minimum of \( 10^5 \) rays.

Using these assumptions, one can calculate the pulse dispersion, the attenuation, and the beam shape outside the waveguide, as functions of the waveguide’s parameters (length and inner diameter) and the coupling conditions (focal length of the coupling lens and off-center displacement of the ray).

The described ray model calculates the new angle of incidence after each time a ray impinges on the waveguide’s wall, taking into account the roughness of the surface, which causes scattering of the impinging ray. Knowing the correct angle of incidence enables the calculation of the ray trajectory along the waveguide and from it, and of the time it takes the ray to exit at the distal tip. The pulse dispersion is then given by

\[
\Delta \tau = \tau_{\text{slow}} - \tau_{\text{fast}}.
\]

Surface roughness influences the ray propagation in hollow waveguides in two ways. The first is increasing the attenuation by backscattering of some of the laser radiation, and the second is by coupling low propagation modes to higher ones by changing the angle of propagation of the mode, since the pulse dispersion depends on the propagation angle. Figure 4 shows the pulse dispersion as a function of the surface roughness. As can be seen, by increasing the surface roughness we increase the pulse dispersion.

The following values were used in the calculations: waveguide length 1 m, bore diameter 0.5 mm. Laser parameters: pulse width 150 fs, \( \lambda = 800 \) nm, and coupling-lens focal length 100 mm.

3 Experimental Setup

There are a few basic methods for measuring a short laser pulse width. One is to use an autocorrelator, where the input beam is split before entering the waveguide and then compared with the output beam using an autocorrelator (Positive Light, Los Gatos, CA). This method was used by Nguyen et al. for studying femtosecond dispersion in waveguides. Another method was suggested by Amirmadhi et al. and used by Pratisto et al. to measure pulse dispersion in waveguides transmitting picosecond pulses from a free-electron laser (FEL). The method is based on two-photon absorption of radiation in Ge, InAs, and Te semiconductors. For nanosecond-range pulses from a Q-switched Er-YAG laser, the dispersion can be calculated by measuring the pulse widths at the waveguide input and at the waveguide output. The pulse dispersion is the difference between them. This method was selected, since theoretical calculations showed that the pulse dispersion would be on the order of nanoseconds, the magnitude of the width of the pulse itself.

Figure 5 shows the experimental setup that was used to measure the pulse width of the laser beam after propagating through the waveguide. For the short-pulse IR laser source, a 70-ns-pulse-width 2.94-\( \mu \)m-wavelength Q-switched Er:YAG laser (MSQ, Israel) was used. The test hollow optical waveguide was manufactured by our group. It was a 2-m-long fused-silica waveguide with inner diameter 0.7 mm. The waveguide was bent at a constant radius (8 cm) at different bending angles, i.e., was turned several times at constant radius. For each bending, the pulse width was measured using a fast IR detector (VIGO, Boston Elec-
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1. The funnel ensured mode homogenizing of the input laser beam distribution, which was converted into a high-quality smooth profile at the funnel output. Figure 6(a) and 6(b) show the energy distribution before and after the funnel, respectively. These measurements were gathered with a beam shape analyzer (Pyrocam by Spiricon, Logan, Utah).

2. The second function of the funnel was to decrease the NA of the output laser beam, because the uncoated hollow taper typically had a low NA due to the direct lens-free laser-to-taper coupling, the lack of sharp focusing, and the formation of a smooth output laser beam profile. The funnel had a low NA of 0.033, corresponding to a very small full angle of divergence of 3.8 deg, measured using far-field intensity distributions.16

3. The funnel provided conditions for achievement of a so-called steady-state or equilibrium-mode distribution (EMD) state of optical power within the test hollow waveguide.16,17 One of the most important aspects of accurate and repeatable pulse dispersion measurement in multimode fibers is the dependence of the performance on the modal power distribution of the input laser beam, since different modes within the fiber have different pulse dispersion—as, for example, happens when a lens-based laser-to-fiber coupling is used, which technique provides sharp focusing of the strongly multimode input laser beam profile, [see Fig. 6(a)] and exciting of high-order modes in the test fiber. At the EMD state, the relative power in the various modes is independent of the fiber length, that is, the mode power distribution at the output is independent of the distribution at the input. One possible method for achievement of an EMD in multimode fibers is to create launch conditions for a limited launch NA, which means in practice creating conditions for excitation predominantly of low-order axial modes rather than of high-order, leaky, and cladding modes.16,17 In our experiment, the hollow-taper-based coupling method ensured perfect conditions within the test hollow waveguide for an EMD regime achieved by a limited launch NA, because the tapered funnel provided a smooth Gaussian-shaped profile of the output laser emission, and it possessed a low output NA and a smaller core diameter than the test fiber core diameter. Thus, in pulse dispersion measurements, the input pulse shape and mode distribution were kept constant (or in an EMD state) during the whole measurement.

4 Results and Discussion

The pulse dispersion and waveguide’s attenuation was measured at various bending angles. The waveguide’s attenuation was found to be 1 dB per 90 deg of bending. This value was obtained using the theoretical model. Figure 7 shows the experimental results (points), the best-fit curve (dotted line), and the theoretical calculations (solid line). In order to calculate the pulse dispersion of the waveguide using the ray model, we assumed the surface roughness was 24 nm.18 This value was measured several times in the past for different fused-silica hollow waveguides using an atomic force microscope. As can be seen, there is good correlation between the experimental and theoretical results (the calculated correlation between the curve fit and the theoretical calculations is 0.92).

