



# Regulating inter-firm agreements: The case of airline codesharing in parallel networks



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## ABSTRACT

We compare aviation markets under conditions of competition, codesharing contracts and anti-trust immune alliances, assuming that demand for flights depends on both fares and the level of frequency offered. Using a hybrid competitive/cooperative game theoretic framework, we show that the stronger the inter-airline agreement on overlapping routes, the higher the producer surplus. On the other hand, consumer surplus and overall social welfare are maximized under limited codesharing agreements. Partial mergers appear preferable to no agreement in ‘thin’ markets, in which both demand and profit margins are relatively low. Inter-governmental agreements are also analyzed and we show that bilaterals create the least favorable market outcomes for consumers and producers. Finally, a realistic case study demonstrates that under asymmetric and uncertain demand, codesharing on parallel links may be preferable to competitive outcomes for multiple consumer types.

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## 1. Introduction

Consumers demonstrate demand patterns that are increasing in the levels of variety offered (Dixit and Stiglitz, 1977). For example, Copeland et al. (2011) analyze the changes in new car prices over time and demonstrate that consumers are willing to pay for higher levels of inventory in a car showroom, which is an indirect measure of variety. In aviation markets, passengers prefer higher frequency because such choice reduces potential schedule delay, defined as the difference between the time that the passenger prefers to travel and the closest scheduled alternative (Douglas and Miller, 1974). In empirical analyses of aviation markets, both Ippolito (1981) and Hansen (1990) demonstrate that passenger utility increases in frequency and Belobaba (2009) shows that higher frequency leads to disproportionately higher market share. In line with this argument, we note that there is a substantial literature that analyzes inventory policies under the assumption of inventory level dependent demand (see e.g., Balakrishnan et al., 2004; Krishnan and Winter, 2010; Sapra et al., 2010; Urban, 2005; Wang and Gerchak, 2001). Consequently, the empirical and theoretical literature indicates that consumers are willing to pay for higher levels of variety, where the airline frequency represents both variety and inventory.

The question then arises as to whether there are benefits to signing contracts between firms in light of variety or inventory level dependent demand. In this paper we examine the aviation industry and analyze the effects of various potential agreements between airlines on overlapping routes in order to understand the impact of partial mergers. Such agreements are signed considering passenger preferences for higher flight frequency, which, in turn, can be viewed as variety level dependent demand. In

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analyzing the potential effect of various aviation agreements in a given market, we aim at identifying the trade-offs between the surpluses of airlines and consumers considering both business and leisure passenger types.

In the airline industry, there are two potential levels of agreement, one between countries and the other between airlines. Agreements between countries will either create a competitive 'open skies' market, in which airlines are free to set frequencies and prices in an unrestricted manner, or a restricted bilateral market in which two or more airlines are designated as carriers, frequencies and/or seat capacity are limited and occasionally pricing policies are defined too. The latter defines the majority of international markets to date (Borenstein et al., 2014). Subsequent to country-level agreements, the airlines can choose to sign a codeshare provided both governments agree, or to ally if anti-trust immunity is granted. We note that the United States and European systems behave differently in that the United States regulator generally carves out the hub-to-hub link in a codeshare agreement, whereas the European regulators are more willing to accept codeshare agreements but add conditions in order to restrict potential reductions in frequencies or increases in prices on parallel routes.

A codeshare agreement between two airlines allows each airline to market flights of the partner airline using its own code name, thus increasing the schedule offered. The advantages of codesharing from the consumers' perspective include access to larger networks, coordination of schedules and flights, more frequent service, wider choice of carriers and greater access to frequent flyer points. A codeshare arrangement may also provide consumers with improved airport facilities including smoother baggage handling, access to lounge facilities overseas, faster transfer times, easier connections etc. Furthermore, codeshare flights facilitate the check-in procedure in the sense that passengers receive boarding passes necessary for all segments of the flight at the origin (Hassin and Shy, 2004). The question with respect to consumer welfare arises from the fact that fares may decrease or increase, dependent on the result of the codeshare with respect to the specific market. The argument is that prices may diminish on beyond gateway trips in which the passenger purchases a single ticket instead of two, but they are likely to rise in gateway-to-gateway markets. The second problem derives from the question as to whether consumers are sufficiently aware of their choices.

The benefits of codesharing to airlines can be broadly considered in terms of increases in market share, traffic and revenue and cost reductions through shared risk and higher load factors. The airlines' economic incentives for codesharing include the following:

1. Increasing market share. Interline itineraries, whereby a passenger must purchase at least two tickets from different carriers in order to travel from an origin to a chosen destination are likely to be more expensive than purchasing comparable online itineraries in which the passenger purchases a single ticket but may travel with more than one operating airline. The comparable gateway market may require a passenger to purchase a single ticket in each direction in order to create their preferred schedule which is also unnecessary under codesharing. This is in part due to double marginalization (Tirole, 1988), an issue that arises when each carrier on an interline itinerary maximizes the profits from its own segment independently of the other carrier. Cooperative pricing therefore encourages the carriers to internalize at least part of the double marginalization externality associated with joint pricing on interline tickets (Brueckner, 2003; Whalen, 1999) whilst earning higher revenues through increased demand for the product. However, this hypothesis does not predict codeshare fares to lie below pure online itineraries (i.e. purchasing a single ticket from one carrier acting both as operating and marketing company), rather somewhere in-between.
2. Network and frequency expansion. Under traditional codesharing, in order to provide coordinated service covering non-overlapping networks, individual airlines increase their capacity and the number of airports served, which is in the interest of the carrier and consumer alike. Codesharing may also be used by a carrier looking to access new markets without incurring the costs associated with launching additional services using its own aircraft. Furthermore, it permits carriers to expand their network which would not be possible under the existing bilateral agreements. However, some of the gains may be at the expense of competing airlines (Park et al., 2001). According to Biggar's OECD roundtable discussion in 2000, the ability to compete effectively on many routes depends on capturing a share of the high-margin time-sensitive business travelers. For time-sensitive travelers, there would appear to be a strong preference for connections between flights of the same airline and airlines with more frequent service. A merger or alliance can allow the merged airline to lower costs and enhance demand for its services by rationalizing hub-and-spoke structures and offering a greater range of connections. Moreover, making use of loyalty programs such as frequent flyer or travel agent commission override programs, provides incentives for travelers and travel agents to focus their bookings on the same airline. These programs are especially targeted at attracting and retaining high-margin, time-sensitive, business travelers.
3. Cost reductions. Expansion of a carrier's network through codesharing provides airlines with economies of scope, scale and density since the airline can serve new markets without increasing fleet size and may enjoy economies in terminal operations through expanded traffic (Gellman Research Associates, 1994). Consequently, these incentives both act to increase fares through reduction in competition and lower fares in order to attract greater market share, potentially possible due to lower costs and higher revenue. Codesharing with a larger airline may improve the position of a smaller airline relative to other large carriers and hence promote competition. On the other hand, codesharing may be considered anti-competitive if large carriers use these arrangements to eliminate smaller rivals.
4. Improved Listings on Computer Reservation Systems. Codeshare flights appear on computer reservation systems (CRS) multiple times, potentially crowding out alternative itineraries onto the second screen. (Humphreys, 1994) has shown that 80% of bookings draw on alternatives from the first screen of a CRS. This could lead to anti-competitive behavior with potentially

**Table 1.1**  
Agreement types.

	No agreement (competition)	Bilateral agreement	Competitive block codeshare agreement	Bilateral block codeshare agreement	Competitive free sale codeshare agreement	Bilateral free sale codeshare agreement	Alliance agreement
Compete	<i>f, p</i>	<i>p</i>	<i>f, p</i>	<i>p</i>	<i>f, p</i>	<i>p</i>	
Coordinate		<i>f</i>		<i>f</i>		<i>f</i>	<i>f, p</i>
Codesharing	<i>no</i>	<i>no</i>	<i>yes</i>	<i>yes</i>	<i>yes</i>	<i>yes</i>	<i>yes</i>
Residual sale	<i>none</i>	<i>none</i>	<i>pre-det.</i>	<i>pre-det.</i>	<i>on demand</i>	<i>on demand</i>	<i>on demand</i>

higher fares resulting (Whalen, 1999). In Europe a particular flight cannot be listed more than twice in the CRS but in the U.S. there are no such controls over the number of times that a codeshare flight may appear.

Within the legal framework set by the countries involved, inter-airline codeshare agreements may be signed according to the following three categories: soft or hard block agreements, free sale agreements and anti-trust immune alliances. Additional types of agreements such as pooling<sup>1</sup> and royalties<sup>2</sup> are no longer permitted in most of the developed markets. Block codeshare agreements require a marketing carrier to swap or purchase a pre-agreed set of seats on the operating carrier's flight, normally per season. The marketing carrier is then responsible for selling the seat inventory and receives all revenues from sales, which in turn increase their schedule offering. As compared to a hard block agreement, soft blocks permit a limited number of seats to be released by the marketing carrier back to the operating carrier, generally up to two weeks prior to the flight. Free sale agreements assume that the carriers codeshare and pay a pre-agreed price or percentage of ticket revenues, provided the operating carrier can accommodate the request and it is worthwhile doing so from an economic perspective. The marketing carrier receives a pre-agreed commission on the revenue collected to cover costs such as ticketing, marketing and frequent flyer bonuses. Under anti-trust immune alliances, airlines maximize profits jointly by coordinating both frequencies and prices.<sup>3</sup> The newest version of an anti-trust immune alliance is a joint venture in which the airlines do not separate between the marketing and operating carriers, referred to as a metal neutral version of the codeshare (Brueckner and Proost, 2010). The joint venture is normally applied to a specific set of routes and all costs and revenues are jointly pooled.

By combining the agreements between countries and inter-airline codeshares, we note that three potential market outcomes are possible under open skies, in which airlines from either country are free to enter any airport pair market between the countries at any level of frequency they deem relevant. The three potential outcomes are (i) competition, (ii) competitive block codeshares, in which seat swaps or purchases are pre-agreed and (iii) competitive free sale codeshares, in which the operating carriers continue to control their own seat inventory. Airfares are supposedly set competitively in all three cases. Under bilateral agreements, home carriers are designated to serve specific markets and frequencies are capped. This leads to three more potential market outcomes, namely (iv) bilaterals, (v) bilateral block codeshares and (vi) free sale codeshares. The two latter markets allow the airlines to share production but are still required to compete in price. Finally, under anti-trust immunity (vii), airlines behave as if they were a single company by setting both frequencies and prices jointly. The final result of these two layers of agreements is seven potential market equilibria outcomes in international markets as defined in Table 1.1. Consequently, frequencies, *f*, and prices, *p*, may be set under competitive or coordinated circumstances, with or without airline codeshare agreements. Furthermore, under the block agreement, seat inventory is shared prior to demand realization (pre-determined), whereas under free sale agreements, inventory is shared on demand, as shown in the last row of Table 1.1 entitled 'residual sale'.

We consider an airline's decision process within a two-stage setting. Facing uncertain demand, airlines choose frequencies offered and whether to sign codesharing agreements. After the uncertainty clears, the airlines set their prices. A two-stage setup is reasonable because product quantities are a medium to long term strategic variable and prices are more easily changed in the short term. Moreover, choosing prices after demand realization is useful in reducing mismatches with the first stage quantities. Such a model is a natural extension of the well studied Cournot game, because Kreps and Scheinkman (1983) show that in a market with homogeneous products and demand certainty, a one-shot Cournot model is equivalent to a two-stage model in which demand is determined by second stage price competition and sales are subject to the quantity constraints generated in the first stage. Similar conclusions hold also in the case of differentiated products (Yin and Ng, 1997).

The novelty of our approach is the combination of a two-stage setting under potential agreements together with the assumption that supply directly impacts the consumer's demand function.<sup>4</sup> Moreover, a hybrid competitive and cooperative model

<sup>1</sup> Pooling agreements set a unit fare (often based on the previous year's revenues collected) and any imbalance between revenues is then transferred to the partner with lower earnings. Frequently the transfer is limited to a maximum percentage transfer. This type of agreement removes competition at the frequency level and also sets a minimum price.

<sup>2</sup> Royalty agreements occur when one airline pays the other airline a percentage of revenue earned on all passengers carried. Royalties of this type simply add further costs to the carrier which is then transferred to the passenger.

<sup>3</sup> We note that one of the authors was privy to reading a series of international codeshare agreements on markets to and from Israel when working with the Israeli Anti-Trust Authority on changing the laws surrounding codeshares. All such agreements existed in the markets and have been taken into account in Section 4.

<sup>4</sup> A substantial difference between our model and previous work analyzing quality choice by firms (e.g., Allon and Federgruen, 2009; Brueckner and Flores-Fillo, 2007) lies in the assumption that all demand is met, irrespective of the capacity chosen, which is not the case in our model.

permits us to analyze both consumer and producer surplus in a differentiated, imperfectly competitive market under various potential agreements. Our analysis is based on two-stage Nash equilibrium and Nash bargaining solution (Nash, 1950), which are computed where relevant for each of the agreement types.

Initially we solve the model analytically for two airlines serving two consumer types under the assumptions of certainty and symmetry in demand and costs. The existence of a unique solution in frequencies and prices is proved for each agreement type. Then the ranking of agreement types with respect to consumer and producer surpluses is characterized, where the summation of the two is referred to as social welfare. We show that airline profitability is always higher when airlines codeshare and is highest under anti-trust immune alliances. We also show that consumer surplus is always maximized when airlines codeshare, provided frequencies and prices are set independently (i.e. governments sign open skies agreements). Moreover, the lowest consumer surplus and social welfare outcome occurs when airlines do not codeshare, compete in prices but set frequencies jointly under restricted bilaterals i.e. capacity agreements are worse than mergers. This conclusion is of particular interest today in light of the fact that many airlines are now arguing that skies should be protected from unfair competition (Tretheway and Andriulaitis, 2015; de Wit, 2014). Given that the Gulf carriers choose not to participate in alliance agreements, both the European and American airlines prefer bilaterals over open skies in order to increase prices and profits, however this would lead to the lowest frequencies and consumer surplus in comparison to all other possible alternatives. Notably, even anti-trust immune alliances are strictly preferable to inter-governmental restricted bilateral agreements with respect to consumer surplus, producer profits and social welfare, yet the latter is significantly more prevalent in international aviation markets to date.

An additional conclusion is that under thin market conditions, any form of inter-airline agreement increases consumer surplus beyond that of the competitive equilibrium outcome. Thin markets arise when the demand magnitude, a measure of market size, as well as the consumers' willingness to pay, are both sufficiently small as compared to variable costs. Normally, it would be argued that a reduction in competition from a duopoly to a quasi-monopoly through an alliance agreement (with or without anti-trust immunity) will likely lead to higher prices that increase firm profits at the expense of consumer surplus. We would therefore argue that the granting of anti-trust immunity to Northwest-KLM on their hub-hub routes but the failure to do so for United-Lufthansa with heavier traffic between hubs (according to Brueckner and Proost, 2010) would appear to be in line with our findings. Clearly, these results ought to be of interest to airline managers and Competition Authorities, as it may be of note when evaluating the potential effects of an alliance a-priori.

