A. Yu. Gelfgat and I. Tanasawa

Numerical Investigation of the Thermocapillary Drift of a Bubble in an Electric Field

Effect of an imposed electric field on the thermocapillary migration of a gas bubble in zero-gravity environment is investigated numerically. An analysis is carried out for an electrically conducting fluid, and two values of the Prandtl number 0.01 and 7. Fluid flow is created by a thermocapillary force, and by an additional electrostatic force which is caused by a non-uniform distribution of the electric charge on the surface of the bubble. It is shown that interaction between the two surface forces may lead either to decrease or to increase of the drift velocity of the bubble. The effect depends on the ratio between both forces and values of the Marangoni and the Prandtl numbers.

1 Introduction

Thermocapillary migration of drops and bubbles under microgravity conditions is one of numerous theoretical problems associated with materials processing in the microgravity environment [1, 2]. Dependence of the surface tension coefficient on the temperature or the concentration leads to appearance of thermocapillary or concentration-capillary force on liquid-liquid or liquid-gas interface whenever a non-uniform distribution of the temperature or the concentration along the interface takes place. In the microgravity environment the thermocapillary force usually becomes the dominant driving source of motion of non-uniformly heated multi-component fluid. Such thermocapillary or concentration-capillary flows arise around drops and bubbles immersed in a non-isothermal fluid, and propel them in the direction parallel to the gradient of the temperature or the concentration.

Since the pioneering work of *Young* et al. [3], where thermocapillary drift of a bubble was studied experimentally and theoretically, the evidence of the thermocapillary drift phenomenon has been approved in several ground-based experiments [1, 4–9]. Several experimental investigations [1, 2, 10–12] launched in spacecrafts and drop towers have shown that the thermocapillary effect is usually the main cause of migration of drops and bubbles in a temperature gradient under microgravity conditions.

A solution of the problem of the thermocapillary drift of a drop in the Stokes limit was obtained by *Young* et al. [3]. Succeeding theoretical works were devoted mainly to analytical analyses of the problem [1, 13–17]. This led to different decompositions of a solution in power series of Marangoni or Reynolds numbers. However, all the known analytical approximations of the solution are limited with rather low values of the Marangoni number.

The numerical investigation of an effect of the thermocapillary force on the motion of a droplet in a uniform fluid flow was started by *Rivkind* and *Sigovtsev* [18]. Formulation of the problem and method of solution used in [18] did not allow to calculate the drift velocity. The first complete numerical analysis, including solution of the full Navier-Stokes and energy equations together with calculation of the drift velocity, was carried out by *Szymzyk* and *Siekmann* [19] for Marangoni and Reynolds numbers not larger than 100, and Prandtl number varying from 0.01 to 100. These results were improved by *Balasubramaniam* and *Lavery* [20] who extended their calculations up to $Ma = 1{,}000$ and $Re = 2{,}000$, and reported dependence of the drift velocity on Marangoni, Reynolds, and Prandtl numbers.

The problem of *e*lectro-*h*ydro*d*ynamic (EHD) fluid motion around an electrically conducting drop, immersed in an electrically conducting fluid and subjected to a uniform electric field, was formulated by *Taylor* et al. [21] who obtained solution of the problem in the Stokes limit, and showed experimental verification of the obtained solution. A system, that consists of an electrically conducting drop surrounded by an electrically conducting fluid in an external electric field, does not contain any inhomogeneous volume charge in the stationary state. On the other hand, non-uniformly distributed electric charges are accumulated on the interface of the two liquids with different electrophysical properties. Interaction of the surface charge with the electric field creates electrostatic force, the tangential component of which produces a fluid flow along the surface.

Griffiths and *Morrison* [22, 23] analyzed the influence of EHD flow described by *Taylor* [21] on heat and mass transfer from a translating drop. *Chang*, *Carleson* and *Berg* [24, 25] investigated the influence of an electric field on the

Mail address: Dr. A. Yu. Gelfgat, Computational Mechanics Laboratory, Department of Mechanical Engineering, Technion–Israel, Institute of Technology, Technion City, Haifa 32000, Israel; I. Tanasawa, Institute of Industrial Science, The University of Tokyo, 7-22-1 Roppongi, Minato-ku, Tokyo 106, Japan.
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terminal velocity of a translating drop, together with heat and mass transfer to the drop. The analysis of *Chang* and *Berg* [25] showed that the terminal velocity of the concentration-capillary drift of a single-drop may either decrease or increase in the presence of the electric field. This analysis included Stokes solution for the velocity and convection-diffusion problem for the concentration.

