THE MAGNETIC FIELD AS IT AFFECTS THE THREE-DIMENSIONAL STRUCTURE OF THE SELF-OSCILLATIONAL REGIMES IN FREE CONVECTION

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The influence exerted by a magnetic field on the thermogravitational convective flows of a nonuniformly heated fluid under conditions characterized by an influx of heat from below has been studied in [1—5]. The more complex case of heating from the side has been covered in [6], which is devoted to a study of the influence exerted by a constant uniform magnetic field on the stability of steady regimes of free convection (as well as the development of nonsteady free-convection regimes) in an electrically conducting fluid contained within a quadratic cavity heated from the side. It was demonstrated that the threshold of stability for the steady convection regimes can be increased through the action of the magnetic field on the free-convective flow being studied, as well as to regularize or suppress the self-oscillations of the convective flow. However, the question of changes in the three-dimensional structure of the oscillating convection flows subjected to the action of a magnetic field has not been dealt with.

In the present study we offer results from investigations into the influence of a constant uniform magnetic field on the three-dimensional structure of the most dangerous infinitely small perturbation which leads to instability in steady convective flow; averaged and pulsating components of supercritical convective flows; the trajectories of fluid particles, governed by oscillatory convection regimes. Studying the action of a magnetic field on various spatial characteristics of oscillatory convective flows allows us to establish, in particular, both the qualitative and quantitative analogous changes in the spatial structure of the convective flows, which occur with an increase in the Hartman number or with a decrease in the Grashof number, as well as of the case for an increase in the Hartman number or an increase in the Prandtl number.

As was done in [6], we examine the problem of the thermogravitational convection within a quadratic region contained within a constant uniform magnetic field. The flow is described by a system of free-convection equations in the Overbeck—Boussinesq approximation in which the electromagnetic force is provided for in induction-free approximation:

$$\partial \mathbf{v}/\partial t+(\mathbf{v}\nabla)\mathbf{v}=-\nabla p+\Delta\mathbf{v}+\mathrm{Gr}\,\theta\mathbf{e}_y+\mathrm{Ha}^2(\mathbf{v}\times\mathbf{B})\times\mathbf{B}; \quad (1)$$

$$\mathrm{div}\,\mathbf{v}=0; \quad (2)$$

$$\partial\theta/\partial t+(\mathbf{v}\nabla)\theta=\Delta\theta/\mathrm{Pr}. \quad (3)$$

Here θ is the temperature; $\theta_0 - \theta_1$ is the characteristic temperature difference; B_0 is the characteristic value for the induction of the external magnetic field; l is the length of the region; $\mathrm{Gr} = g\beta(\theta_0 - \theta_1)l^3/\nu^2$, $\mathrm{Pr} = \nu/\chi$, $\mathrm{Ha} = B_0 l\sqrt{\sigma/\rho\nu}$ represent the Grashof, Prandtl, and Hartman numbers, respectively; the remaining notation is standard. We examine a region with four solid boundaries. Constant and varying values of temperature are specified for the vertical boundaries, while the horizontal boundaries are assumed to be thermally insulated;

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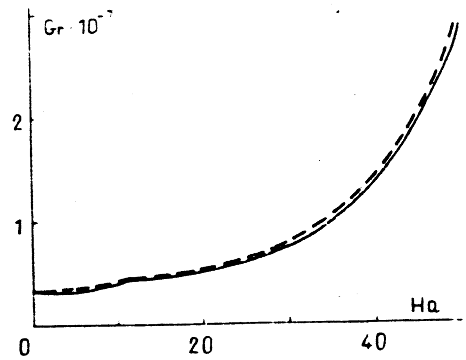


Fig. 1

$$v(x=0;\ 1)=v(y=0;\ 1)=0;\quad \theta(x=0)=1;\quad \theta(x=1)=0;\quad \theta'_y(y=0,\ 1)=0. \tag{4}$$

