# SUPPRESSION OF FORCED AND CONVECTIVE FLOWS OF A STABLY STRATIFIED FLUID IN VERTICAL CHANNELS

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In order to improve certain technological processes, for example, single-crystal growing, it is desirable to be able to control the flow rate in order to influence the heat and mass transfer processes. For this purpose it is usual to employ rotation, an electromagnetic field or reduced gravity [1]. Here, with reference to simple solutions of the system of equations of free convection in infinite vertical channels, it is shown that the problem of reducing the intensity of the flow can be solved given a suitable relation between the degree of stable stratification (with respect to density) and the factors responsible for the flow. The possibility of using temperature stratification is considered, but all the conclusions are also fully applicable to concentration stratification.

We will consider nonisothermal flow in a plane layer bounded by planes $x = \pm d$, at whose edges the linear temperature distribution: $T = kz$ $(x = -d)$, $T = kz + T_0$ $(x = d)$ is given; here $k > 0$ is the stratification parameter and $T_0 > 0$ is the horizontal temperature difference.

In the Boussinesq approximation free convection is described by the following dimensionless equations and boundary conditions:

$$\frac{\partial \mathbf{v}}{\partial t}+\frac{1}{\mathrm{Pr}}(\mathbf{v}\nabla)\mathbf{v}=-\nabla p+\Delta\mathbf{v}+\mathrm{Ra}\,T\mathbf{e}_z,\quad \mathrm{div}\,\mathbf{v}=0,\quad \mathrm{Pr}\frac{\partial T}{\partial t}+(\mathbf{v}\nabla)T=\Delta T \tag{1}$$

$$T=z\quad (x=-1);\quad T=z+\frac{T_0}{kd}\quad (x=1) \tag{2}$$

$$\int_0^1 v_z\,dx=0 \tag{3}$$

Here, $\mathrm{Pr} = \nu/\chi$ is the Prandtl number, $\mathrm{Ra} = g\beta kd^4/\nu\chi$ is the Rayleigh number, $\nu$ is the kinematic viscosity, $\chi$ is the thermal diffusivity, g is the acceleration of free fall, and $\beta$ is the coefficient of volume expansion.

If the motion of the fluid is caused by constant external forces parallel to the z axis, then the solution has the form $\mathbf{v} = (0, 0, v_z(x))$, $T = z + f(x)$.

Natural convection ($v_z(x = \pm 1) = 0$) in such a system was thoroughly investigated in [2, 3]. In [2] it was shown that as k increases the flow velocity $v_z$ decreases as $k^{-1/2}$ and there is a change in the nature of the instability of the free-convection flow. The effect of stable temperature stratification on fluid motion due to other causes has not yet been examined.

We will consider convective flow caused by the thermocapillary force. Let $T_0 = 0$ and

$$\frac{\partial v_z}{\partial x}=\mp\mathrm{Ma}\frac{\partial T}{\partial z}\quad (x=\pm 1);\quad \mathrm{Ma}=\left|\frac{\partial\sigma}{\partial T}\right|\frac{kd^2}{\nu\chi\rho} \tag{4}$$

where $\rho$ is the density, $\sigma$ is the surface tension, and Ma is the Marangoni number. Then the solution of problem (1)-(4) has the form:

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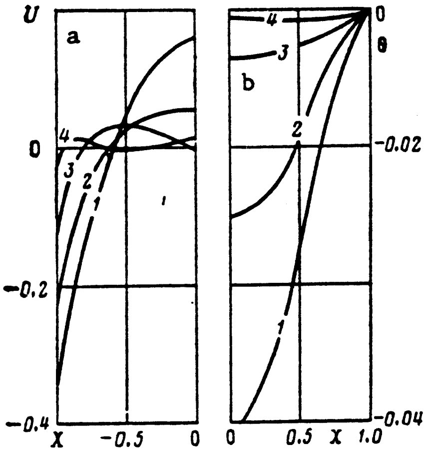


Fig. 1

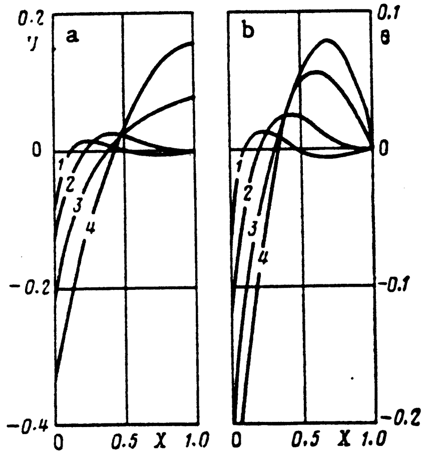


Fig. 2

$$v_z = \frac{Ma}{\sqrt{Ra}} G_1(x), \quad f(x) = \frac{Ma}{\sqrt{Ra}} G_2(x), \quad G_1(x) = \mathrm{Im}\left[\bar\alpha \frac{\mathrm{ch}\,\bar\alpha x}{\mathrm{sh}\,\bar\alpha}\right], \quad G_2(x) = \mathrm{Im}\left[\frac{\mathrm{ch}\,\bar\alpha x - \mathrm{ch}\,\bar\alpha}{\bar\alpha\,\mathrm{sh}\,\bar\alpha}\right] \tag{5}$$

$$\alpha = \sqrt[4]{\frac{Ra}{4}}(1+i), \quad \bar\alpha = \sqrt[4]{\frac{Ra}{4}}(1-i)$$

We will analyze the effect of temperature stratification on the flow obtained. Instead of an increase in the stratification parameter we will consider the more general case of an increase in the Rayleigh number Ra. In this case the quantities $G_1(x)$ and $G_2(x)$ remain bounded, and $v_z$ and $f(x)$ tend to zero as $MaRa^{-1/4}$ and $MaRa^{-3/4}$, respectively.