As a rule, thermocapillary drift of drops and bubbles is not very fast. The issue of increasing the drift velocity is associated with various technological processes of material manufacturing in microgravity, where gas bubbles have to be removed from liquid phase. The present work deals with preliminary numerical estimation of influence of an imposed electric field on the thermocapillary migration of a single bubble in an electrically conducting fluid. Mentioned above surface electric force does not create any drift, but affects the thermocapillary fluid motion in two ways: by the interaction of two surface forces; and by an additional effect of convective heat transfer that alters the temperature distribution on the surface, and hence the thermocapillary force. It is shown that the influence of the electric field on the drift velocity strongly depends on the Prandtl and Marangoni numbers, and the ratio of characteristic values of the thermocapillary and the electric forces.

It should be noticed that there may be at least two additional mechanisms of fluid motion in the electric field. The first one is connected with dependence of surface tension on the electric potential (so-called electrocapillarity [26]), and the second one appears because of a dependence of the electric permeability on the temperature (electroconvection [27, 28]). Nevertheless we believe, that for complete understanding of the whole phenomenon the influence of all the three mechanisms on the thermocapillary migration of drops and bubbles should be studied separately.

2 Formulation of the Problem

The problem of the thermocapillary migration of a single bubble in an electric field is formulated with the following assumptions:

- Shape of the bubble remains spherical.
- Dependence of the surface tension on the temperature is linear. Dependence of all the other properties of the liquid on the temperature is negligibly small.
- Electric field is parallel to the temperature gradient.
- Time necessary to create a stationary distribution of the electric charge on the surface of the bubble is much less than R^*/V^*_{drift} – the characteristic time of the thermocapillary drift.
- Fluid surrounding the bubble is electrically conducting.
- Dependence of all the properties of the liquid on the electric field is negligibly small.
- Electrical conductivity inside the bubble is zero.
- Joule heat and viscous heat dissipation are neglected.
- Viscosity and thermal diffusivity inside the bubble are negligibly small.
- Temperature gradient far from the bubble is constant.
- Electrical field far from the bubble is constant and homogeneous.
- Fluid flow does not influence the electric field.

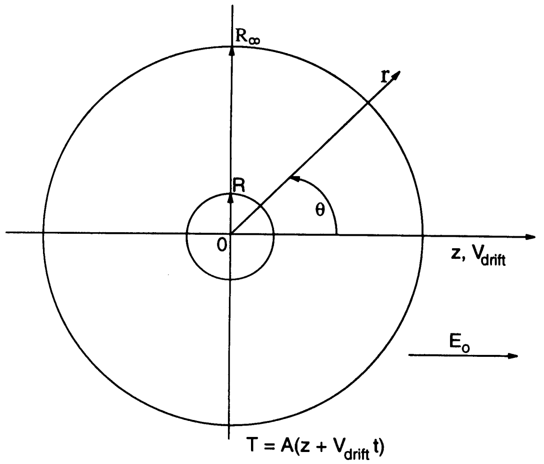


Fig. 1. System of coordinates and sketch of the computational domain

Under these assumptions the spherical bubble migrates parallel to the direction of the temperature gradient with a constant drift velocity V_{drift}. The problem is formulated in a spherical coordinate system with the origin placed at the center of the bubble and the polar axis parallel to the temperature gradient (see fig. 1). The system of coordinates moves together with the bubble in such a way that the origin of the coordinate system always coincides with the center of the bubble. The problem is considered in a computational domain $0 \le r^* \le R^*_\infty$, $0 \le \theta \le \pi$. Temperature at $r^* = R^*_\infty$ is assumed to be equal to $\bar{T}^*_\infty = A^*(z^* + V^*_{drift} t^*)$, where A^* is the external temperature gradient, V^*_{drift} is the drift velocity of the bubble, and t^* is time (here star denotes the dimensionalized variables).