The intuition underlying these results draws from an interesting trade-off with respect to frequency. When airlines cooperate, the reduction in the level of competition would be expected to push prices upwards and frequencies downwards, to the benefit of the producer at the expense of the consumer. On the other hand, as a result of our modeling of frequency dependent demand, both consumers and airlines alike prefer the airlines to codeshare. Since consumers gain from codesharing through joint scheduling, even though total frequency may remain unchanged, their higher willingness to pay in turn encourages airlines to further increase frequency offered. Consequently there are no free riding effects whereby codesharing causes the airlines to reduce frequency by relying on the codesharing agreement. The combined impact of these two effects leads to our finding that, under thin market conditions, the second effect outweighs the first whereas in the remaining markets, airlines and consumers present directly opposing preferences. Airlines demonstrate a strong preference for tighter agreements achieving their highest profits under alliances, whereas consumers prefer codesharing agreements with airlines competing in frequency and price. Hence competitive codesharing agreements maximize social welfare and the bilateral agreement produces the worst market outcome, closely followed by the bilateral codeshare agreements.

Following the analytic conclusions described above, a numerical analysis is used to demonstrate that the results are robust to conditions of demand uncertainty and additional consumer types. We then conduct a case study that accounts also for asymmetric airline markets, thus providing a more realistic analysis of a setting in which these arguments are highlighted. In particular, it is shown that heterogeneous consumers who place emphasis to a different extent on frequency as compared to price, may disagree with respect to their preferences for codesharing versus pure competition in non-thin markets, although overall consumer surplus remains highest under competitive codesharing agreements.

### *Related literature*

International air service agreements were first established during the Chicago Convention of 1944. The framework developed was based on bilateral negotiations between countries and according to Borenstein et al. (2014), liberalization of international agreements began in the late 1970s, modeled after the 1978 U.S.-Netherlands agreement which introduced greater flexibility in frequency and pricing but fell short of being competitively determined. Describing the exact behavior of airlines at the route level is subject to debate but appears to point to pricing between Cournot and Bertrand levels. After analyzing route competition between United and American Airlines, Brander and Zhang. (1993) argued that some periods were characterized by cooperative behavior with price to marginal cost ratios higher than that implied by Cournot behavior and other periods were characterized by competitive behavior, although ratios were higher than those implied by Bertrand expectations. Airlines react to the level of aggressive competition with fares on average 10–20% higher when moving from duopoly to monopoly routes (Borenstein, 1989; Oum et al., 1993). Roller and Sickles (2000) argue that a two-stage differentiated product game better describes the European aviation market and arrive at the conclusion that one-stage specifications would result in biased results with regard to market power.

In terms of networks, two types of codeshares exist; complementary codeshares in which each network serves different routes, and parallel codeshares in which the airlines serve the same set of routes. If two airlines from different continents serve

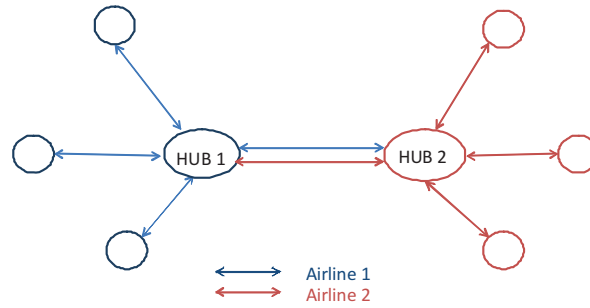


Fig. 1.1. An airline network.

their home markets and each other's hubs, the codeshare is complementary in spoke markets but parallel on the hub-to-hub link, as shown in Fig. 1.1.

Much work has been undertaken to analyze complementary codeshares using econometric analysis, for the most part in the U.S. where data is more accessible, including Borenstein (1990), Brueckner and Whalen (2000), Brueckner (2001), Bamberger et al. (2004), Armantier et al. (2006) and Whalen (1999). Gillen et al. (2002) model and measure the economic effects of bilateral agreements in the Japanese-Canadian market. Theoretical literature includes Hassin and Shy (2004), Heimer et al. (2006) and Bilotkach (2007a) amongst others. This literature appears to agree on one point: passengers on parallel links may be worse off when the carriers enter a codeshare agreement but on complementary links the opposite is true. Brueckner and Whalen (2000) studied airfares and found that overlapping alliance service had a positive, but statistically insignificant, effect on gateway fares. Their point estimates suggested that an alliance between two previously non-allied carriers would raise fares on an overlap route by about 5%. The compensation mechanism embedded in the codeshare agreements relaxes airfare competition, thus although passengers have a greater range of frequencies from which to choose, the airlines are free to extract a higher surplus. However, recent literature including Gayle (2007), Wan et al. (2009), Bilotkach (2007b), Czerny (2009), Brueckner and Proost (2010), Gillespie and Richard (2012) and Brueckner et al. (2011) have begun to question this conclusion. Gayle surmises that if competition between the airlines is weak (based on their elasticities), the equilibrium prices should not change much if they jointly price their products and indeed, he did not find any significant change between collusive and pre-alliance fares. Wan et al. suggest that the price effects of alliances on parallel routes may be either insignificantly different from the pre-alliance case or may even drop. Bilotkach argues that complementary codeshares may not be welfare improving if airlines are not allowed to coordinate in all service dimensions. Czerny reaches a similar conclusion under certain thresholds. Brueckner and Proost argue that under standard alliance agreements, consumers are worse off on parallel links due to higher prices but under joint venture alliances, carve outs would result in excessive efficiency losses by preventing the integration of operations and resultant cost savings. Gillespie and Richard provide evidence that joint ventures adversely affect consumers in comparison to alliances without anti-trust immunity. Brueckner, Lee and Singer provide counter evidence showing that prices set by airlines with a joint venture are lower than those who do not enjoy anti-trust immunity.

Wen and Hsu (2006) present an interactive airline network design procedure to facilitate bargaining interactions required by international codeshare alliance agreements. Wen and Hsu argue that all airlines are strictly better off working under alliance agreements, however they did not include pricing as a direct issue and consequently were unable to analyze the consumers' perspective. Adler and Smilowitz (2007) present a framework to analyze global alliances and mergers in the airline industry under competition. The pressure on airlines to merge or ally would appear to be very strong, a point that is strengthened by the results of this model. Consequently, parallel airline agreements cannot be presumed either positive or negative and need to be assessed with respect to market conditions on a case-by-case basis. Despite the fact that parallel markets represent the vast majority of agreements in international markets, there has been no analysis with respect to their effect on consumer surplus considering both frequency and prices simultaneously. This is one of the achievements of our paper.

The structure of the paper is as follows. In Section 2 we present the proposed model. In Section 3 we characterize analytically the levels and rankings of frequencies, prices, consumer surplus, airline profits and social welfare achieved according to various types of agreements between two airlines serving two consumer types under certainty and symmetry in demand and costs. We also demonstrate numerically that our findings continue to hold under stochastic demand and heterogeneous consumer types in terms of the importance of frequency. In Section 4 we demonstrate our conclusions in a realistic case study of parallel agreements among airlines. The final section presents conclusions and recommendations for future research. Proofs are collected in an Appendix.

## 2. Modeling codesharing agreements under frequency dependent demand

We consider  $n$  airlines serving a single market composed of multiple consumer types. In stage zero, airlines choose whether to sign codesharing agreements, in which case their frequencies are offered jointly. Subsequently in the first stage, airline  $a \in \{1, \dots, n\}$  chooses the frequency  $f_a$  to be offered before the demand realization, and in the second stage prices  $p_{ab}$  per consumer type  $b$  after the demand realization. Denote the vector of frequencies  $f_a$  for all airlines  $a$  by  $\vec{f}$ ; similarly, denote the matrix of



prices  $p_{ab}$  for every airline  $a$  and consumer type  $b$  by  $P$ ; finally, the vector of prices per consumer type  $b$  is denoted by  $P_b$ , and the vector of prices per airline  $a$  is denoted by  $P_a$ .

We first discuss the demand and profit functions and then we present three models to describe the potential contracts. The airlines face uncertain demand that depends linearly on prices (Singh and Vives, 1984), and non-linearly on the frequencies offered and the existence of a codeshare agreement. In addition, the willingness to pay may vary across consumer types depending on home bias, also referred to as country of origin preference (Feit et al., 2010), schedule delay and values of time (Czerny and Zhang, 2011) and frequent flyer programs. For consumer type  $b$ , airline  $a$ 's demand  $d_{ab}(\mathbf{m}_b, \bar{f}, P_b)$  is assumed to be multiplicatively random, where  $\mathbf{m}_b > 0$  is a random variable (the realization of which is denoted by  $m_b$ ) which represents the demand magnitude for consumer type  $b$  (we denote by  $\bar{\mathbf{m}}$  the vector  $\mathbf{m}_b$  per consumer types  $b$ , and by  $\bar{m}$  the vector  $m_b$  per consumer types  $b$ ). Thus the demand function, assuming non-negativity, is

$$d_{ab}(\mathbf{m}_b, \bar{f}, P_b) \equiv \mathbf{m}_b \left( \theta u_{ab} - \sum_{a'} u_{a'b} \right), \quad (2.1)$$

where the parameter  $\theta > 0$  represents the degree of imperfect substitutability between the airlines' services (with higher values corresponding to lower substitutability), and  $u_{ab}$  represents the net value of the service to consumer type  $b$  from airline  $a$ , combining all airline partners' decisions on consumer utility:

$$u_{ab} \equiv \alpha_{ab} + \beta_b \ln \left( 1 + \sum_{a' \in \rho_a} f_{a'} \right) - p_{ab}. \quad (2.2)$$

In our formulation,  $\alpha_{ab} > 0$  represents preferences over airlines (in particular, home bias) for consumer type  $b$ . The effect of codesharing agreements on demand is represented by the set of partners of airline  $a$ , denoted by  $\rho_a$ . In the case of two airlines for example,  $\rho_a = \{1, 2\}$  for each  $a$  when a codesharing agreement has been signed, otherwise  $\rho_a = \{a\}$ . When the partners sign a codeshare, consumers are free to choose among the aggregated schedule. Even if signing an agreement does not affect frequency or price, the degree of imperfect substitutability  $\theta$ , which differentiates own-airline and codeshare flights, implies that customers are better off purchasing from their preferred airline when frequency is offered jointly than when it is not, thus increasing overall demand. Additionally, codesharing airlines are likely to reduce schedule delay by spreading their flights across the day rather than bunching the flights (Dennis, 2000; Pearce and Doernhoefer, 2011). Utilizing both operators on a single ticket is especially valuable to passengers in international markets where the price of a one way ticket is often similar to that of a return. Due to the relatively simplified model, all these considerations are captured through the parameter  $\beta_b \geq 0$ , which represents the value of frequency for the type  $b$  consumer. Passengers prefer higher frequency because such choice reduces potential schedule delay. The preference parameter  $\beta_b$ , sometimes referred to in the literature as demand elasticity with respect to frequency, has been estimated to be strictly significantly positive. For simplicity and in line with empirical research (Copeland et al., 2011; Hansen, 1990; Ippolito, 1981), we assume here log linearity in the frequency offered, in order to reflect the decreasing marginal utility as frequency increases. However, we note that the results presented in this paper are not dependent on this assumption and more general concave formulations could be assumed instead but would require a more complicated analysis.

Based on technological constraints, setting  $f_a$  as the frequency produces  $s_a f_a$  seat capacities, where the plane size  $s_a$  in terms of number of seats is set exogenously prior to agreements, frequency offered and pricing decisions analyzed in this context. Airline decisions include the set of aircraft, the schedule and the prices, all of which are determined over different time frames. The choice of aircraft type is a long term decision because leasing normally requires a five year contract and purchasing means that the airline plans to fly the aircraft in a passenger market for the next twenty years. However, scheduling is a medium term variable and airfares can change on an hourly basis if necessary. We therefore assume in this modeling approach that the airlines' long term decisions are set exogenously and instead we concentrate on the medium to shorter term frequency and price decisions that more directly impact demand.

The airlines maximize their expected profits in two stages, first setting frequency offered  $\bar{f}$  before demand realization, then choosing prices  $P(\bar{m}, \bar{f})$  as a function of the vector  $\bar{m}$  (the vector of demand magnitude realizations), given that both airlines know the first stage decision outcomes. In the second stage, after choosing frequency, the airlines are always limited to selling only up to the frequency to which they have pre-committed. To simplify notation, we suppress arguments and write  $p_{ab}$  instead of  $p_{ab}(\bar{\mathbf{m}}, \bar{f})$ ,  $d_{ab}$  instead of  $d_{ab}(\mathbf{m}_b, \bar{f}, P_b(\bar{\mathbf{m}}, \bar{f}))$ , and quantity sold  $t_{ab}$  instead of  $t_{ab}(\mathbf{m}_b, \bar{f}, P_b(\bar{\mathbf{m}}, \bar{f}))$ .

The first four columns of Table 1.1 (in the Introduction) cover the no-agreement scenario, the inter-government bilateral scenario and the inter-airline block codeshares. We assume that under a block codeshare agreement, when airline  $a$  sells a certain number of its partner's seats, airline  $a$  must provide the partner with exactly the same number of its own seats through a seat swap. Thus the quantity sold in all four cases must satisfy the demand and capacity constraints:

$$t_{ab} \leq d_{ab}, \quad \forall b, \bar{m} \text{ and } \sum_b t_{ab} \leq s_a f_a, \quad \forall \bar{m}.$$

Consequently, airline  $a$ 's expected operating profit (i.e. contribution to fixed costs, taxes and net profits) is given by

$$\mathbb{E}_{\bar{\mathbf{m}}} \sum_b p_{ab} t_{ab} - c_a f_a,$$

where  $\mathbb{E}$  is the expectation operator, taken here with respect to the random vector  $\bar{\mathbf{m}}$  of demand magnitudes, and  $c_a$  represents the unit operating cost of frequency. The assumption of constant marginal costs with respect to frequency is reasonable in the aviation market (Swan and Adler, 2006). The marginal cost  $c_a$  is dependent on the great circle distance and number of seats on the aircraft,  $s_a$ , both of which are set exogenously to the model. One of the potential extensions to the model could endogenize  $s_a$  in order to enable a trade-off between capacity and frequency, however we believe that this would not change the considerations raised in this paper with respect to producer and consumer surplus comparisons across agreement types, thus we ignore this issue. Moreover, the assumption implies that there are no cost advantages from signing codesharing agreements. Goh and Yong (2006) find that codeshare alliances do lower costs, however the magnitude of the reductions are economically immaterial. Including cost savings through codeshare in the model would further strengthen the results, hence the current outcome may be on the conservative side. For purposes of simplification, we concentrate the analysis on the demand side effects of codesharing and leave detailed cost side analysis for future research.