The dissipation of Joule heat is described by the terms $\sigma(v \times B)^2/C_p\rho$ in the equation of temperature transfer, where C_p denotes the heat capacity of the liquid at constant pressure. Subsequent to nondimensionalization, used to derive the system of equations (1)—(3) (see [1]), the coefficient for the indicated term becomes equal to $D = \sigma\nu B_0^2/[C_p\rho(\theta_0 - \theta_1)] = D^*Ha^2/Gr$, where $D^* = g\beta l/C_p$. Of fundamental interest in this problem are the convective flows of liquids with relatively high electrical conductivity (liquid metals and semiconductors), for which $C_p \approx 10^2$ J/(kg·K), $\beta \approx 10^{-3}$ K^{-1}. With $l \leq 1$ m the quantity D^* is on the order of 10^{-4} or lower. In our calculations Ha $\leq 10^2$, Gr $\geq 10^6$, so that consequently $D \leq 10^{-6}$, and therefore in Eq. (3) we can neglect the term describing Joule dissipation.

For the solution of problem (1)—(4) we use a variant of the Galerkin method, whose numerical realization is described in detail in [7]. The solution of problem (1)—(4) is sought in the form

$$v = \sum_{i,j=0}^{N} c_{ij}(t)\varphi_{ij}(x, y);\quad \theta = (1-x) + \sum_{i,j=0}^{K} d_{ij}(t)g_{ij}(x, y), \tag{5}$$

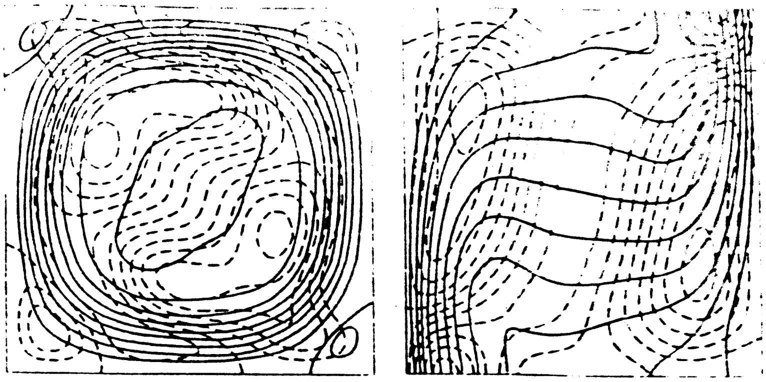
where the functions φ_{ij} and g_{ij} form the bases in the corresponding functional spaces; c_{ij} and d_{ij} are unknowns, dependent on the time coefficient. As a result of the application of the Galerkin method, problem (1)—(4) reduces to a system of ordinary differential equations of the form

$$dX_l/dt = a_{lk}X_k + b_{lmk}X_mX_k + f_l, \tag{6}$$

where X_l is one of the coefficients c_{ij} or d_{ij}.

The test calculations illustrating the suitability of the numerical method used in the present study to solve the system of equations for convection in rectangular areas are described in detail in [8]. Good agreement has been achieved with the results from [9] in studying the stability of steady convective flow of gallium arsenide (Pr = 0.015) in a rectangular cavity heated from the side, with the length-to-height ratio equal to 4. As was done in [6], in the present study the calculations were carried out for Pr = 0.02, characteristic of liquid metals and semiconductors, for six coordinate functions in each spatial direction. The number of coordinate functions is markedly limited by the productivity of the computer being utilized (the calculations were carried out on an ES-1060 computer). However, as demonstrated in [8], even with such a limited number of appropriately constructed spatial modes it is possible to achieve completely satisfactory results.

The stability of the steady solutions for problem (1)—(4) is determined by the stability of the corresponding steady solution for system (6). As a result of the studies into the stability of steady convective flows, such as those conducted in [6] through the calculation of the eigenvalues of the Jacobi matrix in system (6), we derived neutral Gr(Ha) curves (Fig. 1), which show the relationship between the critical Grashof number and the Hartman number in the case of a vertical (solid line) or horizontal (dashed line) magnetic field. These neutral curves consist of two smooth segments with