After introducing a new function T^* as $\bar{T}^* = T^* + A^* V^*_{drift} t^*$, where $\bar{T}^*$ is the temperature of the fluid, and using R^*, R^{*2}/ν, ν/R^*, $\varrho\nu^2/R^{*2}$, A^*R^* and E_0 as scales of the length, the time, the velocity, the pressure, the temperature, and the electric field respectively, the problem in the non-dimensionalized form is described with the momentum equation

$$\frac{\partial \boldsymbol{v}}{\partial t} + (\boldsymbol{v}\nabla)\boldsymbol{v} = -\nabla p + \Delta \boldsymbol{v}, \tag{1}$$

the continuity equation

$$\operatorname{div} \boldsymbol{v} = 0, \tag{2}$$

the energy equation

$$\frac{\partial T}{\partial t} + V_{drift} + (\boldsymbol{v}\nabla)T = \frac{1}{Pr}\Delta T, \tag{3}$$

and the equation for the electric potential

$$\Delta\varphi = 0 \tag{4}$$

where $\boldsymbol{v}$ is the velocity, p is the pressure, $Pr = \nu/\chi$ is the Prandtl number, ν is the kinematic viscosity, and χ is the thermal diffusivity of the fluid, R is the radius of the bubble, ϱ is the density; φ is the potential of the electric field.

The boundary conditions of the problem in non-dimensionalized form are the following:

at the boundary of the computational domain $r = R_\infty$:

$$v_r = V_{drift} \cos(\theta), \quad v_\theta = V_{drift} \sin(\theta), \quad \varphi = \varphi_\infty(R_\infty, \theta) \tag{5}$$

$$T = R_\infty \cos(\theta), \quad \text{for } 0 \le \theta \le \frac{\pi}{2};$$
$$-\frac{\partial T}{\partial z} = -\cos(\theta)\frac{\partial T}{\partial r} + \frac{\sin\theta}{R_\infty}\frac{\partial T}{\partial \theta} = -1, \quad \text{for } \frac{\pi}{2} \le \theta \le \pi; \tag{6}$$

at the axis of symmetry $\theta = 0, \pi$:

$$v_\theta = 0, \quad \frac{\partial v_r}{\partial \theta} = 0, \quad \frac{\partial T}{\partial \theta} = 0, \quad \frac{\partial \varphi}{\partial \theta} = 0; \tag{7}$$

at the surface of the bubble $r = 1$:

$$v_r = 0, \quad \frac{\partial T}{\partial r} = 0, \quad \varphi^{ex} = \varphi^{in}, \quad \frac{\partial \varphi^{ex}}{\partial r} = 0; \tag{8}$$

$$\frac{\partial}{\partial r}\left(\frac{v_\theta}{r}\right) = -Mn\frac{\partial T}{\partial \theta} + El\frac{\partial \varphi^{ex}}{\partial \theta}\left(\frac{\varepsilon^{ex}}{\varepsilon^{in}}\frac{\partial \varphi^{ex}}{\partial r} - \frac{\partial \varphi^{in}}{\partial r}\right). \tag{9}$$

at the centre of the bubble $r = 0$ only one boundary condition for the electric potential is necessary:

$$\varphi(r = 0) = 0. \tag{10}$$

In eqs. (5)–(10) $Mn = -(\partial\gamma/\partial\bar{T})(AR^2/(\varrho\nu^2))$ and $El = \varepsilon_0\varepsilon^{in}E_0^2R^2/(\varrho\nu^2)$ are non-dimensional parameters defined as ratios of the characteristic thermocapillary force and the characteristic electrostatic surface force to the characteristic viscous force. γ is the surface tension coefficient, ε_0 is the electric permeability of the vacuum, ε is the specific electric permeability, and E_0 is the intensity of the electric field far from the bubble. Superscripts *in* and *ex* correspond to areas inside and outside the bubble, respectively. If $\partial\gamma/\partial\bar{T} < 0$ the bubble migrates towards the temperature gradient, and in the opposite direction if $\partial\gamma/\partial\bar{T} > 0$.

The drift velocity V_{drift} has to be defined from the requirement of the balance of all the forces acting on the surface of the bubble. This leads to an additional equation

$$F = 2\pi\int_0^\pi\left[v_\theta(1 - 3\cos^2(\theta)) - \sin(2\theta)\frac{\partial v_\theta}{\partial \theta} - \sin^2(\theta)\frac{\partial v_\theta}{\partial r} - p\cos(\theta)\sin(\theta)\right]_{r=1} d\theta = 0. \tag{11}$$

The formulation of the problem given by eqs. (1)–(11) needs the following comments:

- We do not define the velocity scale with the use of dimensionalized thermocapillary force – $[v] = -(\partial\gamma/\partial\bar{T})(AR/(\varrho\nu))$, and the corresponding Reynolds number $Re = Mn = -(\partial\gamma/\partial\bar{T})(AR^2/(\varrho\nu^2))$ as usually is accepted in problems of thermocapillary convection [20, 29]. This is because in the considered problem two independent forces create the flow, and hence there is no way to choose one of them for definition of the velocity scale. Thus, we do not describe the non-dimensional parameter *Mn*, which is the ratio of the characteristic thermocapillary force to the characteristic viscous force, as the Reynolds number. Definition of the Marangoni number $Ma = Mn \cdot Pr = -(\partial\gamma/\partial\bar{T})(AR^2/(\varrho\nu\chi))$ used here is the same as usual [20, 29].
- The formulation given by eqs. (1)–(11) is not restricted with requirement of high electric conductivity of the surrounding fluid. None of the non-dimensional parameters *Mn*, *Pr*, *El* contains the electric conductivity, so the formulation of the problem may be extended also to fluids that are not perfect conductors of electricity. The stationary state will be correctly described if time, necessary for accumulation of the stationary surface charge, is much less than the characteristic time of the thermocapillary drift. The last one may be estimated as R^*/V^*_{drift}. Reported by *Taylor* et al. [21] experimental observation of EHD flow, which was caused by non-uniform distribution of the surface charge, was done with the use of a silicone oil drop immersed in the mixture of castor oil and corn oil. All the liquids used in this experiment usually are considered as dielectrics.
- The Joule heating of the surrounding fluid is described by the non-dimensionalized source term $(\sigma R^* E_0^2/(Pr \cdot \lambda))(\nabla\varphi)^2$ in the energy equation (3). Here σ is the electric conductivity, λ is the heat conductivity. It is clear, that this term is important for large values of σ and small values of *Pr*, which is characteristic for melts of metals and semiconductors. On the other hand, this term is negligibly small for transparent fluids, such as water or organic oils usually used in experiments. Since the present numerical analysis does not refer to any concrete fluid, and was done to see the distinct interaction of EHD and thermocapillary convective flows in a numerical experiment, we neglected the Joule heating in the mathematical model.

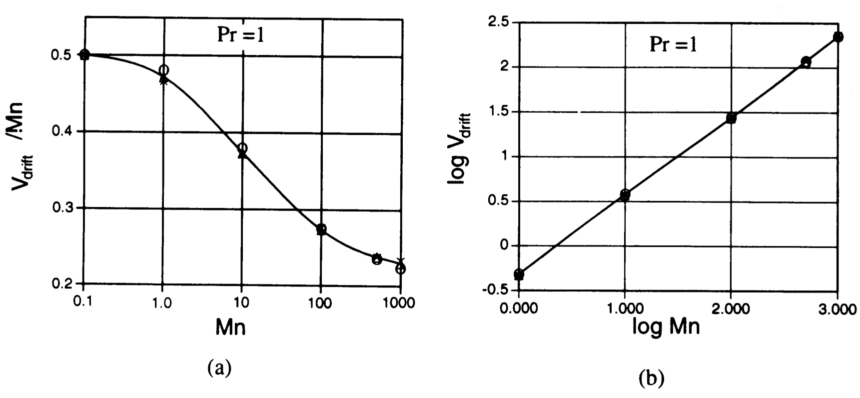
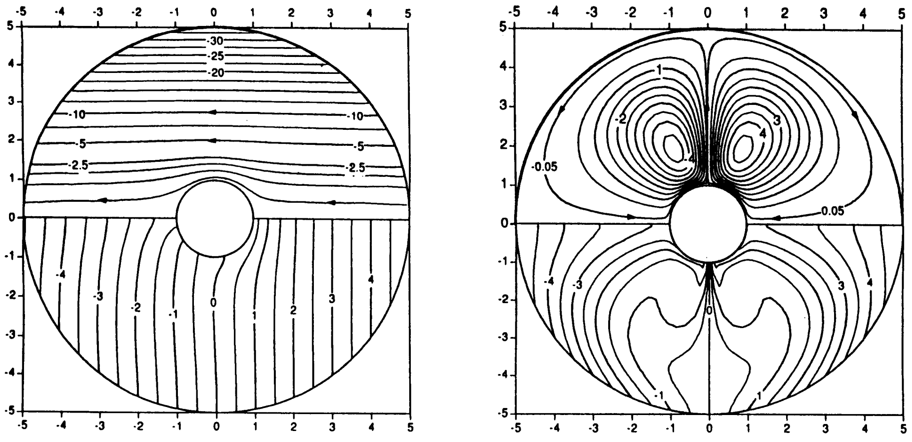


Fig. 2. Comparison of the present results with the results of Balasubramaniam and Lavery [20] for Pr = 1. ○ – results of R. Balasubramaniam and J. Lavery, grid 64 × 32; × – present results, grid 64 × 32; △ – present results, grid 128 × 64