The four agreement types are solved for the second stage in model (2.3). We assume that the airlines compete in the second stage by choosing prices in Nash equilibrium. For any realization of demand and first stage decisions, no airline will set the prices in the second stage such that their total demand is higher than their capacity, because increasing prices in this scenario will simply increase revenues and profits without introducing any additional costs. Consequently, in equilibrium, the quantity sold by an airline is always equal to their demand ( $t_{ab} = d_{ab}, \forall a, b, \bar{\mathbf{m}}$ ), which implies that the total demand is never higher than the number of available seats. Thus airline  $a$ 's best price response to their opponent's prices is given by the constrained optimization:

$$\begin{aligned} \tilde{\pi}_a(\bar{f}) &\equiv \max_{p_a} \mathbb{E}_{\bar{\mathbf{m}}} \sum_b p_{ab} d_{ab} - c_a f_a \\ \text{s.t.} \quad &\sum_b d_{ab} \leq s_a f_a, \quad \forall \bar{\mathbf{m}}. \end{aligned} \tag{2.3}$$

Note that the advantage to airlines from codesharing agreements arises from the frequency dependent demand. There are no additional advantages such as risk pooling in the face of uncertainty because prices are adjusted in the second stage after the demand magnitude  $m_b$  is realized.

After solving for the second stage, we then solve the first stage frequency decisions given the optimal pricing structure, thus maximizing profits  $\tilde{\pi}_a$  for airline  $a$ . Under no agreement and competitive codesharing agreements, the airlines compete by choosing a best response  $f_a$  to their opponent's choice,  $f_{-a}$ , i.e.

$$\max_{f_a} \tilde{\pi}_a(\bar{f}), \tag{2.4}$$

resulting in a Nash equilibrium. Under bilateral agreements the airlines choose their frequency given limits set cooperatively by their governments, leading to the Nash bargaining solution (Nash, 1950), whereby the agreement maximizes the product of profit gains above the disagreement threat point, as in (2.5).

$$\begin{aligned} \max_{\bar{f}} \prod_a [\tilde{\pi}_a(\bar{f}) - \tilde{\pi}_a(\bar{f}^{comp})] \\ \text{s.t.} \quad \tilde{\pi}_a(\bar{f}) \geq \tilde{\pi}_a(\bar{f}^{comp}), \quad \forall a, \end{aligned} \tag{2.5}$$

where  $\bar{f}^{comp}$  refers to the competitive contract solution computed in (2.4). Thus the threat point for a bilateral agreement is no agreement, and the threat point for a bilateral codesharing agreement is the competitive codesharing agreement. As a consequence of this formulation of the Nash bargaining solution, companies reaching agreement will each be (weakly) better off than had no agreement been signed.

The fifth and sixth columns in Table 1.1 cover free sale agreements between airlines, in which airline  $a$  can sell or buy excess seat inventory to/from their codeshare partners for a price  $\tau$  per seat paid to the operating carrier. The price agreed is set with the frequencies in the first stage of the game. Free sale agreements potentially provide greater benefits to airlines over block codeshares due to the higher levels of flexibility with respect to seat inventory sharing, which occurs as demand is realized. To formulate this type of agreement, denote by  $t_{aa'b}$  the number of type  $b$  consumers of airline  $a$  utilizing frequency of airline  $a'$ , to be determined after the realization of demand magnitude  $m_b$  for all consumer types  $b$  (as before, we simplify notation by writing  $t_{aa'b}$  instead of  $t_{aa'b}[\bar{\mathbf{m}}, \bar{f}, P_b(\bar{\mathbf{m}}, \bar{f})]$ ). For simplicity, we denote by  $T$  the array  $t_{aa'b}$  for all airlines  $a$  and  $a'$  and for all consumer types  $b$ . Thus for free sale agreements, airline  $a$ 's expected profit as a function of the decision variables set by all airlines is given by

$$\pi_a(f_a, P_a, T) \equiv \mathbb{E}_{\bar{\mathbf{m}}} \sum_b p_{ab} \sum_{a' \in \rho_a} t_{aa'b} - c_a f_a + \tau \mathbb{E}_{\bar{\mathbf{m}}} \sum_{a' \in \rho_a \setminus \{a\}} \sum_b (t_{a'b} - t_{aa'b}). \tag{2.6}$$

Consequently the second stage, competitive price Nash equilibrium formulation (2.3) becomes

$$\begin{aligned} \tilde{\pi}_a(\bar{f}) &\equiv \max_{P_a, T} \pi_a(f_a, P_a, T) \\ \text{s.t.} \quad &\sum_{a' \in \rho_a} t_{aa'b} \leq d_{ab}, \quad \forall b, \bar{\mathbf{m}} \end{aligned}$$

$$\begin{aligned} \sum_b t_{aab} &\leq s_a f_a, \quad \forall \bar{m} \\ \sum_b t_{aa'b} &\leq \max \left\{ 0, s_{a'} f_{a'} - \sum_b t_{a'a'b} \right\}, \quad \forall a' \in \rho_a \setminus \{a\}, \bar{m}. \end{aligned} \quad (2.7)$$

The first constraint in (2.7) restricts airline  $a$  to selling no more than its own demand, the second constraint prevents airline  $a$  from selling more than its own seats, and the third constraint restricts airline  $a$  to selling no more than the partners' residual seats. Consequently, we assume that an operating carrier will always prefer to sell the seat to its own customers if possible and will share only those seats it judges are likely to remain unsold otherwise. Given the second stage prices, the first stage frequencies are set competitively according to (2.4) or cooperatively as in (2.5). In the competitive agreement, the airlines must agree on the price  $\tau$  prior to setting frequency and prices, hence  $\tau$  is considered an exogenous parameter in this model, for example based on IATA conference interlining agreements. In the bilateral free sale agreement,  $\tau$  is a variable within the model and computed in the first stage together with frequency as part of the Nash bargaining program (2.5).

Finally, under the alliance contract (the last column in Table 1.1), prices are chosen simultaneously to maximize the sum of profits (with the profit function used for the first four agreement types) under the joint frequency constraint,

$$\begin{aligned} \max_p \sum_a \left( \mathbb{E}_{\bar{m}} \sum_b p_{ab} d_{ab} - c_a f_a \right) \\ \text{s.t. } \sum_a \sum_b d_{ab} &\leq \sum_a s_a f_a, \quad \forall \bar{m}, \end{aligned} \quad (2.8)$$

and frequencies offered are optimized simultaneously to maximize the sum of profits, i.e.

$$\max_{\bar{f}} \sum_a \tilde{\pi}_a(\bar{f}), \quad (2.9)$$

where  $\tilde{\pi}_a(\bar{f}) = \mathbb{E}_{\bar{m}} \sum_b p_{ab} d_{ab} - c_a f_a$  is computed from (2.8). Under an alliance, the airlines have complete freedom to share their joint profit in any way, consequently they can ensure that each is better off than under no agreement, provided that the sum of profits is greater than those that could be achieved individually.

In order to compare the results of the different agreement types, we refer to profits, consumer surplus and their sum, social welfare. Consumer surplus is defined as in the literature (see e.g., Hsu and Wang, 2005), given our assumptions with respect to demand:

$$\frac{1}{2} \sum_b \mathbb{E}_{\bar{m}} u_{ab}(\mathbf{m}_b) d_{ab}(\mathbf{m}_b). \quad (2.10)$$

### 3. Effects of agreements under symmetry

In this section we analyze the impact of agreements on frequency, price and welfare, which highlights our main findings even under the simplifying assumptions of symmetric demand and costs. We note that free sale agreements and block codeshares produce the same equilibrium outcomes under symmetric settings, hence findings with respect to free sale agreements are presented separately only in Section 4. For simplicity of presentation we assume deterministic demand and two airlines that serve two consumer types  $b \in \{1, 2\}$ , each representing the home of the corresponding airline. The difference between the two consumer types is represented by their home bias, in the sense that the willingness to pay for the home-based service is weakly higher than that offered by the alternative airline. Under the assumption of symmetry, this can be written as  $\alpha_{11} = \alpha_{22} = \bar{\alpha} \geq \underline{\alpha} = \alpha_{12} = \alpha_{21}$ ,  $\beta_1 = \beta_2 = \beta$ ,  $m_1 = m_2 = m$ , with home bias arising when  $\bar{\alpha} > \underline{\alpha}$ . At the end of this section we demonstrate numerically that our findings continue to hold under stochastic demand and heterogeneous consumer types in terms of the importance of frequency versus price. Symmetry in costs implies  $c_1 = c_2 = c$ . We also assume that  $\frac{\bar{\alpha} + \underline{\alpha}}{2} > \frac{c}{s} > \beta$ , which means that the average direct willingness to pay for the flight,  $\alpha$ , is sufficient to ensure profitability, and the value of frequency,  $\beta$ , is lower than the operating cost per seat. This assumption ensures that we avoid the trivial solution that no frequency is offered because the willingness to pay is too low given the cost of production. In turn, this ensures that the constraint in (2.3) is always binding, hence all seats are filled in this deterministic case, and that the airlines choose strictly positive frequency.

We start by characterizing the second stage prices and operating profits (Proposition A.1 in the Appendix), the first stage frequency offered that result from each of the relevant five agreement types (Proposition A.2) and the consumer surplus and social welfare (Proposition A.3). Then we compare the agreement types in terms of frequencies, prices, profits, consumer surplus and social welfare. Finally we present a numerical analysis of the impact of changes in the willingness to pay on the ranking of contract types.

We find that when the demand is insensitive to the level of frequency, i.e.  $\beta = 0$ , prices exhibit the usual negative slope with respect to frequency. In comparison to this benchmark, any demand sensitivity to frequency offered, i.e.  $\beta > 0$ , causes prices to increase because consumers are willing to pay for the higher frequencies offered. Consequently, prices change non-monotonically with respect to frequency, i.e. increase first, and then decrease as the negative effect dominates the positive one. Similarly, profits



also change non-monotonically with respect to frequency, at first increasing, and then as frequency further increases the profit decreases.

We show that the higher the sensitivity of demand with respect to frequency ( $\beta > 0$ ), the higher is the price in the equilibrium outcome. The intuition for this effect is that passenger demand creates a trade off between price and expected schedule delay, whereby the latter is assumed to be decreasing in frequency. Indeed, demand exists even with zero frequency, in which case it is clearly independent of the importance of schedule delay. By increasing frequency (hence reducing schedule delay), more passengers would be willing to fly. Consequently, were schedule delay to become more important to passengers, any positive frequency levels would further induce demand. Furthermore, prices would rise in equilibrium. Thus, should schedule delay increase in importance, ceteris paribus, demand and prices would be expected to increase and this holds for any level of frequency and price.

A similar result is shown to hold for the equilibrium frequency, and this effect is further enhanced by codesharing, thus plays a central role in the comparison of agreement types (as shown in the propositions below). We find that the expression  $s^{\frac{\bar{\alpha}+\alpha}{2}} - c$  defines a profit margin which directly impacts the preference of firms to compete or cooperate in general. The profit margin measures the difference between the demand magnitude, representing market size, and the consumers willingness to pay, as compared to the variable costs. When the profit margin is sufficiently low, referred to as a thin market, we find that there is a clear preference to cooperate both from the consumers' and the producers' perspectives. Subsequently, we rank all the agreement types according to frequency, price, consumer surplus, producer surplus and social welfare. The ranking of agreement types is dependent on a profit margin threshold determined per market. In other words, if the profit margin lies above a threshold, the frequency and price outcomes and resultant rankings of different contracts will change as compared to those resulting from a profit margin lying below this threshold. The reason for the threshold result is the following: cooperation pushes frequency lower as compared to competition, whereas codesharing provides incentives to increase frequency as compared to no codesharing. Due to the decreasing marginal value of codesharing as frequency expands (represented by the logged frequency in the demand function), codesharing offers higher marginal value to the consumer for lower frequency, which generally occurs when the profit margin is sufficiently low. In this case, the force pushing to increase frequency due to codesharing outweighs the force pushing to decrease frequency due to cooperation. The reverse holds when the profit margin is sufficiently high. This result is summarized in Proposition 3.1, which characterizes the ordering of agreement types with respect to frequency as a function of the profit margin.

**Proposition 3.1.**

- (1) The frequency offered  $f^*$  is increasing in  $\beta$ .
- (2)  $f_{\text{comp codeshare}}^* > f_{\text{no agreement}}^* > f_{\text{bilateral}}^*$  and  $f_{\text{bilateral codeshare}}^* > f_{\text{bilateral}}^*$ .
- (3) There exist a threshold for  $s^{\frac{\bar{\alpha}+\alpha}{2}} - c$  below which  $f_{\text{bilateral codeshare}}^* > f_{\text{comp codeshare}}^*$ , and above which the inequality reverses.
- (4) There exists a threshold for  $s^{\frac{\bar{\alpha}+\alpha}{2}} - c$  below which  $f_{\text{bilateral codeshare}}^* > f_{\text{no agreement}}^*$ , and above which the inequality reverses.
- (5)  $f^*$  is equal under bilateral codesharing and alliance agreements.

Proposition 3.1 also shows that for a sufficiently small profit margin, frequency offered is higher when airlines reach a bilateral codesharing or alliance agreement as compared to the competitive scenario. In all cases, the lowest frequency outcome occurs when governments control the market via bilaterals and the companies do not cooperate beyond requesting certain frequency levels from their government representative.

A similar phenomenon occurs when comparing prices across the agreement types, because the price levels depend on the willingness to pay for frequency ( $\beta$ ), hence are affected by the opposing forces of cooperation and codesharing described in the discussion preceding Proposition 3.1, which in turn depend on the profit margin. Consequently the ranking of contracts with respect to prices depend on a threshold with respect to the profit margin, as shown in Proposition 3.2.

**Proposition 3.2.**

- (1) Price  $p_{ab}^*$  is increasing in  $\beta$
- (2)  $p_{\text{bilateral}}^* > p_{\text{no agreement}}^*$ .
- (3) There exist a threshold for  $s^{\frac{\bar{\alpha}+\alpha}{2}} - c$  above which  $p_{\text{bilateral codeshare}}^* > p_{\text{bilateral}}^*$  and  $p_{\text{bilateral codeshare}}^* > p_{\text{comp codeshare}}^* > p_{\text{no agreement}}^*$ .
- (4) There exists a threshold for  $s^{\frac{\bar{\alpha}+\alpha}{2}} - c$  below which  $p_{\text{comp codeshare}}^* > p_{\text{bilateral}}^*$ , and above which the inequality reverses.
- (5) Given no home bias ( $\bar{\alpha} = \alpha$ ),  $p_{\text{bilateral codeshare}}^* = p_{\text{alliance}}^*$ . With home bias ( $\bar{\alpha} > \alpha$ ), the alliance results in relatively lower prices for local consumers and higher prices for non-local consumers as compared to a bilateral codeshare.

