Magnetohydrodynamics. New York, 1988

EFFECTS OF THE MAGNETIC FIELD MAGNITUDE AND DIRECTION ON THE OSCILLATORY THERMOGRAVITATIONAL CONVECTION REGIMES IN A RECTANGULAR CAVITY

A. Yu. Gel'fgat

UDC 536.25

At the present time great attention is being given to the study of the oscillatory regimes of convective flows. In particular, it is interesting to find out what effect a magnetic field has on the convective flow oscillations in a conducting fluid. This problem has been addressed in [1-5], which studied the magnetic field effect both on stationary and oscillatory convective flows in various systems that were heated from below. Systems that are heated from the side are more difficult to analyze, since in this case it becomes necessary to investigate the stability of the stationary flows arising at arbitrarily small temperature differences. The magnetic field effect on such flows in the stationary case has been investigated in [6, 7]. In the present report, computation results are presented for threshold oscillatory instability and oscillatory convection regimes for a liquid in a uniform horizontal or vertical magnetic field in a square cavity, whose vertical walls are at different temperatures.

The flow is described by a dimensionless system of free convection equations (Oberbeck–Boussinesq approximation), in which the electromagnetic force is accounted for in the inductionless approximation

$$\partial \mathbf{v}/\partial t+(\mathbf{v}\nabla)\mathbf{v}=-\nabla p+\Delta\mathbf{v}+\mathrm{Gr}\,\Theta\mathbf{e}_y+\mathrm{Ha}^2(\mathbf{v}\times\mathbf{B})\times\mathbf{B}, \qquad (1)$$

$$\operatorname{div}\mathbf{v}=0, \qquad (2)$$

$$\partial\Theta/\partial t+(\mathbf{v}\nabla)\Theta=\Delta\Theta/\mathrm{Pr} \qquad (3)$$

subject to the boundary conditions

$$\Theta|_{x=0}=1;\quad \Theta|_{x=1}=\Theta'_y|_{y=0,1}=0;\quad \mathbf{v}|_{x=0,1}=\mathbf{v}|_{y=0,1}=0, \qquad (4)$$

where Θ is the temperature, $\mathrm{Pr} = \nu/\alpha$ is the Prandtl number, and α is the thermal conductivity. The cavity length is taken as the characteristic dimension. The remaining symbols have their customary meaning.

The solution to the problem is sought in the form

$$\mathbf{v}=\sum_{i,j=1}^{M_1} c_{ij}(t)\varphi_{ij}(x,y);\quad \Theta=(1-x)+\sum_{i,j=0}^{M_2} d_{ij}(t)g_{ij}(x,y) \qquad (5)$$

using the Galerkin method. The coordinate functions g_{ij} and φ_{ij} are constructed from linear combinations of Chebyshev polynomials of the first and second kind $T_i(x)$ and $U_j(x)$:

$$g_{ij}(x,y)=(T_i(x)+\alpha_i T_{i+2}(x))(T_j(y)+\beta_j T_{j+2}(y)), \qquad (6)$$

$$\varphi_{ij}(x,y)=\begin{pmatrix} \sum_{l=0}^{4}\frac{f_{li}}{2i}T_{i+l}(x)\sum_{l=0}^{4}q_{lj}U_{j+l-1}(y) \\ -\sum_{l=0}^{4}f_{li}U_{j+l-1}(x)\sum_{l=0}^{4}\frac{q_{lj}}{2j}T_{j+l}(y)\end{pmatrix}. \qquad (7)$$

The coefficients α_i, β_i, $f_{\ell i}$, $q_{\ell j}$ are chosen in such a way as to satisfy all homogeneous boundary conditions. This choice of the coordinate functions makes it possible to exclude

Translated from Magnitnaya Gidrodinamika, No. 3, pp. 70-75, July-September, 1988. Original article submitted June 10, 1987.