(a) (b)

Fig. 3. Streamlines (upper part) and isotherms (lower part) of the flows for $Pr = 7$. (a) $Mn = 10$, $El = 0$, $Ma = 70$, $V_{drift} = 2.75$; (b) $Mn = 0$, $El = 100$, $Ma = 0$, $V_{drift} = 0$

With the assumptions made the problem for the electric field may be solved separately. In the case of the constant and homogeneous external electric field an analytical solution of the problem is known [21]. For the vanishing electric conductivity inside the bubble the following expressions define the electric potential inside and outside the bubble:

$$\varphi^{in} = \frac{3}{2} E_0 r \cos(\theta), \quad \varphi^{ex} = E_0\left(r + \frac{1}{2r^2}\right)\cos(\theta). \tag{12}$$

Using eq. (11), the balance of tangential stress at the surface of the bubble given by eq. (9) may be rewritten as:

$$\left[\frac{\partial}{\partial r}\left(\frac{v_\theta}{r}\right)\right]_{r=1} = -Mn \cdot \left[\frac{\partial T}{\partial \theta}\right]_{r=1} + \frac{9}{4} El \cdot \cos(\theta)\sin(\theta). \tag{13}$$

3 Numerical Method and Test Calculations

The problem (1)–(11) was solved with the finite volume method using predictor-corrector semi-implicit scheme for straight-forward integration in time. The complete numerical procedure is described in detail in [30].

Straight-forward integration in time was carried out until convergence to a stationary solution was reached. An additional formal equation

$$\frac{dV_{drift}}{dt} = \alpha|F|^{\beta}, \quad \text{for } \beta > 0 \tag{14}$$

was added to the problem for obtaining the drift velocity in the same computational process. Here F is defined in eq. (11), α and β are constants to be obtained empirically. It is obvious that a constant value of V_{drift} corresponds to $F = 0$. The additional eq. (14) was used after a correct value of V_{drift} was localized with the secant method. Our numerical experiments showed that convergence of the whole numerical process can be reached with $\beta = 1$ and α of the same order of magnitude with Mn. Δt, where Δt is the time step. Convergence was supposed to be reached when all the unknown functions (the fluid velocity, the pressure, the temperature, and the drift velocity) satisfied the stopping criterion $\|f^{n+1} - f^n\|/\|f^n\| < 10^{-5}$, where f is one of the unknown functions, n is the number of the time step. Superposition of the analytical solution obtained by *Balasubramaniam* and *Chai* [29] for the thermocapillary drift with $Pr = 0$, and the analytical solution of *Taylor* et al. [21] for EHD flow around a bubble in the Stokes limit was used as an initial condition.

The test calculations were done for $Pr = 1$ and $El = 0$. Comparison of the obtained results with the results of *Balasubramaniam* and *Lavery* [20] is shown in fig. 2. The test calculations were carried out for the same value of $R_\infty = 5$, and with the same uniform grid consisting of 64 nodes in r- and 32 nodes in θ-direction, as it was used in [20]. Then calculations were repeated on the twice finer grid consisting of 128 × 64 nodes.

Fig. 2a shows dependence of the non-dimensional drift velocity scaled by Mn on the parameter Mn. The scaled values of V_{drift} correspond to the scaling used in [20]. As it was mentioned above, the Reynolds number defined in [20] coincides with parameter Mn used here. Increase of V_{drift} with the increase of Mn is more apparent with the scaling used in the present work, as is illustrated in fig. 2b, where the logarithmic scales are used for both V_{drift} and Mn. As it is seen from fig. 2, all the results are in good agreement. The largest discrepancy in values of V_{drift} is observed at $Mn = 1$, but even in this case the discrepancy is less than 3 %.

In the case of $El \neq 0$ only Stokes solution obtained in [21] was used for test calculations. Good convergence to the Stokes solution was obtained with the use of zero initial conditions. Taking into account that finite difference approximation of the non-linear terms in the Navier-Stokes and the energy equations was checked in the case of $El = 0$, $Mn \neq 0$, the test calculations were considered to be completed.

4 Results

The numerical analysis was carried out for two values of the Prandtl number, $Pr = 0.01$ and $Pr = 7$, and the ratio El/Mn varying from 0 to 40. A uniform finite difference grid in r-direction was used for $R_\infty \leq 10$, and a non-uniform grid with Chebyshev nodes shifted to the interval $[1, R_\infty]$ for $R_\infty > 10$. The grid in θ-direction was always uniform. The number of nodes varied from 80 to 130 in r-direction and from 40 to 60 in θ-direction.