We show through Proposition 3.2 that the more the consumer values frequency ( $\beta$ ), the higher the prices that will be charged. It also shows that prices are lowest when the two airlines compete fully. Even under codeshare agreements, prices are lowest when the airlines continue to compete in both frequencies and prices. Given home bias, consumers are willing to pay a higher price to their local carrier. Nevertheless, part (5) implies lower price dispersion under alliances as compared to all other agreements, consistent with the impact of market concentration as described in Borenstein and Rose (1994).

Having identified capacities and prices per agreement type, we are now in a position to evaluate their impact on consumer and airline surplus, the sum of which equals social welfare. The levels of frequency and the prices change under the varying agreement

**Table 3.1**  
Ranking summaries (from highest to lowest).

Case	Frequency & consumer surplus	Price & profit
Above threshold	Comp codeshare no agreement alliance, bilateral codeshare bilateral	Alliance, bilateral codeshare bilateral comp codeshare no agreement
Below threshold	Alliance, bilateral codeshare comp codeshare no agreement bilateral	Alliance, bilateral codeshare comp codeshare bilateral no agreement

types, but [Proposition A.3](#) shows that their combined effect on airline surplus, consumer welfare and total social welfare can be captured by changes in the equilibrium frequency alone. Consumer surplus is an increasing function of the difference between the consumer's willingness to pay and the price. In a Cournot model with linear demand, the equilibrium price is equal to the willingness to pay minus an increasing function of the equilibrium quantity. Consequently, although the equilibrium price is an increasing function of the willingness to pay, consumer surplus is also an increasing function of the equilibrium quantity (frequency). Our model maintains this type of comparative statics: the higher the equilibrium frequency, the higher the consumer surplus, despite the potential increases in the equilibrium price. In particular, this implies that an increase in willingness to pay due to codesharing results in higher frequencies, prices and consumer surplus. On the other hand, airline surplus and total social welfare do not increase monotonically in equilibrium frequency, hence their ranking of agreements is different from that of the frequency rankings and they depend on a profit margin threshold as in our previous findings. In [Proposition 3.3](#) we rank the agreement types according to the airline profits, highlighting the differences in ranking as a function of the profit margin.

**Proposition 3.3.**

- (1) Profit  $\pi^*$  is increasing in  $\beta$ .
- (2)  $\pi_{\text{bilateral codeshare}}^* > \pi_{\text{bilateral}}^* > \pi_{\text{no agreement}}^*$  and  $\pi_{\text{bilateral codeshare}}^* > \pi_{\text{comp codeshare}}^*$ .
- (3) There exist a threshold for  $s \frac{\bar{\alpha} + \alpha}{2} - c$  above which  $\pi_{\text{comp codeshare}}^* > \pi_{\text{no agreement}}^*$ .
- (4) There exist a threshold for  $s \frac{\bar{\alpha} + \alpha}{2} - c$  below which  $\pi_{\text{comp codeshare}}^* > \pi_{\text{bilateral}}^*$ , and above which the inequality reverses.
- (5)  $\pi_{\text{alliance}}^* \geq \pi_{\text{bilateral codeshare}}^*$ , with equality if and only if there is no home bias ( $\bar{\alpha} = \alpha$ ).

Part (2) of [Proposition 3.3](#) demonstrates that airlines strictly prefer alliances in all cases when maximizing profits. Coordinating frequency is the next best outcome, through bilaterals and codesharing. Parts (2) and (3) jointly suggest that for most cases, the worst outcome for airlines is no agreement. Part (4) suggests that under thin market conditions it is in the interest of the airlines to produce higher frequency under a codesharing agreement, whilst still competing in frequency and prices, as compared to a bilateral agreement that lowers frequency without the codesharing component. Trivially, part (5) shows that it will always be in the interest of the airlines to request antitrust immunity in order to coordinate frequencies and prices, permitting codesharing and maximizing joint profits.

**Proposition 3.4.**

- (1) The consumer surplus and total social welfare are increasing in  $\beta$ .
- (2) Consumer surplus and social welfare rankings are the same as the frequency ranking for the first four agreement types.
- (3) Consumer surplus and social welfare are at least as high under an alliance agreement than under bilateral codesharing agreements, with equality if and only if there is no home bias ( $\bar{\alpha} = \alpha$ ).

Part (2) of [Proposition 3.4](#) states that the ranking of contracts according to equilibrium frequency match the rankings according to consumer surplus and social welfare. This indicates that the increase in equilibrium frequency more than compensates for any potential increase in equilibrium price. Furthermore, consumer surplus and social welfare are higher under the competitive outcomes as compared to cooperative agreements, because the equilibrium frequency under cooperation is too low. Codesharing increases social welfare by benefiting both producers and consumers alike. In addition, when consumers demonstrate preferences for a specific airline, e.g. their home carrier, part (3) shows that airlines are better able to price discriminate under an alliance. In other words, given equilibrium frequency, the prices for consumers demonstrating home bias drop and the remaining consumers are charged a higher price, resulting in more local customers served and a lower price dispersion within an alliance. An alliance consequently allows each airline to concentrate on their home market to the advantage of airlines and consumers alike, in the symmetric, deterministic case. In summation, competitive codesharing is the most preferable alternative in terms of consumer surplus and social welfare in standard sized markets with relatively low home bias and an alliance is the best outcome for thin markets.

[Table 3.1](#) summarizes the ranking described in the propositions above. We would like to note that multiple thresholds may exist, depending on the market under analysis, thus we further depict these rankings using a numerical analysis and the crossover thresholds are shown in [Fig. 3.1](#).

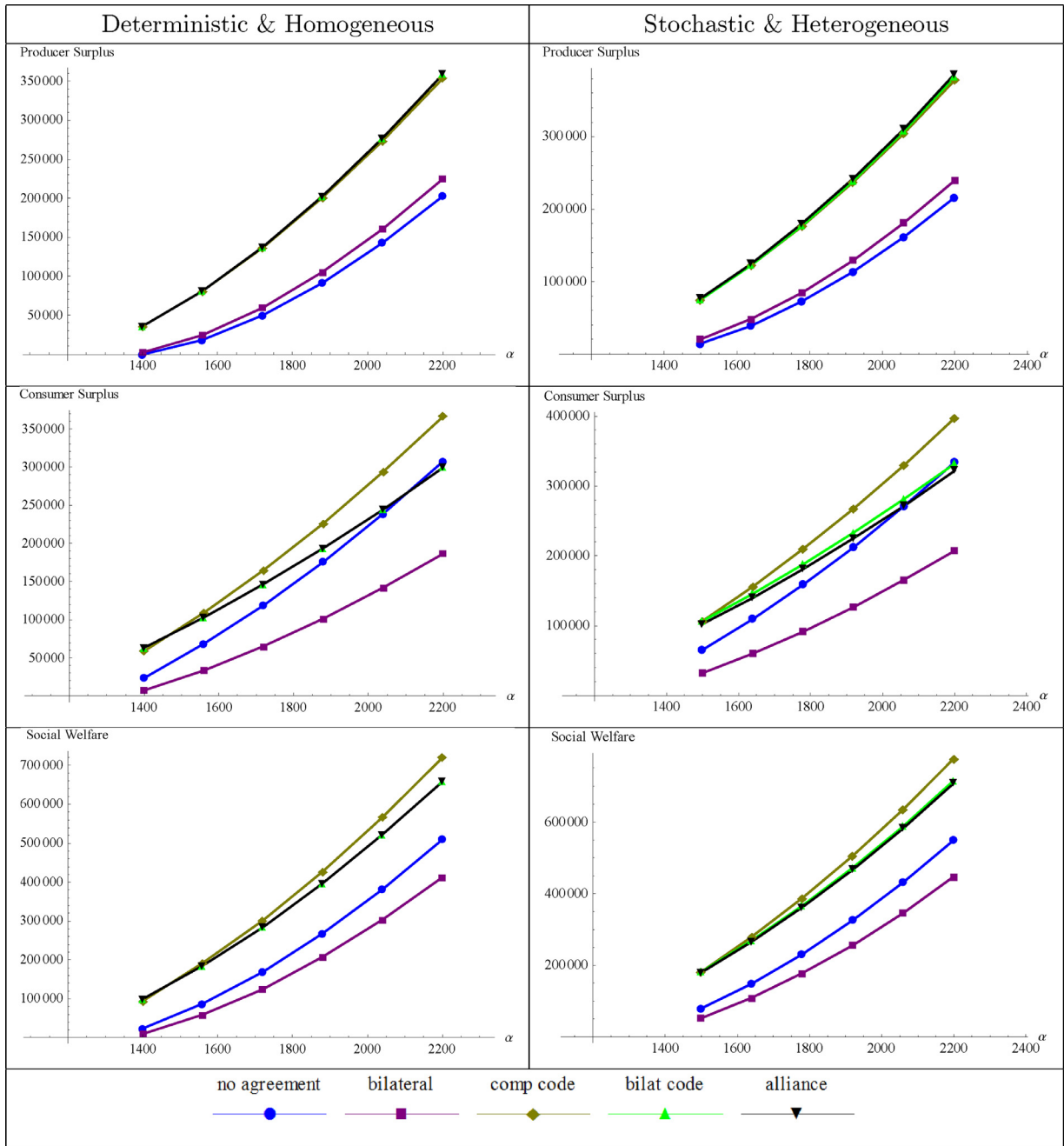


Fig. 3.1. Numerical analysis results.

We now present a numerical analysis of a symmetric, two airline market demonstrating our findings in this section. The results of the model were computed based on parameters drawn randomly according to uniform distributions as detailed in Table 3.2. Simulation shows that the results for all instances are qualitatively similar, thus we show the analysis for two of them. One instance is based on deterministic demand and homogeneous consumers, matching the propositions described in this section. The other instance is based on stochastic demand and heterogeneous consumers, for which there are five demand magnitude realizations of equal probability, and there are four consumer types  $b \in \{1B, 1L, 2B, 2L\}$ , where 1 or 2 represents the origin of the passenger's trip and B or L represent business and leisure, depending on their home base and reason for trip. As can be seen in the table, we assume that business passengers have a higher willingness to pay ( $\bar{\alpha}_B \geq \bar{\alpha}_L$ ,  $\underline{\alpha}_B \geq \underline{\alpha}_L$ ) and a higher value for level of service i.e. frequency ( $\beta_{1B} = \beta_{2B} = \beta_B \geq \beta_L = \beta_{1L} = \beta_{2L}$ ), but are a smaller market overall ( $m_{1B} = m_{2B} = m_B \leq m_L = m_{1L} = m_{2L}$ ). To enable a clear comparison, the parameter values for the deterministic, homogeneous instance were computed

**Table 3.2**  
Parameters for numerical analysis.

Parameter	Distribution	Realized values	
		Deterministic & homogeneous	Stochastic & heterogeneous
$\alpha_B$	Unif(1500, 2000)	1479.44	1678.9, 1706.7
$\alpha_L$	Unif(1000, 1500)		1359.0, 1383.6
$\beta_B$	Unif(150, 200)	131.85	185.1
$\beta_L$	Unif(100, 150)		104.9
$m_B$	Unif(0.5, 1)	1.02	0.94, 0.57, 0.71, 0.65, 0.55
$m_L$	Unif(1, 1.5)		1.20, 1.33, 1.47, 1.36, 1.39
$\theta$		3	
$c$	Unif(100000, 200000)	159172	159172
$s$	Unif(100, 500)	475	475

as  $m$ -weighted averages of the realized parameter values for the stochastic, heterogeneous instance, and the parameter  $\theta$  in the demand function, defining the degree of imperfect substitutability across the travel alternatives, was set for both instances to 3, the number of alternatives (including no travel).

The computation results are depicted in Fig. 3.1, where the three graphs on the left show the producer surplus (sum of profits), consumer welfare and total social welfare, respectively, for the deterministic, homogeneous instance, and the three graphs on the right show the results for the stochastic, heterogeneous instance, respectively. The horizontal axis represents the values of the willingness to pay ( $\alpha$ ), which we vary around those of Table 3.2. In the stochastic and heterogeneous case the willingness to pay requires a computation because there are four such values. Consequently, we compute in this case the expected weighted sum of the willingness to pay,  $\mathbb{E}[\mathbf{m}_B(\bar{\alpha}_B + \underline{\alpha}_B) + \mathbf{m}_L(\bar{\alpha}_L + \underline{\alpha}_L)]$ .

The results show that the rankings described in the propositions are robust to changes in the assumptions, including the addition of multiple consumer types and stochastic demand. We note that there is one change in the ranking of frequencies due to the stochasticity in demand, which we discuss further in Section 4.

#### 4. Case study of three airline markets

We present a case study of three airline markets in order to ascertain the effects of such agreements on airlines, consumers and overall social welfare. To allow a sufficiently realistic and interesting analysis, we include three airlines,  $a \in \{0, 1, 2\}$ , two of which serve a direct link between their respective hub nodes 1 and 2, and a third airline 0 that flies indirectly between these nodes. We assume that airline 0's indirect service frequency is exogenously fixed based on network considerations that were ignored in the current model, but prices  $p_{ab}$  remain endogenous decision variables. We assume there are four consumer types  $b \in \{1B, 1L, 2B, 2L\}$ , where 1 or 2 represents the origin of the passenger's trip and  $B$  or  $L$  represent business and leisure, depending on their home base and reason for trip. For flights serving consumer type  $b$ , the parameter  $\alpha_{ab}$  in airline  $a$ 's demand function,  $d_{ab}$ , takes into account the additional utility due to frequent flyer programs or disutility for  $a = 0$  due to indirect itineraries. The third airline offering indirect service is in a position to serve all three markets via their hub. We assume that consumers prefer direct connections and have a preference for their home carrier, i.e.  $\alpha_{1,1B} > \alpha_{2,1B} > \alpha_{0,1B}$ ,  $\alpha_{1,1L} > \alpha_{2,1L} > \alpha_{0,1L}$ ,  $\alpha_{2,2B} > \alpha_{1,2B} > \alpha_{0,2B}$  and  $\alpha_{2,2L} > \alpha_{1,2L} > \alpha_{0,2L}$ . We also assume that business travelers place a greater value on frequency and less on price than their leisure counterparts, i.e.  $\beta_{1B} = \beta_{2B} = \beta_B > \beta_L = \beta_{1L} = \beta_{2L}$ .

The three markets analyzed in this section include Tel Aviv to London, Brussels and Bangkok respectively. Except for the highly asymmetric market, Tel Aviv–Bangkok, the results of the free sale agreements were identical to those of the block codeshares, hence are presented only for this destination. Based on direct demand levels for March 2007, as reported by the Israeli Ministry of Transport, the Israeli–English market was relatively symmetric in size with 6,300 passengers carried per week. The Israeli–Belgian market was asymmetric to the extent that one third of travellers fly Brussels–Tel Aviv–Brussels and two thirds vice versa, in a 1,500 passenger per week market. Israel–Thailand is the most extreme example with over 95% of passengers traveling Tel Aviv–Bangkok–Tel Aviv in an 1,800 passenger per week market. Since the Israel–EU open skies agreement was only signed in 2013, bilateral agreements existed in all three markets, although the Israeli–English bilateral was considered relatively weak because the frequency ceiling was higher than the number of flights operated. No codeshare agreement exists on the London link and British Airways (BA) flew twice daily in each direction and El AL (LY) flew 11 flights in each direction per week in March 2007. El Al had codeshare agreements with both S.N. Brussels (SN) and Thai (TG) airlines, the former resulting in both airlines serving the link approximately 5 times a week and the latter with only El Al operating a service 3 times a week (according to El Al's website in March 2007). In each market analyzed, we model three airlines, namely El Al and the foreign carrier serving the market directly, and Lufthansa providing indirect services.