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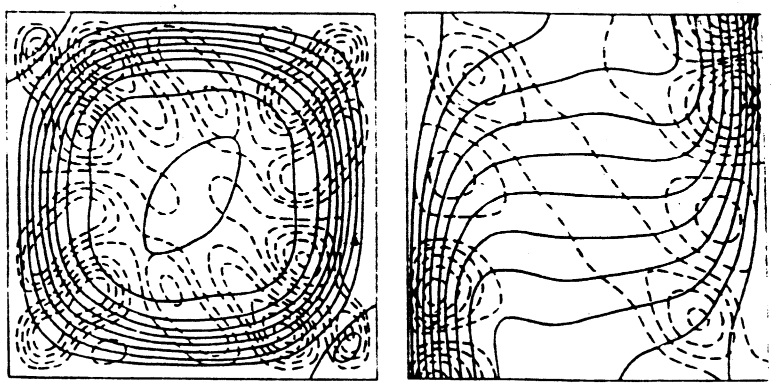

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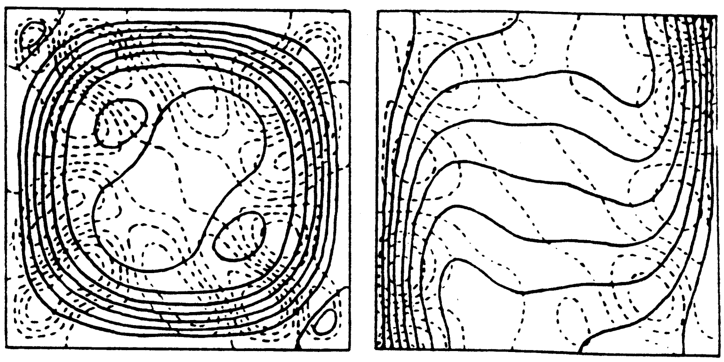


c

Fig. 2

a break at Ha ≈ 12. At this breaking point on the neutral curves we have a jumplike change in the physical mechanism responsible for the oscillational instability of the convective flow. Analysis of the structure of the most dangerous infinitely small perturbation, which is determined by the eigenvector of system (6), calculated for the critical values of the parameters and by means of the corresponding eigenvalue, with a nonnegative real part, provides some idea as to the mechanisms responsible for convective instability. The values of the coefficients c_{ij} and d_{ij}, equal to the values of the corresponding components of the eigenvector of system (6), in conjunction with formulas (5), determine (with accuracy to multiplication by the constant) the most dangerous perturbations in velocity and temperature. Figures 1 and 2 show the streamlines and isotherms of the convective flows for critical values of the Grashof number (solid lines) and the isolines of the most dangerous infinitely small perturbations in the functions of current and temperature (dashed lines). At the instant at which oscillational instability sets in, the convective flow is made up of the sum of the constant term represented by the solid lines (see Fig. 2) and the pulsating portion represented by the dashed lines, the latter with an exponentially increasing amplitude. Figure 2a, b shows the results for Pr = 0.02 and for the Hartman and Grashof