The patterns of calculated flows are plotted in figs. 3–5. Streamlines of the flow are shown in the upper halves of the plots, and the isotherms of the shifted temperature $T = \bar{T} - V_{drift}t$ are shown in the lower parts. Arrows on the streamlines indicate the direction of fluid flow in the coordinate system connected with the bubble.

Fig. 3 illustrates thermocapillary flows without electric field (fig. 3a), and EHD flows without thermocapillary

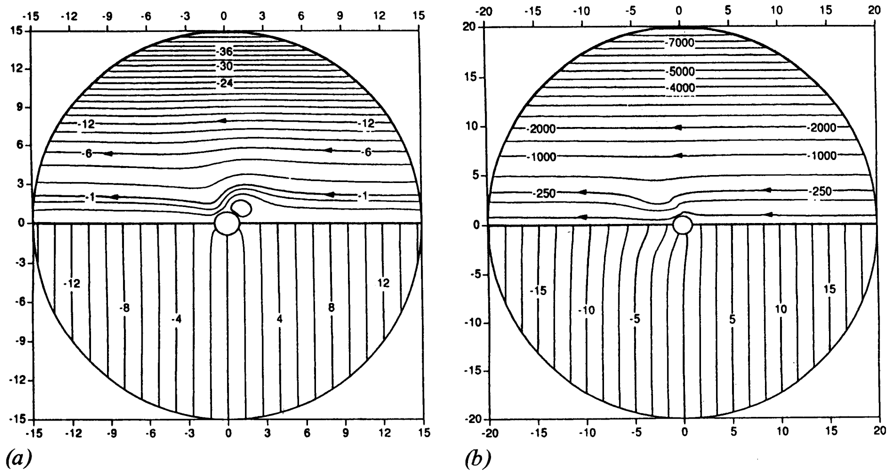


Fig. 4. Streamlines (upper part) and isotherms (lower part) of the flows for $Pr = 0.01$. (a) $Mn = 1$, $Ma = 0.01$, $El = 10$, $V_{drift} = 0.397$; (b) $Mn = 100$, $Ma = 1$, $El = 2{,}000$, $V_{drift} = 41.1$

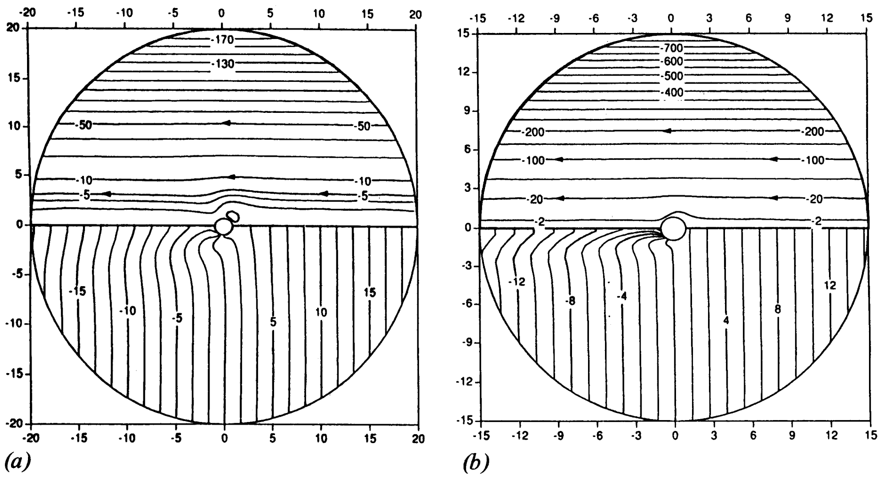


Fig. 5. Streamlines (upper part) and isotherms (lower part) of the flows for $Pr = 7$. (a) $Mn = 1$, $Ma = 7$, $El = 20$, $V_{drift} = 0.939$; (b) $Mn = 10$, $Ma = 70$, $El = 100$, $V_{drift} = 7.19$

force (fig. 3b) for $Pr = 7$. Comparison of the two plots shown in fig. 3 leads to two main conclusions.

(1) The direction of EHD flow consisting of two toroidal vortices generally does not coincide with the direction of the thermocapillary flow, which is non-uniform only near the bubble. This may lead to distortion of the thermocapillary flow in the vicinity of the bubble and, as it will be shown, to slow down the thermocapillary drift.