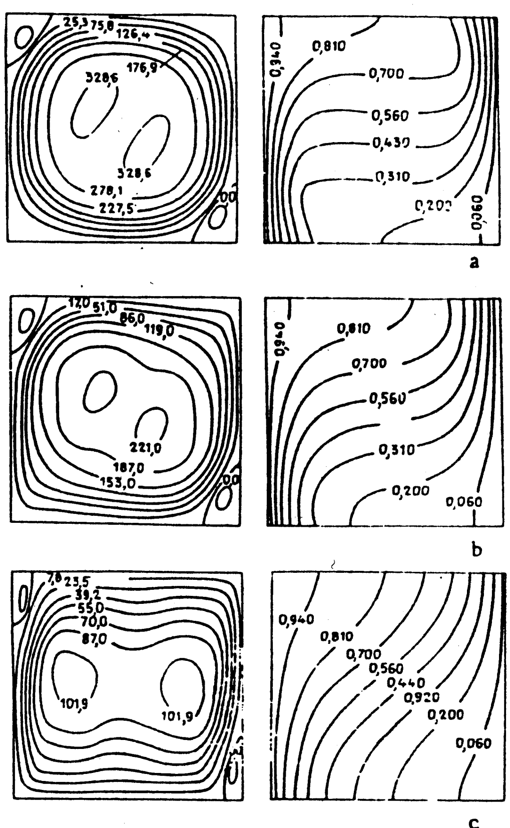

Fig. 1

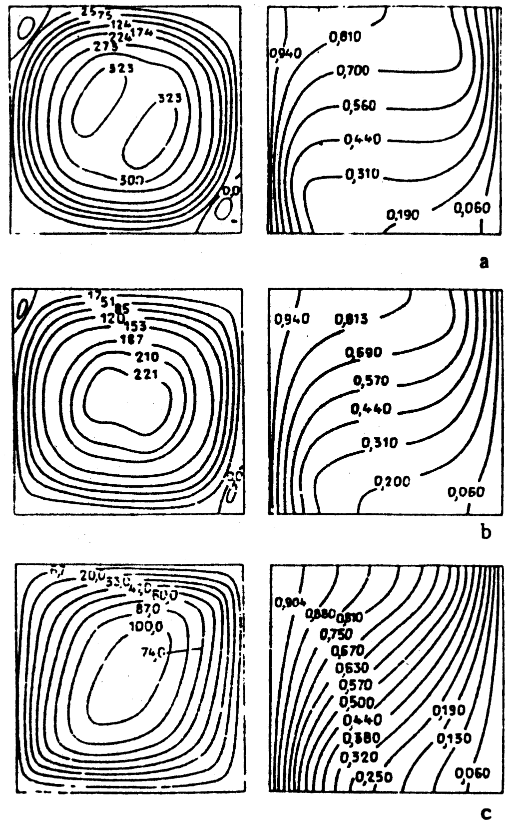

Fig. 2

TABLE 1

Hartmann number	Max. stream function value		Hartmann number	Max. stream function value	
	vertical field	horizontal field		vertical field	horizontal field
0	462	462	100	106	107
10	429	428	150	54.8	51.3
25	311	335	200	31.1	29.3
50	229	229	300	14.1	13.5
75	151	163	500	5.19	5,04

the pressure from the Navier—Stokes equations and to transform the problem described by Eqs. (1) to (4) into a system of ordinary differential equations of the form

$$dX_i/dt = a_{ij}X_j(t) + b_{ijk}X_j(t)X_k(t) + F_i, \quad i = 1, 2, \ldots, N, \tag{8}$$

where $X_i(t)$ is one of the coefficients c_{ij} or d_{ij}. The utilized variant of the Galerkin method is presented in greater detail in [8].

The computations were carried out for six coordinate functions ($M_1 = M_2 = 5$) along each direction ($N = 72$). The limitations on M_1 and M_2 are tied in with the dimensions of the three-dimensional matrix b_{ijk} containing N^3 quantities. However, even for such a (relatively small) number of coordinate functions it is possible to obtain sufficiently accurate results. In addition, the possibility of a detailed investigation of the stability of fixed points and the relatively fast numerical time integration of the system (8) make it possible to arrive at some qualitative conclusions which are unobtainable with various kinds of finite difference, or finite element, methods. All computations were carried out for Pr = 0.02, which characterizes liquid metals or semiconductors.

The fixed points of system (8) corresponding to a steady-state solution of the system (1) to (4) were determined by the Newton method.