Figure 7 also shows that the pulse dispersion increases as the angle of bending increases. This is due to mode coupling between lower modes of propagation and higher ones. The mode coupling is caused by two mechanisms. The first is scattering from a rough surface. Each time a ray impinges on the waveguide wall, it scatters, which means that its angle of reflection is different from the angle of incidence. Since each angle corresponds to a different mode of propagation, mode coupling occurs. The second mecha-
nism is bending. Changing the radius of curvature changes the angle of incidence, hence changing the mode of propagation.

The two mechanisms cause lower-order modes to couple to higher-order ones. The mode coupling causes a longer optical path and thus significant pulse dispersion.

The preceding measurements and calculations correspond to the optical funnel. However, calculations can be made for each input angle, which is selected by a focusing lens. Usually, lenses with focal lengths between 50 and 150 mm were used. Calculations were extended to an even broader range of 25 to 200 mm. Pulse broadening was found to be a function of focal length. This can be seen in the graph in Fig. 8. A smaller focal length leads to larger pulse dispersion. The theoretical pulse dispersion is about 10% of the original (70 ns) for a 25-mm focal length, and almost negligible compared to that of the original pulse at focal lengths greater than 100 mm. These theoretical calculations are in agreement with the experimental results, which were obtained by Pratisto et al.13 The pulse dispersion of micropulses from an FEL (picosecond range) was measured. An increase of about 50% was found in the pulse width for short focal lengths, and a negligible increase at longer ones.

In order to further validate the described theoretical model, these theoretical calculations were compared with experimental results obtained by Matsuura et al.19 and Nguyen et al.11 Table 1 shows the theoretical pulse dispersion (with and without roughness) compared with Matsuura et al.’s measurements \((f=300 \text{ mm}, \ l=1 \text{ m, ID}=1 \text{ mm, pulse width } 196 \text{ fs, and } \lambda=775 \text{ nm})\). Table 2 shows the comparison of theoretical calculations (with and without roughness) with measurements by Nguyen et al. \((f=100 \text{ mm}, \ l=1 \text{ m, ID}=0.5, 0.75, \text{ and } 1 \text{ mm, pulse width } 150 \text{ fs, and } \lambda=800 \text{ nm})\). As can be seen in both tables, there is good agreement between the experimental results and the theoretical calculations. Theoretically, the results showed 127-fs broadening, which is very close (less than 5% difference) to Matsuura’s 133-fs average measurement.

In Table 2, measurements are reported on straight and bent waveguides, with and without roughness. Table 1 compares the pulse dispersion calculations with Matsuura et al.’s measurements for a waveguide with a radius of curvature of 50 cm.

### Table 1 Comparison of pulse dispersion calculation with measurements made by Matsuura et al.19

<table>
<thead>
<tr>
<th>Waveguide</th>
<th>Without roughness</th>
<th>With roughness</th>
<th>Matsuura et al.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Straight</td>
<td>21</td>
<td>25</td>
<td>17</td>
</tr>
<tr>
<td>Bent ((R=50 \text{ cm}))</td>
<td>46</td>
<td>127</td>
<td>Max.: 178</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Av.: 133</td>
</tr>
</tbody>
</table>

![Fig. 7 Calculated pulse dispersion as a function of the waveguide bending angle.](image1)

![Fig. 8 Pulse dispersion as a function of focal length of a conventional coupling lens, instead of the optical funnel.](image2)
bent waveguides with different internal diameters. The results of the calculations and measurements were of the same order of magnitude, with as little as 8% difference between them. The bigger differences could be attributed to the mode structure of the laser used in the experiments in comparison with the clean Gaussian beam assumed in the theoretical calculations, and to the surface roughness that was taken into account in the simulations. The differences between the results obtained by Matsuura et al. and by Nguyen et al. could be due to differences in wavelengths, types of waveguides (different roughness, which may cause different dispersion), and optical setups (mainly the use of the funnel instead of coupling lenses).

5 Conclusions
The temporal behavior of an IR laser pulse after transmission through a hollow optical waveguide was studied. A simple method was developed, based on a refined ray-tracing program, to predict pulse dispersion in these waveguides. Pulse broadening was experimentally measured and compared with theoretical calculations. In addition, the theoretical findings were compared with measurements from two other groups. The experimental results were in agreement with the calculations. These results help to understand laser-pulse behavior in hollow waveguides. They will help to characterize laser pulse transmitted to tissue and may assist in designing better surgical or diagnostic procedures.

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References

Israel Gannot received his BSc degree in electrical engineering from the Technion—Israeli Institute of Technology, Israel, in 1981, and the MSc and PhD degrees in biomedical engineering from Tel-Aviv University, Tel-Aviv, Israel, in 1989 and 1994, respectively. Between 1994 and 1997 he held a National Academy of Sciences postdoctoral fellowship in the Electro-optics Branch of the Office of Science and Technology of the U.S. Food and Drug Administration. Since 1997 he has been on the faculty of the Biomedical Engineering Department, School of Engineering, Tel-Aviv University. Since 2002 he has also been a visiting senior research scientist at the National Institutes of Health. Dr. Gannot is a member of SPIE, and fellow of the American Society for Laser Medicine and Surgery and of the American Institute of Medical and Biological Engineering. He is the author of more than 30 peer-reviewed scientific papers, about 70 proceedings papers, 3 book chapters, and 8 patents. His research fields are optical fibers and waveguides, thermal imaging, laser tissue interaction, and optical diagnostic methods.

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