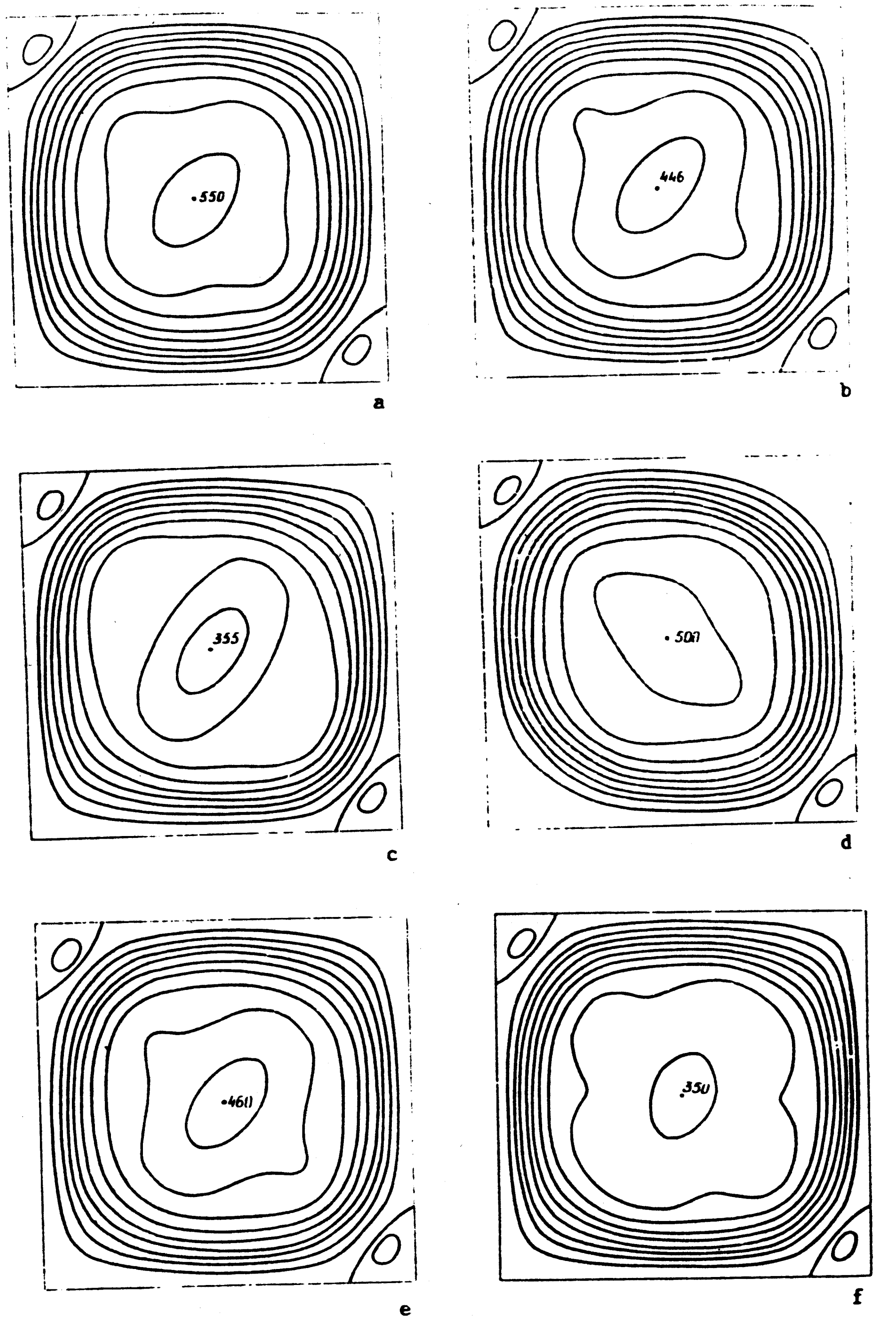
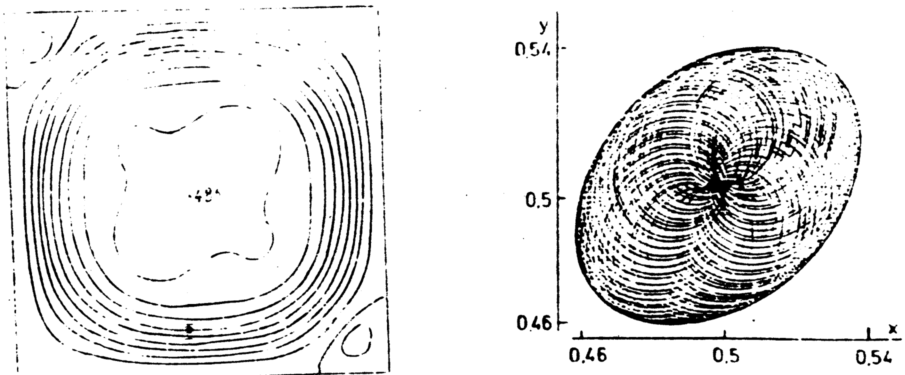


Fig. 3

numbers of various smooth segments of the neutral curve, i.e., Ha = 10, Gr = $4.3 \cdot 10^6$ (see Fig. 2a) and Ha = 20, Gr = $5.3 \cdot 10^6$ (see Fig. 2b). Figure 2c shows the results for the case in which there is no magnetic field (Ha = 0) for Pr = 0.04, Gr = $2.5 \cdot 10^6$. As we can see from Fig. 2a, b, the isolines of the most dangerous perturbations for Ha = 10 and 20 differ qualitatively. The change in the three-dimensional structure of the most dangerous perturbation occurs in a jump on transition through the break in the neutral curve at Ha $\approx$ 12. At the same time, comparison of Figs. 2b and 2c shows that the isolines of the most dangerous perturbations in these two cases exhibit identical shape. This indicates that the instability of the convective flow, with an increase in the Hartman number and with an increase in the Prandtl number, is generated by analogous physical mechanisms.