The case study also includes demand uncertainty in the first stage, which then becomes known prior to pricing in the second stage, with five demand magnitude realizations of equal probability per passenger type. To develop the relevant parameters, we collected data on the total number of passengers carried to represent one scenario and then increased or decreased each

**Table 4.1**  
Parameters for case study.

Parameter	Value								
	London–Tel Aviv			Brussels–Tel Aviv			Bangkok–Tel Aviv		
	a = 0	a = 1	a = 2	a = 0	a = 1	a = 2	a = 0	a = 1	a = 2
$\alpha_{a,1B}$	2981	5181	3586	2500	4700	3270	4116	4416	4316
$\alpha_{a,1L}$	1331	1881	1482	850	1400	1043	1716	1916	1816
$\alpha_{a,2B}$	2651	4081	4521	2280	3600	4260	4416	4666	10620
$\alpha_{a,2L}$	1248	1606	1716	795	1125	1290	2016	2106	3492
$\beta_B$	600								
$\beta_L$	150								
$m_{1B}$	0.14 ± 50%			0.06 ± 50%			0.001 ± 50%		
$m_{1L}$	0.85 ± 50%			0.34 ± 50%			0.01 ± 50%		
$m_{2B}$	0.15 ± 50%			0.12 ± 50%			0.06 ± 50%		
$m_{2L}$	0.87 ± 50%			0.71 ± 50%			0.34 ± 50%		
$\theta$	4								
c	146, 378	91, 626	91, 626	134, 523	84, 428	84, 428	366, 336	236, 691	236, 691
s	300	150	150	300	150	150	300	300	300

market by 50% such that the correlation of demand across the four markets were close to zero (Swan, 1993). We also assume that approximately one seventh of the market is willing to purchase business class tickets based on aircraft configurations and relevant load factors. The parameter  $\theta$  in the demand function, defining the degree of imperfect substitutability across the travel alternatives, is set to 4, the number of alternatives (including no travel). The parameters of the model have been calibrated such that the relevant equilibria outcome approximately reflects the results of the market for this season in terms of frequency and prices, permitting a what-if analysis of the potential market outcomes were different agreements to be signed among the relevant parties. Demand information has been collected from the Israeli Ministry of Transport and the Israeli Central Bureau of Statistics websites and prices were purchased from an MIDT seller for the relevant time frame.

In order to compute airline operating costs, (Swan and Adler, 2006) found that great circle distance,  $GCD$ , and the number of seats on an aircraft,  $s_a$ , are the two main factors affecting aircraft trip costs. Two market-based equations were developed based on average length of haul and aircraft size, taking into account direct and indirect operating costs based on ICAO definitions and inflation (hence multiplied by a factor of 2.2). Eq. (4.1) presents the cost function for short to medium haul markets (i.e. less than 5, 000 km). Eq. (4.2) provides the cost function for long haul markets (more than 5, 000 km). In order to consider the return route, each equation is further multiplied by a factor of 2 (as specified in Swan and Adler (2006)).

$$c_a^{short} = [(GCD + 722)(s_a + 104)0.019] \cdot 2.2 \cdot 2 \tag{4.1}$$

$$c_a^{long} = [(GCD + 2200)(s_a + 211)0.0115] \cdot 2.2 \cdot 2 \tag{4.2}$$

The two airlines offering direct service are assumed to be symmetric in costs. We assume that Lufthansa uses a wide body aircraft with 300 seats, as was true in 2007, because it serves multiple destinations through its hub. Since we are concentrating our analysis on three specific markets instead of a full network, we limit the plane size of the direct operating carriers to 150 in order to more easily calibrate this model. The parameters of the model relevant for the case study are summarized in Table 4.1.

The results, presented in Tables 4.2–4.7, first present the consumer surpluses per directional market and airline surpluses for the two direct carriers only. The indirect airline has an exogenously set frequency since we assume that it carries passengers to a substantial number of destinations not considered within the current framework but prices remain endogenous. Subsequently, the tables present demand-weighted average prices and frequencies. In the London–Tel Aviv market, presented in Table 4.2, it becomes clear that business travelers prefer the competitive codeshare agreement in which airlines compete for frequency and price but agree to a block codeshare based on seat swap. Leisure travelers, however, strictly prefer the pure competitive situation and the airlines (British Airways and El Al) maximize profits under an alliance. It should be noted that under an anti-trust immune alliance, the two carriers coordinate both frequency and price in order to maximize joint profits and subsequently could split the frequencies such that only one airline serves the market up to sharing the flights evenly, which is the solution presented in Table 4.2.

It is also possible to track the changes in prices and frequencies across the different agreement types and Table 4.3 presents the changes in these values with respect to the competitive scenario. Social welfare is maximized under the competitive codeshare scenario and the trade-off between the different agreement types becomes clear. Bilaterals severely dampen frequency (17% reduction) and increase prices (on average by 7% ), such that the consumer surplus is reduced by 12% . The airlines prefer the stronger agreements (namely codeshare or alliance), since this permits higher prices and higher profits (up to 31% beyond those achieved in the competitive solution).



**Table 4.2**

London–Tel Aviv airline market (per week).

	Competitive			Bilateral			Competitive codeshare			Bilateral codeshare			Alliance		
Surplus (in \$000s):															
UK–Israel business	3495			3188			3709			3580			3158		
UK–Israel leisure	1903			1550			1817			1677			1736		
Israel–UK business	3323			3014			3550			3421			2964		
Israel–UK leisure	1829			1476			1742			1603			1648		
Consumer surplus	10550			9228			10818			10281			9506		
Producer surplus	5335			5518			6689			6718			6993		
Social welfare	15885			14746			17508			16999			16499		
Average prices (\$):	LH	BA	LY	LH	BA	LY	LH	BA	LY	LH	BA	LY	LH	BA	LY
UK–Israel business	891	2584	1590	969	2681	1670	840	2776	1864	870	2813	1,898	982	3095	2297
UK–Israel leisure	481	1030	773	540	1154	887	492	1144	915	516	1192	962	512	1167	968
Israel–UK business	756	2009	2177	834	2105	2258	704	2201	2451	735	2237	2486	847	2593	2813
Israel–UK leisure	447	887	920	506	1010	1034	458	1000	1062	482	1049	1109	478	1042	1097
Frequency	12.0	27.3	22.0	12.0	22.8	17.8	12.0	26.5	21.5	12.0	24.8	19.9	12.0	22.4	22.4

**Table 4.3**

Effect of potential BA–El Al agreements as compared to the competitive scenario.

	Bilateral			Competitive codeshare			Bilateral codeshare			Alliance		
Business surplus	−9.0%			6.5%			2.7%			−10.2%		
Leisure surplus	−18.9%			−4.6%			−12.1%			−9.3%		
Consumer surplus	−12.5%			2.5%			−2.6%			−9.9%		
Producer surplus	3.4%			25.4%			25.9%			31.1%		
Social welfare	−7.2%			10.2%			7.0%			3.9%		
Prices business	3.1%			10.4%			11.6%			23.9%		
Prices leisure	8.2%			10.2%			13.1%			13.7%		
Frequency	−17.6%			−2.5%			−9.4%			−8.9%		

In the solution presented in Table 4.2, ranking of the agreement types based on the average frequency across airlines leads to the following:

$$\begin{aligned}
 f_{\text{no agreement}}^* &> f_{\text{comp codeshare}}^* > f_{\text{alliance}}^* \\
 &> f_{\text{bilateral codeshare}}^* > f_{\text{bilateral}}^*
 \end{aligned} \tag{4.3}$$

Comparing this ranking with the analytical results of Proposition 3.1 shows that the only difference lies in the swapping of no agreement and competitive codesharing. The reason for the difference lies in the fact that uncertainty in demand caused the load factor to drop below one, in which case the frequency constraint is no longer binding. As a result of uncertainty, competitive codesharing allows airlines to reduce excess frequency further than would be optimal in the no agreement setting. The codesharing and bilateral frequencies do not swap under uncertainty because the frequencies are already relatively low due to the cooperative setting and so do not contain the buffer available in the competitive setting. The prices are almost entirely consistent with the results of Proposition 3.2.

Table 4.4 presents a sensitivity analysis of the Tel-Aviv–London market with respect to the indirect airline's schedule and reduced consumer preference for frequency. If the indirect service was reduced by half i.e. Lufthansa's frequency was halved to one flight daily, consumer surplus drops by 11 to 17% whereas the two direct airlines enjoy increases in profit of 13 to 16%, due to a substantial rise in airfares and insignificant change in frequency. Alternatively, were the indirect carrier permitted to double frequencies, consumers would enjoy a 5 to 16% increase in surplus due to a reduction in prices despite a small decrease in direct service, whereas the direct carriers would lose approximately 9% in profits. The last part of Table 4.4 analyses the importance of frequency in the consumer's utility function by reducing  $\beta$  by 20%. The result (in comparison to the original base run) shows a decrease in frequency of around 11% and a reduction in prices of around 5%, independent of agreement type.

Table 4.5 analyses the Brussels–Tel Aviv market in which the two current carriers, S.N. Brussels and El Al, have a codeshare agreement. The market is strictly limited by the bilateral that each country signed in which a single designated carrier per country is defined limiting frequency between the two cities per season. The general pattern of results is similar to that of the Tel Aviv–London market. The codeshare agreement reduces frequencies by 11% as compared to the competitive situation and increases prices by approximately 10%. The bilateral, on the other hand, reduces frequencies by 40% and increases prices by approximately 5% over the competitive solution. Consequently, if the transportation authorities were forced to choose between pure bilaterals

**Table 4.4**  
Sensitivity analysis on London–Tel Aviv market with respect to frequency.

Lufthansa frequency halved	Competitive	Bilateral	Competitive	Bilateral codeshare	Alliance codeshare
Business surplus	−7.7%	−10.9%	−7.2%	−10.0%	−11.4%
Leisure surplus	−18.6%	−29.6%	−19.5%	−27.4%	−27.0%
Consumer surplus	−11.5%	−17.0%	−11.3%	−15.5%	−17.0%
Producer surplus	15.6%	16.5%	13.0%	13.6%	13.2%
Social welfare	−2.4%	−4.5%	−2.0%	−4.0%	−4.2%
Prices business	9.5%	12.1%	9.8%	11.5%	11.1%
Prices leisure	16.9%	24.1%	17.3%	22.1%	21.9%
Frequency	1.2%	−0.3%	1.6%	−1.8%	−1.9%
Lufthansa frequency doubled					
Business surplus	5.6%	7.1%	5.2%	6.3%	7.6%
Leisure surplus	11.2%	16.5%	11.8%	15.0%	14.6%
Consumer surplus	7.6%	10.2%	7.4%	9.1%	10.1%
Producer surplus	−9.6%	−9.7%	−8.4%	−8.5%	−8.4%
Social welfare	1.8%	2.8%	1.4%	2.1%	2.3%
Prices business	−4.8%	−5.3%	−5.3%	−5.6%	−5.6%
Prices leisure	−6.8%	−8.4%	−7.2%	−8.1%	−8.0%
Frequency	−3.4%	−3.2%	−3.6%	−2.4%	−2.3%
Reduced importance of frequency (20%)					
Business surplus	−15.4%	−14.8%	−16.2%	−16.3%	−16.2%
Leisure surplus	−14.5%	−13.0%	−14.0%	−14.2%	−14.5%
Consumer surplus	−15.1%	−14.2%	−15.5%	−15.6%	−15.6%
Producer surplus	−13.8%	−14.1%	−16.7%	−16.7%	−16.6%
Social welfare	−14.7%	−14.2%	−16.0%	−16.0%	−16.0%
Prices business	−5.0%	−5.0%	−6.5%	−6.3%	−6.7%
Prices leisure	−2.5%	−3.7%	−4.5%	−4.6%	−4.4%
Frequency	−11.2%	−10.9%	−11.1%	−11.8%	−11.8%

**Table 4.5**  
Brussels–Tel Aviv airline market (per week).

	Competitive			Bilateral			Competitive codeshare			Bilateral codeshare			Alliance		
Surplus (in \$000s):															
Belgium–Israel business	1110			1012			1169			1156			1019		
Belgium–Israel leisure	451			361			414			403			424		
Israel–Belgium business	2153			1960			2274			2248			1953		
Israel–Belgium leisure	807			651			739			719			750		
Consumer surplus	4520			3984			4596			4525			4146		
Producer surplus	1920			2000			2579			2580			2714		
Social welfare	6440			5984			7175			7106			6859		
Average prices (\$):															
	LH	SN	LY	LH	SN	LY	LH	SN	LY	LH	SN	LY	LH	SN	LY
Belgium–Israel business	771	2169	1349	833	2231	1405	730	2396	1579	738	2404	1586	839	2662	1947
Belgium–Israel leisure	306	769	561	347	863	653	321	902	697	326	914	709	321	885	707
Israel–Belgium business	681	1575	1950	742	1637	2006	640	1803	2180	647	1810	2187	749	2144	2474
Israel–Belgium leisure	283	621	711	324	714	803	298	753	848	304	765	859	298	756	839
Frequency	12.0	11.5	11.6	12.0	9.3	9.1	12.0	11.0	10.7	12.0	10.7	10.4	12.0	10.6	10.6

or codeshares, the latter would be strictly preferable. In this market, the competitive codeshare agreement maximizes social welfare, whilst the anti-trust immune alliance maximizes the two direct airlines profits. Again, the different consumer types show disagreement, with leisure passengers preferring a competitive market outcome and business passengers maximizing surplus through the competitive codeshare agreement.