In Fig. 1a we have plotted $u = v_z/Ma$ and in Fig. 1b $\theta = f(x)/Ma$ for Rayleigh numbers Ra = 0, 100, $10^3$, and $10^4$ (curves 1-4). Starting from Ra = $10^4$ the flow is displaced towards the edge of the layer, while in the central part of the layer the velocity fluctuates about the value $v_z = 0$, the number of roots of the function $v_z(x)$ increasing as $Ra^{1/4}$. The mean-square value of the velocity

$$\langle v\rangle = \int_0^1 |v_z|^2\,dx$$

decreases as $MaRa^{-3/8}$.

We will consider the case where the planes $x = \pm 1$ move at the constant velocity W, i.e., at $x = \pm 1$ $v_z = -W$, but the vertical temperature gradient remains unchanged and the flow closure condition (3) is satisfied. Then when $T_0 = 0$ we have

$$v_z = W\frac{\mathrm{Im}[\bar\alpha\,\mathrm{sh}\,\alpha\,\mathrm{ch}\,\bar\alpha x]}{\mathrm{Im}[\alpha\,\mathrm{sh}\,\bar\alpha\,\mathrm{ch}\,\alpha]} \tag{6}$$

$$f(x) = \frac{W}{\sqrt{Ra}}\frac{\mathrm{Im}[\bar\alpha\,\mathrm{sh}\,\alpha(\mathrm{ch}\,\bar\alpha x - \mathrm{ch}\,\bar\alpha)]}{\mathrm{Im}[\alpha\,\mathrm{sh}\,\bar\alpha\,\mathrm{ch}\,\alpha]} \tag{7}$$

Here, an increase in the Rayleigh number does not affect the maximum value of the velocity, since at the boundaries the fluid velocity is fixed. However, the mean-square value of the velocity decreases as $W\,Ra^{-1/8}$. As before, the function f(x) decreases as $W\,Ra^{-1/2}$.

In Fig. 2 we have shown that the quantities $u = v_z/W$ and $\theta = f(x)/W$ vary with increase in the Rayleigh number Ra = $10^4$, $10^3$, 100, and 0 (curves 1-4). As Ra increases, the flow is displaced towards the edge. In this case, as follows from Fig. 2, the velocity in the central part of the layer falls more rapidly than in the previous example. Thus, here too, there is a flow suppression effect.

Similar results can also be obtained for a right circular vertical cylinder of radius d on whose lateral surface the temperature T = kz is given, at (r = d). In this case the solution of system of equations (1) takes the form $v = (0, 0, v_z(r))$, T = z + f(r).

For thermocapillary convection we obtain

$$\frac{\partial v_z}{\partial r} = -\mathrm{Ma}\frac{\partial T}{\partial r} \quad (r=1), \qquad v_z = \frac{\mathrm{Ma}}{2\beta}\left[\frac{J_0(\beta r)}{J_1(\beta)} + i\frac{J_0(i\beta r)}{J_1(i\beta)}\right] \tag{8}$$

$$f(r) = \frac{\mathrm{Ma}}{2\beta^3}\left[\frac{J_0(\beta) - J_0(\beta r)}{J_1(\beta)} + i\frac{J_0(i\beta) - J_0(i\beta r)}{J_1(i\beta)}\right] \tag{9}$$

If the velocity is given on the surface of the cylinder, we have

$$v_z = -W \quad (r=1), \qquad v_z = -W\frac{J_1(\beta)J_0(i\beta r) + iJ_1(i\beta)J_0(\beta r)}{J_1(\beta)J_0(i\beta) + iJ_1(i\beta)J_0(\beta)} \tag{10}$$

$$f(r) = \frac{W}{\beta^3}\left[J_1(i\beta)\left(J_0(\beta r) - J_0(\beta)\right) - J_1(\beta)\left(J_0(i\beta r) - J_0(i\beta)\right)\right]\left(J_1(\beta)J_0(i\beta) + iJ_1(i\beta)J_0(\beta)\right)^{-1} \tag{11}$$

where $\beta = \sqrt[4]{-\mathrm{Ra}}$ and $J_0$ and $J_1$ are Bessel functions.

The values of (8)-(9) and (10)-(11) can also be depressed by imposing stable temperature stratification. In this case the conclusions reached above continue to hold: f(r) (9), (11) decreases in proportion to $\mathrm{Ra}^{-3/4}$; for thermocapillary (8) convection $v_z(r)$ decreases as $\mathrm{MaRa}^{-3/4}$; in both cases the number of roots of the function increases as $\mathrm{Ra}^{1/4}$. The mean-square value of the velocity decreases as $\mathrm{MaRa}^{-3/4}$ and $W\,\mathrm{Ra}^{-1/4}$ for (8) and (11), respectively.

Similar results can also be obtained by considering a vertical cylindrical layer with a radial temperature difference, moving surfaces and a thermocapillary force on one or both surfaces.

We note that from (5), (7), (9), and (11) there follows: $f'(1) = 0$.

This means that the conclusions also remain valid when Newton's heat transfer condition or the Stefan–Boltzmann condition is imposed on the lateral surface.

The above examples show that stable (with respect to density) stratification leads to suppression not only of natural thermoconvective flows but also of flows caused by any other factor.

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