(2) Two toroidal vortices of the EHD flow drive cold and hot fluid towards the line $\theta = \pi/2$ (fig. 3b). This increases the temperature gradient along the surface of the bubble, and consequently increases the thermocapillary force. Thus, changes of the temperature on the surface of the bubble, caused by EHD convective heat transfer, may intensify the thermocapillary convection and increase the drift velocity. The resulting effect of the electric field on the drift velocity of the bubble depends on which of two tendencies prevails.

The calculated dependence of the drift velocity on the electric field is illustrated in fig. 6 where ratio $V_{drift}(El \neq 0)/V_{drift}(El = 0)$ is plotted versus the ratio El/Mn. The parameter El/Mn is the ratio of the characteristic electrostatic surface force to the characteristic thermocapillary force, and characterizes the relative contribution of both forces to the resulting flow. It is clear that flow with a small ratio El/Mn will be close to the thermocapillary flow without an electric field.

4.1 Case of $Pr = 0.01$

In the case of $Pr = 0.01$ (fig. 6a), the dependence of V_{drift} on the ratio El/Mn strongly depends on the value of Mn or, since $Ma = Mn \cdot Pr$, on the value of Ma. When the Marangoni number is relatively small, the drift velocity decreases with the increase of the electric field at least for the considered values of El/Mn. But for larger values of Ma the drift velocity decreases when the ratio El/Mn is less than approximately 8, and then increases with the increase of El/Mn.

The difference in the dependence of V_{drift} on El/Mn for different values of Ma is caused by two different tendencies of the effect of the electric field on the thermocapillary convection around a bubble. This may be figured out from the analysis of flow patterns plotted in figs. 4 and 5.

In the case of small Marangoni number (e.g., $Mn = 1$, $Ma = Mn \cdot Pr = 0.01$) and the ratio El/Mn less than approximately 10, the streamlines of the flow remain the same shape as is illustrated in fig. 3a. After the ratio El/Mn exceeds 10, the interaction between the two forces leads to appearance of a small vortex ahead of the migrating bubble (fig. 4a). On the part of the bubble surface, connected with this small vortex, the surface velocity is opposite to the direction of the drift. This leads to a decrease of the drift velocity seen in the curve corresponding to $Ma = 0.01$ in fig. 6a. The isotherms of the flow in the case $Ma = 0.01$ remain symmetric with respect to the plane $\theta = \pi/2$, which means that the effect of convective heat transfer is weak.

When the parameter Mn is larger (e.g., $Mn = 10$, $Ma = Mn \cdot Pr = 0.1$) the thermocapillary force is stronger and the additional vortex does not appear. Distribution of the temperature when $El/Mn \leq 10$ is also close to symmetric with respect to the line $\theta = \pi/2$, which indicates that the heat transfer is still driven mainly by heat conduction. As in the case of $Ma = 0.01$ the electric force slows down the thermocapillary drift (fig. 6a). On the contrary, when the parameter El increases such that $El/Mn > 10$, the symmetry of the isotherms becomes visibly broken near the bubble (fig. 4b). This means that influence of the thermocapillary force on the convection of heat becomes larger. Since the Marangoni number is maintained constant, one can conclude that the growth of the thermocapillary force is caused by the increased temperature gradient on the surface of the bubble. The increase of the temperature gradient is a sequence of the convection created by the electric force. The growth of the thermocapillary force affects the drift velocity too. V_{drift} begins to grow with the growth of the electric field when $El/Mn > 10$ (fig. 6a).

Similar changes in the patterns of the isolines and the isotherms are observed for larger values of Mn (fig. 6a). Since both forces are larger than those in the previous cases, both effects of decrease and increase of the drift velocity are stronger, and in the case $Mn = 100$ an abrupt change from decreasing to increasing V_{drift} takes place at $El/Mn \approx 8$ (fig. 6a).

In the case of $Mn = 1{,}000$ we could calculate only flows with $El/Mn \leq 15$. Larger values of El/Mn gave rise to time-wise oscillations of the numerical solution. It should be noticed that among oscillatory instabilities of the thermocapillary or EHD flow there may be an additional instability mechanism connected with the two tendencies of increase or decrease of the drift velocity. The mathematical model used here does not describe a non-stationary flow with oscillating drift velocity, so appearance of an oscillatory numerical solution was considered as a limit of applicability of the mathematical model or the numerical method.