Finally, in Tables 4.6 and 4.7, we analyze the Bangkok–Tel Aviv market, in which the demand is rather weak and one sided, with a market of around 100,000 passengers annually of which over 95% originates in Israel. This is a reasonably standard picture for a tourist destination. The introduction of stochastic demand changes the total number of flights offered, specifically the London

**Table 4.6**  
Bangkok–Tel Aviv airline market (per week).

	Competitive			Bilateral			Competitive block codeshare			Bilateral free sale ( $\tau = \$370$ )			Alliance		
Surplus (in \$000s):															
Thailand–Israel business	23			22			26			26			24		
Thailand–Israel leisure	17			16			18			18			17		
Israel–Thailand business	2770			2706			2888			2886			2621		
Israel–Thailand leisure	1497			1422			1524			1528			1486		
Consumer surplus	4306			4166			4456			4458			4147		
Producer surplus	3441			3451			3364			3404			3538		
Social welfare	7748			7616			7820			7862			7685		
Average prices (\$):															
	LH	TG	LY	LH	TG	LY	LH	TG	LY	LH	TG	LY	LH	TG	LY
Thailand–Israel business	1716	1390	1955	1739	1417	2000	1597	1939	1873	1609	1988	1859	1684	2131	2081
Thailand–Israel leisure	705	942	1036	722	978	1089	681	1097	1031	689	1151	1022	683	1080	1030
Israel–Thailand business	1153	798	4823	1176	825	4868	1034	1347	4741	1046	1396	4727	1225	2066	5043
Israel–Thailand leisure	644	819	1761	662	855	1815	620	973	1757	629	1027	1748	649	1092	1785
Frequency	12.0	0.2	6.3	12.0	0.1	5.9	12.0	0.7	6.1	12.0	0.1	6.5	12.0	3.0	3.0

**Table 4.7**  
Effect of potential Thai–El Al agreements as compared to the competitive scenario.

	Bilateral			Competitive block codeshare			Bilateral block codeshare			Bilateral free sale			Alliance		
Business surplus	–2.3%			4.3%			2.0%			4.3%			–5.3%		
Leisure surplus	–5.0%			1.9%			–2.9%			2.1%			–0.7%		
Consumer surplus			–3.3%			3.5%			0.3%			3.5%			–3.7%
Producer surplus			0.3%			–2.2%			–2.0%			–1.1%			2.8%
Social welfare			–1.7%			0.9%			–0.7%			1.5%			–0.8%
Prices business			0.7%			–6.8%			–6.2%			–5.9%			4.9%
Prices leisure			1.2%			–2.2%			–1.0%			0.0%			2.9%
Frequency			–6.7%			5.3%			–1.0%			1.7%			–6.4%

and Brussels to Tel Aviv markets offer a higher number of flights under the no agreement outcome as compared to competitive block and free sale codeshares. Without this uncertainty, we would expect the opposite to be true as shown in Table 3.1. However, the Bangkok–Tel Aviv market represents a thin market and in this case, the sum of frequencies is lower without an agreement.

Social welfare is maximized in this market under the free sale agreement although consumers are fairly ambivalent to the codeshare or free sale agreement and the direct carriers would still prefer to ally. Both consumer and airline surplus are higher under free sale or codeshare agreements as compared to the competitive outcome (Table 4.7). It should be noted that the Israel–Belgian market is slightly smaller hence low demand is not a sufficient reason for permitting airlines to ally, rather the costs of serving the market also need to be considered. The costs to fly Tel Aviv–Bangkok are twice as high due to the greater distances involved, although once seat inventory is taken into account, the costs to fly to Bangkok are only 40% higher per seat. In this case, it could be argued that regulatory authorities or departments of transport should permit the airlines to sign an agreement on this gateway-to-gateway link since it is in the interests of both consumers and airlines alike.

## 5. Conclusions and future directions

The modeling framework developed in this research accounts for the level of frequency and price in the consumer's demand function. The impact of this change leads to new insights into the effects of inter-airline and inter-governmental agreements on consumer surplus, suggesting that not all anti-trust immune alliances are necessarily detrimental to consumers. In particular, benefits to consumers from codesharing agreements may outweigh disutilities arising from frequency reduction by colluding airlines. Conditions under which these effects occur were characterized analytically, alongside the ranking of various agreement types with respect to frequency offered, prices and surpluses of both airlines and consumers.

In a case study section, we analyze the impacts of country and airline agreements on frequency and prices on gateway-to-gateway routes in the aviation industry. The hybrid competitive and cooperative model permits analysis under asymmetric and uncertain demand with respect to business and leisure travellers' surplus, airline profits and overall social welfare. It is clear that there is a trade-off in the travelers' utility function between frequency (the higher the better) and airfare (the lower the better). The traveler may also have carrier preferences which are accounted for as vertical differentiation variables in the utility function. Competition authorities interested in understanding which agreement is most appropriate on parallel links, can observe that the bilateral agreements are consistently the worst of all possibilities, drastically reducing frequencies and increasing prices.

We note that a competitive codeshare is the most preferable outcome with respect to overall consumer surplus, but is only relevant on domestic flights or under ‘open-skies’ policies. If a competitive codeshare is not an alternative, then in most cases the competitive outcome is the most appropriate with respect to consumer surplus, however this is not always true. On a thin route, it is possible to identify cases in which bilateral block codeshares or free sale agreements are the most preferable market outcomes. Furthermore, whilst leisure passengers in markets with sufficient demand prefer the no agreement outcome, business passengers prefer competitive or bilateral codeshares and overall consumer surplus is also maximized under codeshares. Airlines strictly prefer alliances and in terms of overall social welfare, this outcome may still be preferable to that of competition, dependent on market parameter values.

In conclusion, there is a need to analyze each codeshare request on an individual basis. The modeling approach presented here is relatively simple from a computational perspective and could be applied to such a request when necessary. Airline managers could also use such an analysis in order to prove their case for a codeshare or anti-trust immune alliance when required to do so by their relevant regulatory authority. Up until recently, the literature argued that complementary codeshares are considered positive whereas parallel agreements are always onerous on consumers. This modeling approach shows that such results are over-simplistic. Indeed frequencies may be lower than the competitive scenario (anywhere between 2 and 10% lower) and prices might increase (from 10% for codeshares up to 20% for anti-trust immune alliances), yet the trade-off between opposing interests of consumer types and the airlines suggests that competitive codeshares are not always a bad alternative on the gateway-to-gateway links, dependent on amongst other things the size of the market. It may also be worthwhile pushing departments of transport towards the relaxation of bilateral agreements which consistently proved to be the worst market outcome, suggesting that multi-laterals or opening the skies are strictly preferable for all consumers. One could also argue that neither the European nor the United States regulators are correct in their response to requests to codeshare on hub-to-hub links. We would argue that codeshare agreements should be permitted on these links as opposed to the United States stance, and do not need to be restricted as occurs in the European Union. However, anti-trust immunity increases producer surplus at the expense of consumers, which should be avoided except in the case of thin markets.

In considering future directions, clearly this model is rather simplistic and requires assumptions which it may be interesting to reduce or remove. For example, on the theoretical level, the assumption of linear demand which could be adapted to include a logit style function, the log of the frequency offered which could be replaced with a concave increasing function, and the consideration of asymmetric information reflecting uncertainty about the type of consumer purchasing the ticket may enrich the analysis in new directions. We also have not considered the potential cost advantages airlines may gain when reaching an agreement that would further strengthen the results arrived at here. It would also be of considerable interest to extend the analysis to consider networks, which would enable an evaluation of the trade-offs between parallel and complementary codeshares with respect to consumer surplus and overall social welfare. This would create theoretical understanding of the type of regulation necessary (or not) for international airline markets. Furthermore, the introduction of point-to-point network competition with full service network carriers within this framework would help to shed further light on, for example, the impact of subsidies on international aviation markets.

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**Appendix A**

**Proposition A.1.** *For each agreement type, there exists a unique solution in prices, as a function of the frequency offered, under binding capacity constraints (in formulation (2.3)), defined by*

$$\tilde{p}_{ab}(\bar{f}) = -\frac{s}{2m\theta} \left[ (1-y)f_a + \left( \frac{1}{\theta-2} + \frac{y}{2} \right) \sum_{a'} f_{a'} \right] + \beta \ln \left( 1 + \sum_{a' \in \rho_a} f_{a'} \right) + \frac{(\theta+y-1) \frac{\bar{\alpha}+\alpha}{2} + \theta\alpha_{ab}}{2\theta+y-1}. \tag{A.1}$$

Given these prices, airline a’s profit as a function of the frequency offered is

$$\tilde{\pi}_a(\bar{f}) = -\frac{s^2}{2m\theta} \left( f_a + \frac{1}{\theta-2} \sum_{a'} f_{a'} \right) f_a + \left( s\beta \ln \left( 1 + \sum_{a' \in \rho_a} f_{a'} \right) + s \frac{\bar{\alpha}+\alpha}{2} - c \right) f_a + \frac{m(\bar{\alpha}-\alpha)^2\theta^2(\theta+y-1)}{2(2\theta+y-1)^2}. \tag{A.2}$$

Note that the fifth contract covering anti-trust immune alliance agreements is the only contract that permits cooperation in pricing,  $y = 1$ , hence for the first four contracts  $y = 0$ . The proposition assumes binding capacity constraints, which is justified below in [Proposition A.2](#).

To characterize frequency offered under the five agreement types, let  $l = 1$  represent the existence of a codesharing agreement and  $l = 0$ , no codesharing. Also, let  $r = 1$  represent frequency cooperation and  $r = 0$ , frequency competition. Thus, for example,  $(l = 0, r = 0, y = 0)$  represents no agreement,  $(l = 1, r = 1, y = 0)$  refers to a bilateral codesharing agreement and  $(l = 1, r = 1, y = 1)$  represents an alliance agreement. Throughout the analysis we assume  $\beta > 0$ . After solving for all agreement types, we can summarize the set of solutions as demonstrated in Proposition A.2 and A.3.

**Proposition A.2.** For each agreement type, a unique symmetric solution exists in frequency and prices as characterized by the first order conditions

$$H \equiv -\frac{s^2(2\theta - 1 + r)f^*}{2m\theta(\theta - 2)} + s\beta \left[ \ln(1 + (1 + l)f^*) + \frac{(1 + rl)f^*}{1 + (1 + l)f^*} \right] + s\frac{\bar{\alpha} + \alpha}{2} - c = 0, \tag{A.3}$$

where  $f^*$  represents the frequency offered by each airline, and the prices are

$$p_{ab}^* \equiv -\frac{sf^*}{2m(\theta - 2)} + \beta \ln(1 + (1 + l)f^*) + \frac{(\theta + y - 1)\frac{\bar{\alpha} + \alpha}{2} + \theta\alpha_{ab}}{2\theta + y - 1}. \tag{A.4}$$

The capacity constraints (in (2.3)) for these solutions are always binding.

**Proposition A.3.** For each agreement type, the profit of each airline equals

$$\pi^* \equiv -\frac{s^2(f^*)^2}{2m(\theta - 2)} + \left( s\beta \ln(1 + (1 + l)f^*) + s\frac{\bar{\alpha} + \alpha}{2} - c \right) f^* + \frac{\theta^2(\theta + y - 1)m(\bar{\alpha} - \alpha)^2}{2(2\theta + y - 1)^2}, \tag{A.5}$$

the consumer net value equals

$$u_{ab}^* \equiv \frac{sf^*}{2m(\theta - 2)} - \frac{(\theta + y - 1)(\frac{\bar{\alpha} + \alpha}{2} - \alpha_{ab})}{2\theta + y - 1}, \tag{A.6}$$

the consumer welfare equals

$$\frac{s^2(f^*)^2}{2m(\theta - 2)} + \frac{\theta(\theta + y - 1)^2m(\bar{\alpha} - \alpha)^2}{2(2\theta + y - 1)^2}, \tag{A.7}$$

and the total social welfare equals

$$-\frac{s^2(f^*)^2}{2m(\theta - 2)} + 2\left( s\beta \ln(1 + (1 + l)f^*) + s\frac{\bar{\alpha} + \alpha}{2} - c \right) f^* + \frac{\theta(\theta + y - 1)(3\theta + y - 1)m(\bar{\alpha} - \alpha)^2}{2(2\theta + y - 1)^2}. \tag{A.8}$$

**Proof of Proposition A1.** Under the first four agreements, each airline's profit function can be formulated with the frequency constrained Lagrange multiplier  $\lambda_a$ , representing the value of having an additional unit of frequency, as

$$\sum_b m \left( k_{ab} - \theta p_{ab} + \sum_{a'} p_{a'b} \right) p_{ab} - c f_a - \lambda_a \left( \sum_b d_{ab} - s f_a \right),$$

where  $k_{ab} \equiv \theta[\alpha_{ab} + \beta \ln(1 + \sum_{a' \in \rho_a} f_{a'})] - \sum_{a'} [\alpha_{a'b} + \beta \ln(1 + \sum_{a'' \in \rho_{a'}} f_{a''})]$  depends on the first stage choices, but not on the second stage prices. The first order conditions for airline  $a$  and consumer type  $b$  are

$$0 = m \left( k_{ab} - (2\theta - 1)p_{ab} + \sum_{a'} p_{a'b} + (\theta - 1)\lambda_a \right).$$

Solving this system of linear equations for  $a$  and  $b$ , each equation can be rewritten as  $(2\theta - 1)p_{ab} - \sum_{a'} p_{a'b} = k_{ab} + (\theta - 1)\lambda_a$ . Summing for  $a$ ,  $\sum_{a'} p_{a'b} = \frac{1}{2\theta - 3} \sum_{a'} k_{a'b} + \frac{\theta - 1}{2\theta - 3} \sum_{a'} \lambda_{a'}$ . Substituting and rearranging terms we find

$$p_{ab} = \frac{1}{2\theta - 1} \left[ k_{ab} + \frac{1}{2\theta - 3} \sum_{a'} k_{a'b} + (\theta - 1)\lambda_a + \frac{\theta - 1}{2\theta - 3} \sum_{a'} \lambda_{a'} \right].$$

We assume here that the frequency constraint is binding for each  $a$  (this is verified in the proof of Proposition A.2). Substituting  $p_{ab}$  into the binding constraint  $\sum_b d_{ab} = s f_a$ , we have  $\lambda_a \theta - \frac{\theta - 1}{2\theta - 3} \sum_{a'} \lambda_{a'} = \frac{1}{2m} \gamma_a$ , where  $\gamma_a \equiv m \sum_b (k_{ab} + \frac{1}{2\theta - 3} \sum_{a'} k_{a'b}) - \frac{2\theta - 1}{\theta - 1} s f_a$ . Summing for  $a$ ,  $\sum_{a'} \lambda_{a'} = \frac{1}{2m} \frac{2\theta - 3}{(2\theta - 1)(\theta - 2)} \sum_{a'} \gamma_{a'}$ . Substituting and rearranging terms,

$$\lambda_a = \frac{1}{2\theta m} \left[ \gamma_a + \frac{\theta - 1}{(2\theta - 1)(\theta - 2)} \sum_{a'} \gamma_{a'} \right].$$

Since  $\sum_{a'} \lambda_{a'} = \frac{1}{2m} \frac{2\theta - 3}{(2\theta - 1)(\theta - 2)} \sum_{a'} \gamma_{a'}$ , we can substitute to find

$$p_{ab} = \frac{1}{2\theta - 1} \left[ k_{ab} + \frac{1}{2\theta - 3} \sum_{a'} k_{a'b} + \frac{1}{2m} \frac{\theta - 1}{\theta} \gamma_a + \frac{1}{2m} \frac{\theta - 1}{\theta(\theta - 2)} \sum_{a'} \gamma_{a'} \right].$$