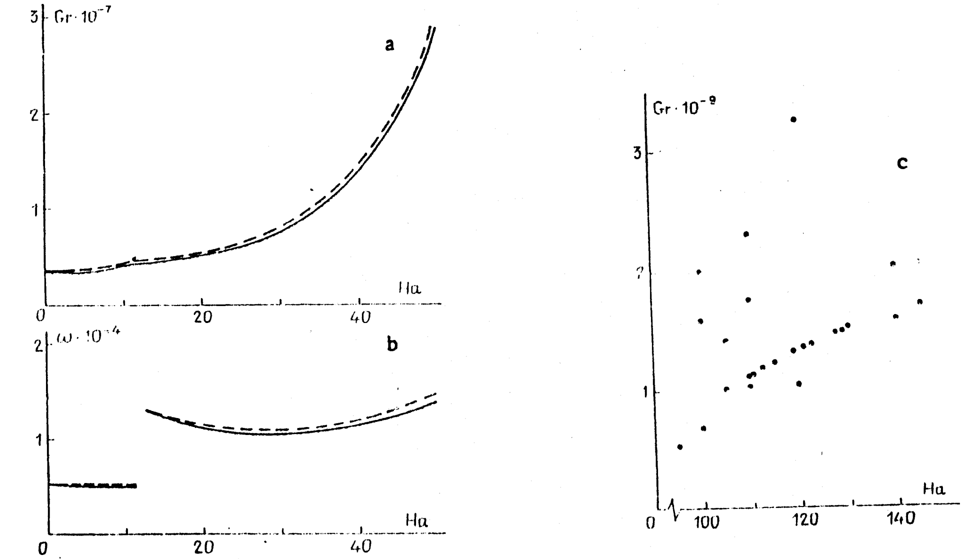

Fig. 3

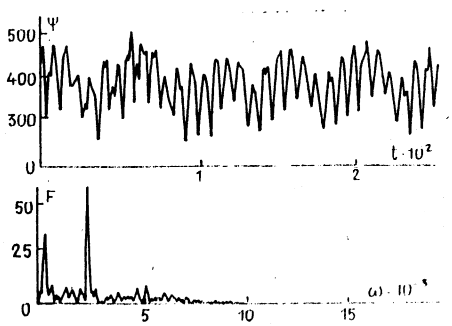

Fig. 4

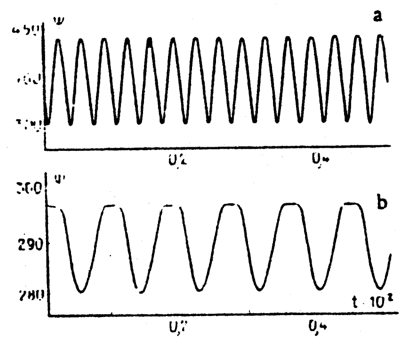

Fig. 5

Table 1 shows the variation of the maximum value of the stream function ψ ($v_x = \partial\psi/\partial y$, $v_y = \partial\psi/\partial x$) at the calculated fixed points with increasing magnetic field in the case Gr = 10^7. It is evident from the results presented that the convective flow intensity is weakly dependent on the magnetic field direction. For Ha = 75 the greatest difference of the maximum ψ value amounts to about 8%. On the other hand, strong magnetic fields influence the convective flow structure in different ways. Shown in Figs. 1 and 2 are streamlines (on the left) and isotherms (on the right) for Gr = 10^7 and Ha = 25, 50, and 100 (shown in a, b, and c, respectively) in a vertical (Fig. 1) and horizontal (Fig. 2) magnetic field. It is evident from these figures that an increase in the Hartmann number in the horizontal magnetic field leads to the merging of two stream function maxima in the core flow, whereas in the vertical field the difference between the stream function maxima increases.

The investigation of the stability of the fixed points in the system (8) with respect to infinitely small perturbations leads to the determination of the critical Grashof number at which at least one eigenvalue of the Jacobian matrix of the system (8), evaluated at the pertinent fixed point, has a nonnegative, real part. The computations, carried out using the BIFOR2 program [9], have shown that such eigenvalues appear in complex conjugate pairs; this points to the oscillatory instability of the steady state solution (1)-(4) [1, 9].

Shown in Fig. 3a is the neutral curve, indicating the dependence of the critical Grashof number on the Hartmann number in vertical (continuous) and horizontal (dashed line) magnetic fields. Shown below, in Fig. 3b, is the corresponding dependence of the dimensionless flow oscillation frequency on the Hartmann number at the critical point. The neutral curves consist of two parts, defined by different eigenvalues of the Jacobian matrix. The left section of the curve at 0 < Ha < 12 corresponds to the instability of a free convection flow in a weak magnetic field which displays insignificant influence upon the character and stability of the flow. The right neutral curve branches at Ha > 12 describe a regime in which the magnetic field is the major determinant. In this instance, a slight increase in the magnetic field (12 < Ha < 30) abruptly changes the oscillatory flow frequency (Fig. 3b), and consequently, also the nonstationary flow structure. At Ha > 30 the critical Grashof number begins to increase rapidly.

TABLE 2

Ha	Vertical field				Horizontal field			
	ψ_{cr}	θ_{cr}	A_ψ	A_θ	ψ_{cr}	θ_{cr}	A_ψ	A_θ
10	320	0.316	0.181	0.139	389	0.310	0.133	0.041
20	311	0.298	0.128	0.036	307	0.298	0.130	0.035
30	255	0.293	0.085	0.032	242	0.292	0.099	0.034
40	210	0.305	0	0	190	0.302	0	0

TABLE 3

Ha	Vertical field				Horizontal field			
	ψ_{cr}	θ_{cr}	$A_\psi \cdot 10^2$	$A_\theta \cdot 10^1$	ψ_{cr}	θ_{cr}	$A_\psi \cdot 10^2$	$A_\theta \cdot 10^1$
10	484	0.5	1.42	0.133	530	0.5	0.117	0.195
20	430	0.5	0.096	0.179	430	0.5	0.095	0.177
30	341	0.5	0.045	0.126	346	0.5	0.045	0.129
40	280	0.5	0	0	288	0.5	0	0

The indicated frequency jump in the oscillating convective flow for increasing Hartmann numbers can be used as a criterion in experimental verification of the obtained results. Furthermore, as evidenced from Fig. 3, the magnetic field direction has a very weak effect on the critical Grashof number value and the flow oscillation frequency. This fact constitutes an important qualitative conclusion which easily lends itself to experimental verification. The stronger suppression of oscillatory instability by the horizontal magnetic field can be explained by the different interactions of the variously oriented field with flow in the boundary layer found on the vertical cavity wall.