The results of these investigations into the stability of the subject convective flow, in the absence of a magnetic field, are covered in [10]. The observed analogy between the increase in the Hartman and Prandtl numbers allows us to draw the conclusion that it is possible to simulate the effects observed in [10], the latter associated with the continuous



a b

Fig. 4

increase in the Prandtl number, by studying the motion of the current-conducting fluid with unchanging properties within a constantly increasing magnetic field.

Analysis of the properties derived as part of the nonsteady calculations of the self-oscillation convection regimes was conducted in the following manner: the values of the coefficients $c_{ij}(t)$ and $d_{ij}(t)$ remained the same at the instants of time t_n at various intervals Δt. After accumulating information from 2M points, we approximated the coefficients $c_{ij}(t)$ and $d_{ij}(t)$, being components of the vector $X_k(t)$, in conjunction with the rapid Fourier transform, by trigonometric sums of the form

$$X_k(t) = \bar{X}_k^0 + \sum_{l=1}^{M-1} \left[\tilde{X}_k^l \cos \frac{2\pi l}{L} t + \hat{X}_k^l \sin \frac{2\pi l}{L} t \right], \tag{7}$$

where $L = 2M\Delta t$ is the time interval in which the calculations were carried out. Expansion (7) in these calculations was carried out with the use of 512 points ($M = 256$).

Averaging over the time interval L leads to the following expressions for the averaged ($\bar{v}$, $\bar{\theta}$) and pulsation (v', θ') components of motion:

$$\bar{v} = \sum_{i,j=0}^{N} \bar{c}_{ij}^0 \varphi_{ij}(x, y); \tag{8}$$

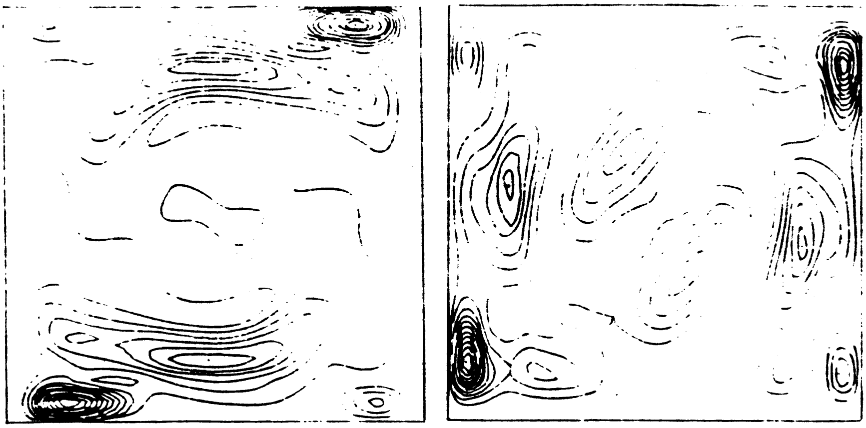
$$\bar{\theta} = (1-x) + \sum_{i,j=0}^{K} \bar{d}_{ij}^0 g_{ij}(x, y); \tag{9}$$

$$v' = \sum_{i,j=0}^{N} \sum_{l=1}^{M-1} \left[\tilde{c}_{ij}^l \cos \frac{2\pi l}{L} t + \hat{c}_{ij}^l \sin \frac{2\pi l}{L} t \right] \varphi_{ij}(x, y); \tag{10}$$

$$\theta' = \sum_{i,j=0}^{K} \sum_{l=1}^{M-1} \left[\tilde{d}_{ij}^l \cos \frac{2\pi l}{L} t + \hat{d}_{ij}^l \sin \frac{2\pi l}{L} t \right] g_{ij}(x, y). \tag{11}$$

Thus, the Fourier transform (7) allows us to obtain explicit expressions for the pulsation and averaged flow components. Expressions for the mean-square pulsations $\overline{v'^2}$ and $\overline{\theta'^2}$, as well as for other moments of second and higher orders, can be derived from (10) and (11) by direct integration over time.