Substituting  $\sum_{a'} \gamma_{a'} = m \frac{2\theta-1}{2\theta-3} \sum_b \sum_{a'} k_{a'b} - \frac{2\theta-1}{\theta-1} s \sum_{a'} f_{a'}$  and  $\sum_{a'} k_{a'b} = (\theta-2) \sum_{a'} [\alpha_{a'b} + \beta \ln(1 + \sum_{a'' \in \rho_{a'}} f_{a''})]$  and rearranging terms, the expression for  $\tilde{p}_{ab}(f_a, f_{-a})$  is proved for  $y = 0$ . A similar derivation proves the expression for  $y = 1$ . Substituting these prices into the profit function, we arrive at the expression for  $\tilde{\pi}_a(f_a, f_{-a})$ .  $\square$

**Proof of Proposition A2.** Assuming symmetry, we look for a symmetric solution, i.e. the frequencies offered  $f_a$  are equal for both airlines. Under no agreement and the competitive codesharing agreement, the frequencies offered are determined in Nash equilibrium. Differentiating  $\tilde{\pi}_a(f_a, f_{-a})$ , since  $\frac{\partial \tilde{\pi}_a}{\partial f_a}(0, 0) = s \frac{\bar{\alpha} + \alpha}{2} - c > 0$ ,  $f_a = 0$  for both  $a$  cannot be in equilibrium. Since

$$\frac{\partial \tilde{\pi}_a}{\partial f_a} = -\frac{s^2[(2\theta-3)f_a + \sum_{a'} f_{a'}]}{2m\theta(\theta-2)} + s\beta \left[ \ln \left( 1 + \sum_{a' \in \rho_a} f_{a'} \right) + \frac{f_a}{1 + \sum_{a' \in \rho_a} f_{a'}} \right] + s \frac{\bar{\alpha} + \alpha}{2} - c$$

is concave in  $f_a$  and attains negative values for sufficiently high  $f_a$ , for any opponent's frequency offered,  $f_{-a}$ , there exists a unique frequency offered  $f_a(f_{-a}) > 0$  such that  $\frac{\partial \tilde{\pi}_a}{\partial f_a} = 0$  and  $\frac{\partial^2 \tilde{\pi}_a}{\partial (f_a)^2} < 0$ , thus forming the best response function of airline  $a$  to  $f_{-a}$ . Since this function is continuously decreasing in  $f_{-a}$ , it has a unique fixed point  $f^*$ , forming a Nash equilibrium and satisfying  $\frac{\partial \tilde{\pi}_a}{\partial f_a}(f^*, f^*) = 0$ . This proves that the first order condition  $H = 0$  characterizes the equilibrium for  $r = 0, y = 0$  and  $l = 0$  or  $1$ .

Under bilateral and bilateral codesharing agreements, frequencies offered are determined according to the Nash bargaining solution. The derivative of  $\Pi_a[\tilde{\pi}_a(f_a, f_{-a}) - \tilde{\pi}_a(f_a^{comp}, f_{-a}^{comp})]$  at a symmetric point equals  $\sum_{a'} \frac{\partial \tilde{\pi}_{a'}}{\partial f_a}(f, f)[\tilde{\pi}_{-a'}(f, f) - \tilde{\pi}_{-a'}(f^{comp}, f^{comp})]$ . Since symmetry implies that  $\tilde{\pi}_1(f_1, f_2) = \tilde{\pi}_2(f_2, f_1)$ ,  $\sum_{a'} \frac{\partial \tilde{\pi}_{a'}}{\partial f_a}(f, f) = \frac{\partial [\tilde{\pi}_a(f, f)]}{\partial f}$ . Since

$$\frac{\partial [\tilde{\pi}_a(f, f)]}{\partial f} = -\frac{s^2(2\theta)f}{2m\theta(\theta-2)} + s\beta \left[ \ln(1 + (1+l)f) + \frac{(1+l)f}{1 + (1+l)f} \right] + s \frac{\bar{\alpha} + \alpha}{2} - c$$

is strictly concave in  $f$ , there exist  $f'$  close to  $f^{comp}$  such that  $\tilde{\pi}_a(f', f') > \tilde{\pi}_a(f^{comp}, f^{comp})$  for both  $a$ , thus  $f^{comp}$  cannot be a solution. Adding that  $\frac{\partial [\tilde{\pi}_a(f, f)]}{\partial f}(0) > 0$ , a symmetric solution must satisfy  $\frac{\partial [\tilde{\pi}_a(f, f)]}{\partial f} = 0$ . Concavity implies that there exists a unique  $f^* > 0$  satisfying this condition as well as  $\frac{\partial^2 [\tilde{\pi}_a(f, f)]}{\partial f^2}(f^*) < 0$ , thus proving that the first order condition  $H = 0$  characterizes the solution for  $r = 1, y = 0$  and  $l = 0$  or  $1$ .

It remains to be proved that the first order condition under an alliance, i.e. for  $l = 1, r = 1$  and  $y = 1$ , characterizes the solution. An argument similar to the one for the Nash bargaining solution can be used to show that since the derivatives of  $\sum_a \tilde{\pi}_a(f_a, f_{-a})$  are positive at  $(0, 0)$ , symmetry implies that a symmetric solution must satisfy  $\frac{\partial [\tilde{\pi}_a(f, f)]}{\partial f} = 0$ , the existence of which was already established. Note that this derivative does not depend on  $y$ , verifying the condition in the proposition.

We now show that the frequency constraint is binding for each  $a$  under  $H = 0$ , as assumed in Proposition A.1. We first show that given the solution characterized above, the case where the constraint is not binding for  $a$  but is binding for its opponent leads to a contradiction. In this case the prices are

$$\begin{aligned} p_{ab} &= \frac{1}{2\theta-2} (k_{ab} + p_{(-a)b}) \\ &= -\frac{sf}{4m(\theta-1)(\theta-2)} + \frac{1}{2} \left[ \frac{2\theta\alpha_a^b - \frac{\bar{\alpha} + \alpha}{2}}{2\theta-1} + \beta \ln(1 + (1+l)f) \right], \end{aligned}$$

thus  $\sum_b d_{ab} = -\frac{s(2\theta-3)f}{2(\theta-2)} + m(\theta-1) \left[ \frac{\bar{\alpha} + \alpha}{2} + \beta \ln(1 + (1+l)f) \right]$ . Since  $H = 0$ ,

$$\begin{aligned} \sum_b d_{ab} - sf &= \sum_b d_{ab} - sf + \frac{m}{s}(\theta-1)H \\ &= \frac{(\theta-1+r)(\theta-1) - \theta(2\theta-3)}{2\theta(\theta-2)} sf + m(\theta-1) \left[ \frac{c}{s} - \beta \frac{(1+rl)f}{1+f(1+l)} \right] \\ &> 0 \end{aligned}$$

because  $\frac{c}{s} > \beta$ , violating the frequency constraint. Thus the constraint must be binding for both  $a$ .

Now we show that there is no equilibrium where the constraint is not binding for both  $a$ . In this case, the prices are  $p_{ab} = \frac{1}{2\theta-1} [k_{ab} + \frac{1}{2\theta-3} \sum_{a'} k_{a'b}]$  (see proof of Proposition A.1). With these prices, a similar analysis to the one given above for binding constraints leads to the following first order condition for the frequency offered:

$$\begin{aligned} &\frac{(\theta-2)(\theta-1)[(\theta-1)(\theta-2) + (\theta-1)(1-r)(1-l)](1+rl)}{(1+(1+l)f)(2\theta-3)^2(2\theta-1)} \\ &\times 4m\beta \left[ \frac{\bar{\alpha} + \alpha}{2} + \beta \ln(1 + (1+l)f) \right] - c = 0. \end{aligned}$$

Moreover, total demand for airline  $a$  equals

$$\begin{aligned} \sum_b d_{ab} &= \sum_b m \left( k_{ab} - \theta p_{ab} + \sum_{a'} p_{a'b} \right) \\ &= \frac{2m(\theta - 2)(\theta - 1)}{2\theta - 3} \left[ \frac{\bar{\alpha} + \underline{\alpha}}{2} + \beta \ln(1 + (1 + l)f) \right]. \end{aligned}$$

Substituting the first order conditions we have  $\sum_b d_{ab} = \frac{c(1+f)(2\theta-3)(2\theta-1)}{2\beta[(2\theta-1)(\theta-2)+(1-r)(\theta-1)]}$  for  $l = 0$  and  $\frac{c(1+2f)(2\theta-3)}{2\beta(\theta-2)(1+r)}$  for  $l = 1$ , where  $r = 0$  or  $1$ . Both expressions are greater than  $\frac{cf}{\beta}$ . Since  $\frac{c}{s} > \beta$ , in both cases the demand is higher than  $sf$ , violating the constraint. Therefore the constraint must be binding for both  $a$ .

The expression for  $p_{ab}^*$  follows immediately from  $\tilde{p}_a(f_a, f_{-a})$  in Proposition A.1 under equal frequencies offered by both airlines.  $\square$

**Proof of Proposition A3.** The expression for  $\pi^*$  follows immediately from  $\tilde{\pi}_a(f_a, f_{-a})$  in Proposition A.1 under equal frequencies offered by both airlines. Similarly, the expression for  $u_{ab}^*$  follows from  $p_{ab}^*$  in Proposition A.2 and the definition of  $u_{ab}$ . The consumer surplus is then

$$\begin{aligned} \sum_b \frac{m}{2} \left[ \theta \sum_{a'} (u_{a'b}^*)^2 - \left( \sum_{a'} u_{a'b}^* \right)^2 \right] &= \frac{m}{2} \left[ \theta \left[ 4 \left( \frac{sf^*}{2m(\theta - 2)} \right)^2 + \left( \frac{\theta + y - 1}{2\theta + y - 1} \right)^2 \left( 4 \left( \frac{\bar{\alpha} + \underline{\alpha}}{2} \right)^2 \right. \right. \right. \\ &\quad \left. \left. \left. - 4 \frac{(\bar{\alpha} + \underline{\alpha})^2}{2} + 2(\bar{\alpha}^2 + \underline{\alpha}^2) \right) \right] - 2 \left( \frac{2sf^*}{2m(\theta - 2)} \right)^2 \right] \\ &= \frac{s^2(f^*)^2}{2m(\theta - 2)} + \frac{\theta(\theta + y - 1)^2 m(\bar{\alpha} - \underline{\alpha})^2}{2(2\theta + y - 1)^2}. \end{aligned}$$

The total social welfare then follows from summing the profits of both airlines and the consumer surplus.  $\square$

**Proof of Proposition 3.1.** To show (1), note that total differentiation of  $H$  from Proposition A.2 implies that  $\frac{\partial f^*}{\partial \beta} = -\frac{\partial H / \partial \beta}{\partial H / \partial f^*}$ . Since  $H$  is positive at  $f^* = 0$  and strictly concave in  $f^*$ ,  $H = 0$  implies  $\frac{\partial H}{\partial f^*} < 0$ . Since  $\frac{\partial H}{\partial \beta} > 0$ ,  $\frac{\partial f^*}{\partial \beta} > 0$ .

For (2), we will show that there exist  $\hat{m}(\beta) > 0$  and  $\hat{h}(m, \beta) > 0$  such that  $f_{\text{bilateral codeshare}}^* > f_{\text{comp codeshare}}^*$  for  $\frac{s}{2\beta\theta(\theta-2)} < m < \hat{m}(\beta)$  and  $s\frac{\bar{\alpha}+\underline{\alpha}}{2} - c < \hat{h}(m, \beta)$ ; the opposite is true otherwise. Note that

$$\frac{1}{s\beta} (H_{\text{comp codeshare}} - H_{\text{no agreement}}) = \ln \left( \frac{1 + 2f}{1 + f} \right) + \frac{f}{1 + 2f} - \frac{f}{1 + f}$$

equals 0 at  $f = 0$ , increases for  $0 < f < \frac{1+\sqrt{5}}{2}$ , decreases for  $f > \frac{1+\sqrt{5}}{2}$  and converges to  $\ln 2 - \frac{1}{2} > 0$  as  $f \rightarrow \infty$ , thus is positive for  $f > 0$ , implying that  $f_{\text{comp codeshare}}^* > f_{\text{no agreement}}^*$ . Similarly,

$$\frac{1}{s\beta} (H_{\text{bilateral codesharing}} - H_{\text{bilateral}}) = \ln \left( \frac{1 + 2f}{1 + f} \right) + \frac{2f}{1 + 2f} - \frac{f}{1 + f}$$

equals 0 at  $f = 0$  and increases for  $f > 0$ , thus is also positive for  $f > 0$  implying that  $f_{\text{bilateral codesharing}}^* > f_{\text{bilateral}}^*$ . Since  $H_{\text{no agreement}} - H_{\text{bilateral}} = \frac{s^2}{2m\theta(\theta-2)} f > 0$  for  $f > 0$ ,  $f_{\text{no agreement}}^* > f_{\text{bilateral}}^*$ .

For (3), we will show that there exist  $\tilde{m}(\beta) > 0$  and  $\tilde{h}(m, \beta) > 0$  such that  $f_{\text{bilateral codeshare}}^* > f_{\text{no agreement}}^*$  for  $\frac{s}{4\beta\theta(\theta-2)} < m < \tilde{m}(\beta)$  and  $s\frac{\bar{\alpha}+\underline{\alpha}}{2} - c < \tilde{h}(m, \beta)$ ; The opposite is true otherwise. Consider the difference

$$\delta(f) \equiv H_{\text{bilateral codesharing}} - H_{\text{comp codeshare}} = -\frac{s^2 f}{2m\theta(\theta - 2)} + \frac{s\beta f}{1 + 2f}.$$

Since  $\delta(0) = 0$  and  $\delta(f)$  is strictly concave for  $f > 0$ , a necessary condition for  $\delta(f) > 0$  is  $\frac{\partial \delta}{\partial f}(f = 0) = s\beta - \frac{s^2}{2m\theta(\theta-2)} > 0$ , i.e.  $m > \frac{s}{2\beta\theta(\theta-2)}$ . Under this necessary condition, since  $\frac{\partial \delta}{\partial f} < 0$  for  $f$  sufficiently large, there exists  $\bar{f} > 0$ , increasing in  $m$  and  $\beta$ , such that  $\delta = 0$  at  $f = \bar{f}(m, \beta)$ , and  $\delta(f) > 0$  if and only if  $0 < f < \bar{f}(m, \beta)$ . Thus

$$f_{\text{bilateral codeshare}}^* > f_{\text{comp codeshare}}^* \text{ if and only if } H(f = \bar{f}(m, \beta)) < 0.$$

Let  $\bar{H} = H - (s\frac{\bar{\alpha}+\underline{\alpha}}{2} - c)$ . Since  $\frac{\partial \bar{H}_{\text{bilateral codesharing}}}{\partial f}(f = 0) = 4s\beta - \frac{s^2}{m(\theta-2)}$ , it equals  $-2\beta s(\theta - 2) < 0$  for  $m = \frac{s}{2\beta\theta(\theta-2)}$ . Since  $\bar{H}_{\text{bilateral codesharing}}$  is concave in  $f$  and continuous in  $m$ ,  $\bar{H}_{\text{bilateral codesharing}}(f = \bar{f}(m, \beta)) < 0$  for sufficiently small  $m > \frac{s}{2\beta\theta(\theta-2)}$ . Since  $\bar{H}_{\text{bilateral codesharing}}(f = \bar{f}(m, \beta))$  increases in  $m$  above some point where it is still negative, there exists  $\hat{m} > 0$ , that depends on  $\beta$ ,

such that  $\bar{H}(f = \bar{f}(m, \beta)) < 0$  if and only if  $\frac{s}{2\beta\theta(\theta-2)} < m < \hat{m}(\beta)$ . For  $m$  in this range, letting  $\hat{a}(m, \beta) = -\bar{H}(f = \bar{f}(m, \beta))$ , we have

$$s\frac{\bar{\alpha} + \alpha}{2} - c < \hat{a}(m, \beta) \text{ if and only if } H(f = \bar{f}(m, \beta)) < 0,$$

thus proving (3).