A further increase in the Hartmann number (Ha > 70) at fixed Gr and Pr numbers leads to the appearance of several mutually slightly differing fixed points of the system of equations (8). This means that for strong magnetic fields and sufficiently large Grashof numbers there exist several close branches of the steady-state solution to the problem posed by Eqs. (1)-(4), each of them corresponding, for fixed Pr and Ha values, to a certain critical Grashof number value. In this case, the appearance of one of the steady or oscillatory regimes depends on the process history. This is illustrated in Fig. 3c, which shows the computed points at which stability is lost in various steady regimes at Ha > 100 in a vertical magnetic field. It should be noted that, according to Fig. 3 some of the steady regimes indicated in Table 1 and in Figs. 1 and 2 are unstable or not unique and that they merely indicate the general character of the convection flow variation when the magnetic field is increased.

The oscillatory regimes were investigated for the cases of Gr = 10^7 and Pr = 0.02 in magnetic fields varying both in magnitude and direction. For the indicated parameter values in the absence of the magnetic field aperiodic convective flow oscillations were observed. Shown in Fig. 4 are the stream function oscillations at the points having the coordinates (0.25, 0.25), and below it, the frequency spectrum density of these oscillations. Upon the application of a weak magnetic field with Ha = 10, the oscillations become periodic, and their amplitude diminishes. This finding is illustrated in Fig. 5, which shows the stream function oscillations at the same point at Ha = 10 (a – for the vertical, and b – for the horizontal magnetic field). It is evident from the figure that the oscillatory convection regimes in magnetic fields having different orientations are substantially different. At the same time, it follows from the linear stability analysis (Fig. 3a, b) that near the point where the stability is lost, the flow oscillations in variously oriented fields proceed at close frequencies.

Listed in Tables 2 and 3 are average values and relative oscillation amplitudes

$$A_f = |f_{max} - f_{min}| / |f_{max} + f_{min}| \quad (9)$$

of the stream function and temperature at various magnetic field strengths for the point with coordinates x = 0.25 and y = 0.25 (in Table 2) and x = 0.5 and y = 0.5 (in Table 3). It follows from these results that for Ha > 20 the average values of the functions and oscillation amplitudes in the horizontal and vertical magnetic fields become close to each other, which confirms the conclusions derived by linear stability analysis of steady-state flows. As previously indicated, in weak magnetic fields at Ha < 20, the characteristics of the oscillatory convection regimes also depend on the magnetic field direction.

LITERATURE CITED

1. G. Z. Gershuni and E. M. Zhukhovitskii, Convective Stability of Incompressible Liquid [in Russian], Moscow (1972).
2. M. R. E. Proctor and N. O. Weiss, "Magnetoconvection," Rep. Prog. Phys., 45, 1317-1379 (1982).
3. S. Fauve, C. Laroche, and A. Libchaber, "Horizontal magnetic field and the oscillatory instability onset," J. Phys. Lett., No. 45, L-101-L-105 (1984).
4. L. E. Sorokin, E. L. Tarunin, and M. I. Shliomis, "Monotonic and oscillatory regimes in conducting medium convection in a magnetic field," Magn. Gidrodin., No. 4, 22-30 (1975).
5. M. I. Shliomis, "Oscillatory convective instability of a conducting liquid in a magnetic field," Prikl. Mat. Mekh., 28, No. 4, 678-683 (1964).
6. M. Seki, H. Kawamura, and K. Sanokawa, "Free convection of mercury in a magnetic field parallel to the gravity force vector," Teploperedacha, No. 2, 45-51 (1979).
7. G. M. Oreper and J. Szekely, "The effect of an externally imposed magnetic field on buoyancy driven flow in a rectangular cavity," J. Cryst. Growth, 64, 505-515 (1983).
8. A. Yu. Gel'fgat, "Variational solution method for the problem of the dynamics of a viscous liquid in rectangular regions," in: Applied Problems in Mathematical Physics [in Russian], Riga (1987), pp. 14-24.
9. D. B. Hassard, N. D. Kazarinoff, and I. Wen, Theory of Application of Hopf Bifurcations [Russian translation], Moscow (1985).