Analysis of the time structure of the oscillations in the convective flow, such as that conducted in [6], demonstrated that the nonlinear development of oscillatory instability occurs differently in variously directed magnetic fields. This conclusion confirms the analysis of the averaged flow components. Figure 3 shows the streamlines for the averaged flow in the case of $Pr = 0.02$ and $Gr = 10^7$ in the vertical (see Fig. 3a-c) and horizontal (see Fig. 3d-f) magnetic fields for the case in which $Ha = 10$ (a, d), 20 (b, e), and 30 (c, f). The greatest differences in the streamlines corresponding to the same parameter values and various directions of the magnetic field are observed near the center of the cavity. An exception is represented by the case $Ha = 20$, for which the shape of the streamline and the intensity of the convective flow in the variously directed magnetic fields are close to each other. Figure 4a shows the streamlines of the averaged



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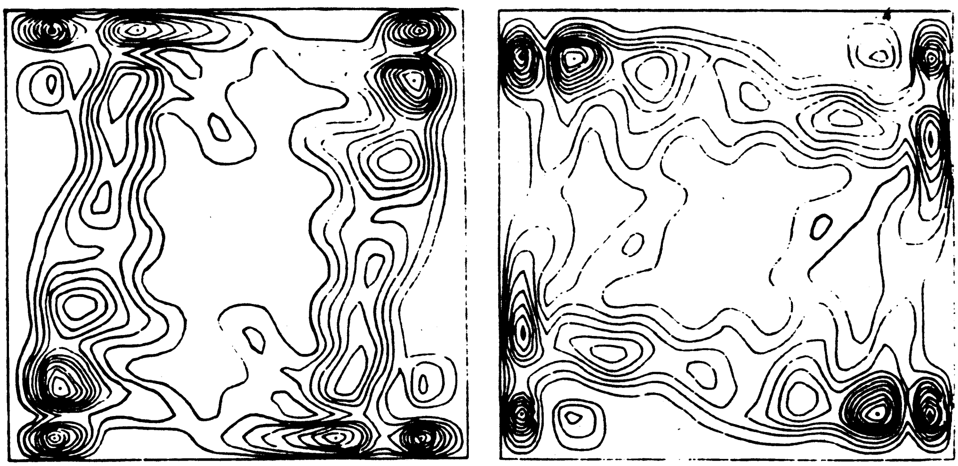

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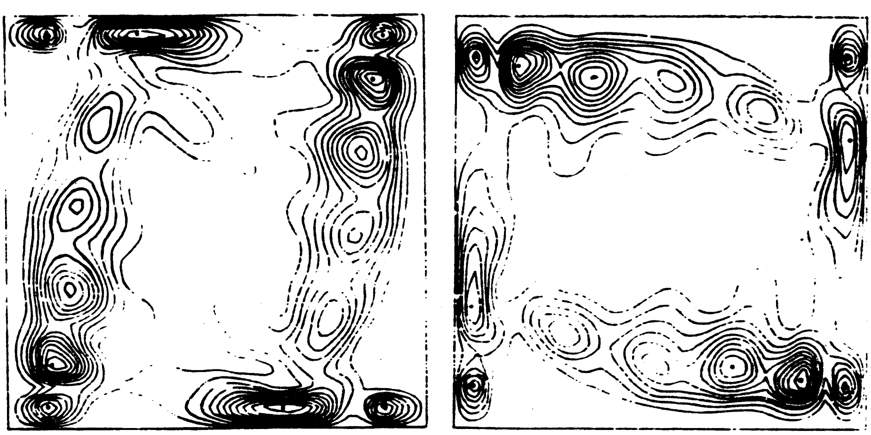