4.2 *Case of Pr = 7*

In the case of $Pr = 7$ (fig. 5) the convective heat transfer is stronger, and the effect of increase of the drift velocity in the increasing electric field prevails. As it is seen from fig. 6b the growth of V_{drift} with the growth of El/Mn is faster for larger values of Ma. The asymmetry of the isotherms, caused by the thermocapillary convection, is noticeable for $Mn \geq 1$. Small vortex ahead of the moving bubble appears in the case $Mn = 1$, $El \geq 1$ ($Ma = 7$, see also fig. 5a) as it took place for $Pr = 0.01$ (fig. 4a). As it was noticed, appearance of this additional vortex slows down the thermocapillary drift. But in the case of $Pr = 7$ the EHD convection increases the temperature gradient on the rear part of the moving bubble (fig. 5a), and hence increases the thermocapillary force there. This leads to the prevailing effect of increase of the drift velocity.

In the case of $Mn = 10$ ($Ma = 70$, see fig. 5b) the temperature gradient in the rear part of the bubble increases sharply with the increase of El/Mn. This leads to very intense thermocapillary convection of heat, and hence to the rapid increase of the drift velocity, as it can be seen from fig. 6b.

5 Conclusions

The described analysis shows that the electric field has dual influence on the thermocapillary drift of a single bubble. The thermocapillary flow around the bubble may be slowed down by the electrohydrodynamic (EHD) flow. On the other hand the convective heat transfer, created by the EHD flow, increases the temperature gradient along the surface of the bubble, and hence increases the thermocapillary force.

In the case of small Prandtl number, $Pr = 0.01$, the thermocapillary drift is slowed down by the interaction between the thermocapillary and EHD flow, when the Marangoni number is small (of the order of 10^{-2} or less). For larger values of the Marangoni number, at least for $0.1 \leq Ma \leq 1$, the slowing down effect exists only for small values of the ratio El/Mn, and is replaced by intensifying one with the increase of El/Mn. The mathematical model and the numerical method used in the present work do not allow to extend last conclusion for larger values of the Marangoni number, such as $Ma = 10$ ($Mn = 1{,}000$) or more.

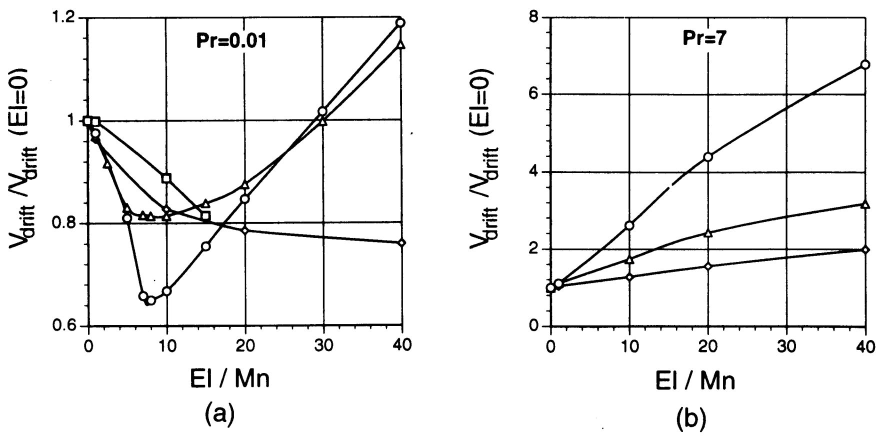


Fig. 6. Dependence of the drift velocities on the ratio El/Mn for different values of the Marangoni number. (a) $Pr = 0.01$, ◇ $Mn = 1$, $Ma = 0.01$, △ $Mn = 10$, $Ma = 0.1$, ○ $Mn = 100$, $Ma = 1$, □ $Mn = 1{,}000$, $Ma = 10$; (b) $Pr = 7$, ◇ $Mn = 0.1$, $Ma = 0.7$, △ $Mn = 1$, $Ma = 7$, ○ $Mn = 10$, $Ma = 70$

In the case of moderate Prandtl number, $Pr = 7$, the convective heat transfer caused by electrohydrodynamic flow is stronger and intensification of the thermocapillary drift prevails for all considered values of the Marangoni number. The growth of the drift velocity with the increase of El/Mn in this case becomes faster for larger values of Ma.

The preliminary results presented here approve the proposition that the terminal velocity of the thermocapillary drift of a gas bubble may be increased by an imposed electric field. The noticeable growth of the drift velocity may take place in fluids with relatively large Prandtl number. On the other hand, decrease of the drift velocity in an imposed electric field may occur in low-Prandtl-number fluids.

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