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Fig. 5

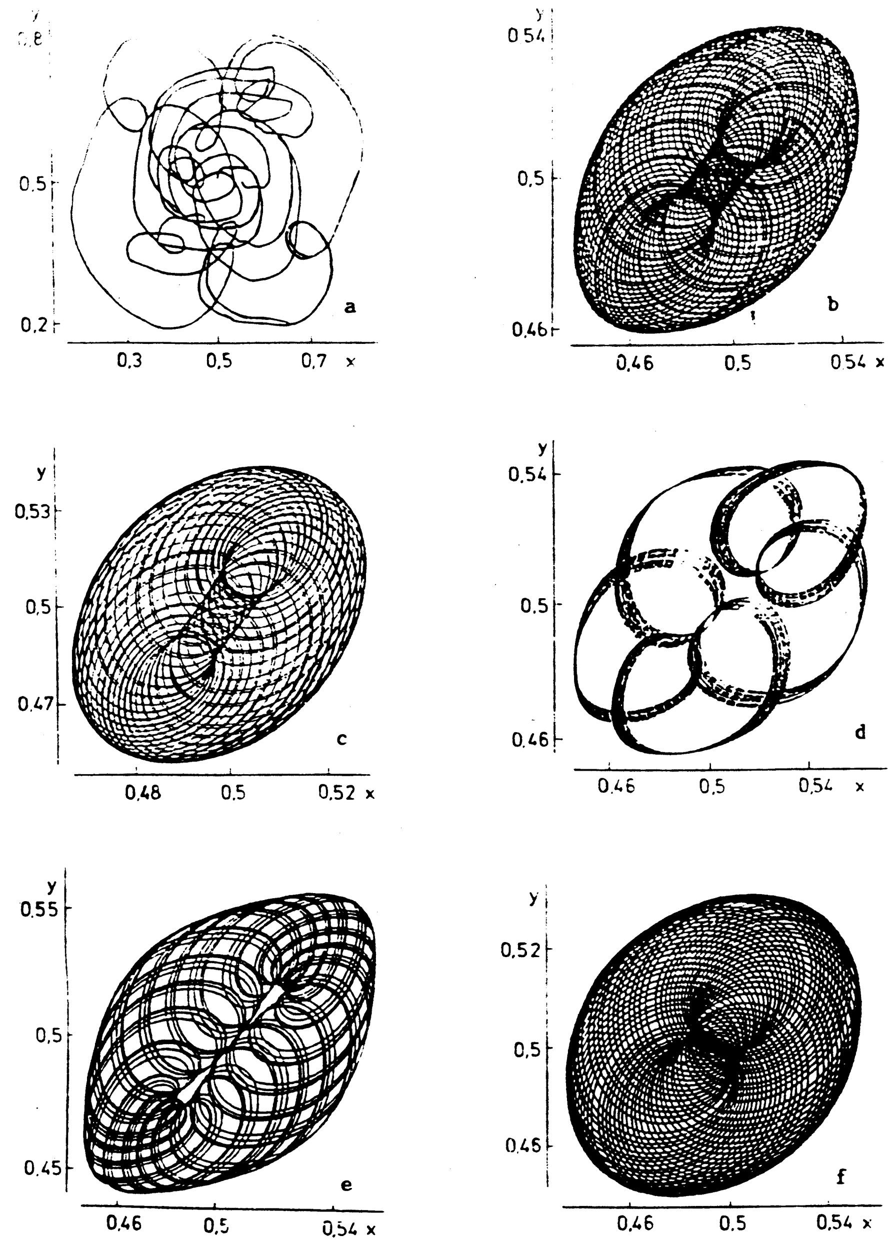

Fig. 6

flow for Pr = 0.02 and Gr = $6 \cdot 10^6$ in the absence of a magnetic field. Comparison of Fig. 3b, e and Fig. 4a shows that in all three of the cases under consideration the structure and intensity of the averaged flows are close to each other. Thus, the development of convective flows with an increase in the Hartman number and with a decrease in the Grashof number occurs in analogous fashion. Consequently, by means of the magnetic field it becomes possible to simulate certain effects associated with the conditions of reduced gravitation.

Unlike the frequency spectra and averaged characteristics of oscillating convection regimes, the distribution of the magnitudes of mean-square pulsations in velocity and temperature is independent of the direction of the magnetic field. Figure 5a, b shows the isolines of the quantities $\overline{v_x'^2}$ and $\overline{v_y'^2}$ for Pr = 0.02, Gr = 10^7, Ha = 10, and Ha = 30, respectively. The change in the shape of the isolines of the mean-square pulsations occurs in the vicinity of Hartman number values Ha $\approx$ 12, corresponding to the break in the neutral curves (see Fig. 1). The changes in the spatial distribution of the convective-flow pulsations are associated with the change in the structure of the most dangerous infinitely small perturbation (see Figs. 1 and 2b). Figure 5c shows the isolines for the pulsations of each of the velocity components in the absence of a magnetic field for Pr = 0.1 and Gr = $4.1 \cdot 10^6$. Comparison of Fig. 5b and c showed that the distribution in the velocity pulsations through the flow region changes in a manner similar to the case in which the

Hartmann number increases or in the absence of a magnetic field for an increase in the Prandtl number. It was noted earlier that the shape of the isolines for the mean-square pulsations and the most dangerous perturbations does not change with the change in the direction of the magnetic field.

Thus, the existence of analogy between the increase in the Hartman and Prandtl numbers is associated with the structure of the pulsation component of the nonsteady convective flow (for example, with the structure of the most dangerous perturbation for the spatial distribution of the mean-square pulsations) and it is independent of the direction of the magnetic field.

The quantitative characteristics for the suppression of convective-flow oscillations by means of a magnetic field can be found in Table 1, where the change in the maximum values of the mean-square pulsations in velocity and temperature (over the flow region) is shown as functions of the Hartman number for $Gr = 10^7$.

As noted in [10], the instability of this convective flow (for $Pr = 0.02$) is accompanied by a disruption of the properties of central symmetry: the isolines of the steady convection regimes are symmetrical with respect to a rotation through 180° about the center of the cavity (see Fig. 2a), while the supercritical oscillating flows do not exhibit this property. At the same time (see Figs. 3 and 5) the streamlines and isotherms of the time-averaged flows, as well as the isolines of the mean-square pulsations, are centrally symmetrical. This means that the pulsation components of the described self-oscillation convection regimes at the centrally symmetric points of the flow region carry opposite signs.

The flow approximation defined by expressions (5) and (7), continuous over time and space variables, allows us to integrate the equations for the trajectories of the liquid particles

$$dx/dt = v_x(x, y, t); \quad dy/dt = v_y(x, y, t)$$

with sufficiently high accuracy. Of particular interest is the trajectory passing through the center of the cavity $x = y = 0.5$. In the case of stable steady flow, the particle located at the center of the cavity is nonmoving. After the loss of stability due to disruption of central flow symmetry the trajectory passing through the center of the cavity changes into the curve shown in Fig. 4b. This curve takes on the appearance of a quasiperiodic winding about a two-dimensional torus and is characteristic of the weakly supercritical convection regimes, whether in or without the presence of a magnetic field.

The change in the shape of the trajectory passing through the center of the cavity for the case in which $Gr = 10^7$ and $Pr = 0.02$, given a constant increase in the magnetic field, is shown in Fig. 6. In the absence of a magnetic field, the fluctuations in the flow are nonperiodic (see [6, 10]), but they are regularized as early as $Ha = 10$. However, in the horizontal magnetic field passing through the center of the cavity, the trajectory retains its irregular shape (see Fig. 6a). With an increase in the magnetic field, the trajectory becomes regular (see Fig. 6b—f). In this case, we have a gradual reduction in the dimensions of the region into which enters the liquid particle situated at the center of the cavity.

CONCLUSIONS

1. The direction of the magnetic field exerts considerable influence on the structure of the averaged components of the flow and has virtually no influence on the stability characteristics of the steady flows and the spatial properties of the pulsation components in the fluctuating flows.

TABLE 1

Ha	Vertical field			Horizontal field		
	$\overline{v_x'^2}\cdot 10$ [illegible]	$\overline{v_y'^2}\cdot 10$ [illegible]	$\overline{\theta'^2}$	$\overline{v_x'^2}\cdot 10$ [illegible]	$\overline{v_y'^2}\cdot 10$ [illegible]	$\overline{\theta'^2}$
0	0.96	0.87	0.015	0.96	0.87	0.015
10	0.57	0.56	0.0043	0.925	0.837	0.017
20	0.35	0.35	0.0034	0.36	0.35	0.0035
30	0.13	0.14	0.0016	0.12	0.13	0.0016
34.5	0.0014	0.0013	0.000018	0	0	0
40	0	0	0	0	0	0

2. Analogies exist between the increase in the Hartmann number and the reduction in the Grashof number, as well as between the increase in the Hartman and Prandtl numbers.

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