The proof of (4) is similar to (3), with the changes:

$$\begin{aligned} \delta(f) &\equiv H_{\text{bilateral codeshare}} - H_{\text{no agreement}} \\ &= -\frac{s^2 f}{2m\theta(\theta-2)} + s\beta\left[\ln\left(\frac{1+2f}{1+f}\right) + \frac{2f}{1+2f} - \frac{f}{1+f}\right], \end{aligned}$$

$$\frac{\partial \delta}{\partial f}(f=0) = 2s\beta - \frac{s^2}{2m\theta(\theta-2)} > 0, m > \frac{s}{4\beta\theta(\theta-2)}, \text{ and } \frac{\partial \bar{H}_{\text{bilateral codeshare}}}{\partial f}(f=0) = -4\beta s(\theta-1) < 0 \text{ for } m = \frac{s}{4\beta\theta(\theta-2)}.$$

(5) follows immediately because  $H$  does not depend on  $y$ .  $\square$

**Proof of Proposition 3.2.** (1) follows from total differentiation of price with respect to  $\beta$  since  $\frac{df^*}{d\beta} > 0$  by Proposition 3.1.

To prove (2) first note that

$$p_{ab}^* = \frac{1}{sf^*} \left[ \pi^* - \frac{\theta^2(\theta+y-1)m(\bar{\alpha}-\alpha)^2}{2(2\theta+y-1)^2} \right] - \left( \frac{\bar{\alpha} + \alpha}{2} - \frac{c}{s} \right) + \frac{(\theta+y-1)\frac{\bar{\alpha}+\alpha}{2} + \theta\alpha_{ab}}{2\theta+y-1}.$$

Thus (2) follows because  $\pi_{\text{bilateral}}^* > \pi_{\text{no agreement}}^*$  and  $f_{\text{bilateral}}^* < f_{\text{no agreement}}^*$  by Propositions 3.3 and 3.1. A similar argument shows that for any  $m, \beta > 0$  there exist  $\hat{h}(m, \beta) > 0$  such that  $p_{\text{bilateral codeshare}}^* > p_{\text{comp codeshare}}^*$  for  $s\frac{\bar{\alpha}+\alpha}{2} - c > \hat{h}(m, \beta)$ . To prove the remainder of (3) we show that for any  $m, \beta > 0$  there exist  $\hat{h}(m, \beta) > 0$  such that  $p_{\text{comp codeshare}}^* > p_{\text{no agreement}}^*$  and  $p_{\text{bilateral codeshare}}^* > p_{\text{bilateral}}^*$  for  $s\frac{\bar{\alpha}+\alpha}{2} - c > \hat{h}(m, \beta)$ . Note that since  $H = 0$ ,

$$\begin{aligned} p_{ab}^* &= p_{ab}^* - \frac{1}{s}H \\ &= \frac{s(\theta-1+r)f^*}{2m\theta(\theta-2)} - \beta \frac{(1+rl)f^*}{1+(1+l)f^*} - \left( \frac{\bar{\alpha} + \alpha}{2} - \frac{c}{s} \right) + \frac{(\theta+y-1)\frac{\bar{\alpha}+\alpha}{2} + \theta\alpha_{ab}}{2\theta+y-1}. \end{aligned}$$

Then

$$\begin{aligned} p_{\text{comp codeshare}}^* - p_{\text{no agreement}}^* &= \left( \frac{s(\theta-1)}{2m\theta(\theta-2)} - \frac{\beta}{1+2f_{\text{comp codeshare}}^*} \right) f_{\text{comp codeshare}}^* \\ &\quad - \left( \frac{s(\theta-1)}{2m\theta(\theta-2)} - \frac{\beta}{1+f_{\text{no agreement}}^*} \right) f_{\text{no agreement}}^*. \end{aligned}$$

Since  $f_{\text{comp codeshare}}^* > f_{\text{no agreement}}^*$  by Proposition 3.1, and  $f^*$  is increasing in the profit margin  $s\frac{\bar{\alpha}+\alpha}{2} - c$ , the price difference is positive for sufficiently large  $s\frac{\bar{\alpha}+\alpha}{2} - c$ . This proves the existence of a threshold  $\hat{h}(m, \beta)$ . A similar argument applies to the comparison

$$\begin{aligned} p_{\text{bilateral codeshare}}^* - p_{\text{bilateral}}^* &= \left( \frac{s}{2m(\theta-2)} - \frac{2\beta}{1+2f_{\text{bilateral codeshare}}^*} \right) f_{\text{bilateral codeshare}}^* \\ &\quad - \left( \frac{s}{2m(\theta-2)} - \frac{\beta}{1+f_{\text{bilateral}}^*} \right) f_{\text{bilateral}}^*. \end{aligned}$$

For (4) we will show that for any  $m, \beta > 0$  there exist  $\tilde{h}(m, \beta) > 0$  such that  $p_{\text{comp codeshare}}^* > p_{\text{bilateral}}^*$  for sufficiently large values of  $s\frac{\bar{\alpha}+\alpha}{2} - c$ . for  $s\frac{\bar{\alpha}+\alpha}{2} - c < \tilde{h}(m, \beta)$ ; The opposite is true otherwise. Note that since  $f_{\text{no agreement}}^* - f_{\text{bilateral}}^* > 0$  by Proposition 3.1,

$$-\frac{s^2}{2m\theta(\theta-2)} [(2\theta-1)f_{\text{no agreement}}^* - (2\theta)f_{\text{bilateral}}^*] < H_{\text{no agreement}} - H_{\text{bilateral}} = 0,$$

thus  $\frac{2\theta}{2\theta-1} f_{\text{bilateral}}^* < f_{\text{no agreement}}^* < f_{\text{comp codeshare}}^*$ . Therefore

$$p_{\text{comp codeshare}}^* - p_{\text{bilateral}}^* > \left[ \left( \frac{s(\theta-1)}{2m\theta(\theta-2)} - \frac{\beta}{1+2\frac{2\theta}{2\theta-1}f_{\text{bilateral}}^*} \right) \left( \frac{2\theta}{2\theta-1} \right)^2 - \left( \frac{s\theta}{2m\theta(\theta-2)} - \frac{\beta}{1+f_{\text{bilateral}}^*} \right) \right] f_{\text{bilateral}}^*.$$

The right hand side is positive when  $f_{\text{bilateral}}^*$  is sufficiently low, which holds for sufficiently small  $s\frac{\bar{\alpha}+\alpha}{2} - c$ . For large values of  $s\frac{\bar{\alpha}+\alpha}{2} - c$ ,  $f_{\text{comp codeshare}}^* - \frac{2\theta}{2\theta-1} f_{\text{bilateral}}^*$  approaches a positive constant and  $p_{\text{comp codeshare}}^* - p_{\text{bilateral}}^*$  approaches  $\frac{s}{2m\theta(\theta-2)}$

$[(\theta - 1)(f_{\text{comp codeshare}}^*)^2 - \theta(f_{\text{bilateral}}^*)^2]$ . Since  $(\theta - 1)(\frac{2\theta}{2\theta-1})^2 - \theta = -\frac{\theta}{(2\theta-1)^2} < 0$ , we have  $p_{\text{comp codeshare}}^* < p_{\text{bilateral}}^*$ . This proves the existence of a threshold  $\tilde{h}(m, \beta)$  that determines where the price ranking reverses.

Finally, (5) is verified because  $\bar{\alpha} > \underline{\alpha}$  implies that as  $y$  changes from 0 to 1,  $\frac{(\theta+y-1)\frac{\bar{\alpha}+\underline{\alpha}}{2} + \theta\alpha_{ab}}{2\theta+y-1}$  decreases for  $\alpha_{ab} = \bar{\alpha}$  and increases for  $\alpha_{ab} = \underline{\alpha}$ .  $\square$

**Proof of Proposition 3.3.** (1) follows from total differentiation of profit with respect to  $\beta$  since  $\frac{df^*}{d\beta} > 0$  by Proposition 3.1.

To prove (2) note that by Proposition 3.1,  $f_a = f_{\text{bilateral codeshare}}^*$  for both  $a$  maximizes the sum of profits  $\sum_a \tilde{\pi}_a(f_a, f_{-a})$  under the bilateral codesharing agreement. Thus  $\pi_{\text{bilateral codeshare}}^* > \pi_{\text{bilateral}}^*$ , because for any  $f \geq 0$ , in particular for  $f = f_{\text{bilateral}}^*$ , the difference between  $\sum_a \tilde{\pi}_a(f, f)$  under the bilateral codeshare and bilateral agreements equals  $2s\beta f_{\text{bilateral}}^* \ln(\frac{1+2f}{1+f}) > 0$ . According to the Nash bargaining solution definition,  $\pi_{\text{bilateral}}^* \geq \pi_{\text{no agreement}}^*$  and  $\pi_{\text{bilateral codeshare}}^* \geq \pi_{\text{comp codeshare}}^*$ . The proof of Proposition A.2 shows that the inequalities are strict because there exist  $f'$  close to  $f^{\text{comp}}$  such that  $\tilde{\pi}_a(f', f') > \tilde{\pi}_a(f^{\text{comp}}, f^{\text{comp}})$  for both  $a$ .

To prove (3) and (4) first note that since  $H = 0$ ,

$$\begin{aligned} \pi^* &= \pi^* - f^*H \\ &= \left[ \frac{s^2(\theta - 1 + r)}{2m\theta(\theta - 2)} - s\beta \frac{(1 + rl)}{1 + (1 + l)f^*} \right] (f^*)^2 + \frac{\theta^2(\theta + y - 1)m(\bar{\alpha} - \underline{\alpha})^2}{2(2\theta + y - 1)^2}. \end{aligned}$$

To prove (3) we will show that for any  $m, \beta > 0$  there exists  $\hat{h}(m, \beta) > 0$  such that  $\pi_{\text{comp codeshare}}^* > \pi_{\text{no agreement}}^*$  for  $s\frac{\bar{\alpha}+\underline{\alpha}}{2} - c \geq \hat{h}(m, \beta)$ . Note that

$$\begin{aligned} \pi_{\text{comp codeshare}}^* - \pi_{\text{no agreement}}^* &= \left( \frac{s^2(\theta - 1)}{2m\theta(\theta - 2)} - \frac{s\beta}{1 + 2f_{\text{comp codeshare}}^*} \right) (f_{\text{comp codeshare}}^*)^2 \\ &\quad - \left( \frac{s^2(\theta - 1)}{2m\theta(\theta - 2)} - \frac{s\beta}{1 + f_{\text{no agreement}}^*} \right) (f_{\text{no agreement}}^*)^2. \end{aligned}$$

Since  $f_{\text{comp codeshare}}^* > f_{\text{no agreement}}^*$  by Proposition 3.1, and  $f^*$  is increasing in the profit margin  $s\frac{\bar{\alpha}+\underline{\alpha}}{2} - c$ , the profit difference is positive for sufficiently large  $s\frac{\bar{\alpha}+\underline{\alpha}}{2} - c$ , thus proving the existence of a threshold  $\hat{h}(m, \beta)$ .

For (4) we will show that for any  $m, \beta > 0$  there exists  $\tilde{h}(m, \beta) > 0$  such that  $\pi_{\text{comp codeshare}}^* > \pi_{\text{bilateral}}^*$  for  $s\frac{\bar{\alpha}+\underline{\alpha}}{2} - c < \tilde{h}(m, \beta)$ ; The opposite is true otherwise. Note that since  $f_{\text{no agreement}}^* - f_{\text{bilateral}}^* > 0$  by Proposition 3.1,

$$-\frac{s^2}{2m\theta(\theta - 2)} [(2\theta - 1)f_{\text{no agreement}}^* - (2\theta)f_{\text{bilateral}}^*] < H_{\text{no agreement}} - H_{\text{bilateral}} = 0,$$

thus  $\frac{2\theta}{2\theta-1}f_{\text{bilateral}}^* < f_{\text{no agreement}}^* < f_{\text{comp codeshare}}^*$ . Therefore

$$\begin{aligned} \pi_{\text{comp codeshare}}^* - \pi_{\text{bilateral}}^* &> \left[ \left( \frac{s^2(\theta - 1)}{2m\theta(\theta - 2)} - \frac{s\beta}{1 + 2\frac{2\theta}{2\theta-1}f_{\text{bilateral}}^*} \right) \left( \frac{2\theta}{2\theta - 1} \right)^2 \right. \\ &\quad \left. - \left( \frac{s^2\theta}{2m\theta(\theta - 2)} - \frac{s\beta}{1 + f_{\text{bilateral}}^*} \right) \right] (f_{\text{bilateral}}^*)^2. \end{aligned}$$

The right hand side is positive when  $f_{\text{bilateral}}^*$  is sufficiently low, which holds for sufficiently small  $s\frac{\bar{\alpha}+\underline{\alpha}}{2} - c$ . For large values of  $s\frac{\bar{\alpha}+\underline{\alpha}}{2} - c$ ,  $f_{\text{comp codeshare}}^* - \frac{2\theta}{2\theta-1}f_{\text{bilateral}}^*$  approaches a positive constant and  $\pi_{\text{comp codeshare}}^* - \pi_{\text{bilateral}}^*$  approaches  $\frac{s^2}{2m\theta(\theta-2)} [(\theta - 1)(f_{\text{comp codeshare}}^*)^2 - \theta(f_{\text{bilateral}}^*)^2]$ . Since  $(\theta - 1)(\frac{2\theta}{2\theta-1})^2 - \theta = -\frac{\theta}{(2\theta-1)^2} < 0$ ,  $\pi_{\text{comp codeshare}}^* < \pi_{\text{bilateral}}^*$ . This proves the existence of a threshold  $\tilde{h}(m, \beta)$  that determines where the profit ranking reverses.

Finally, to verify (5), since  $f_{\text{alliance}}^* = f_{\text{bilateral codeshare}}^*$ ,

$$\pi_{\text{alliance}}^* - \pi_{\text{bilateral codeshare}}^* = \frac{\theta m(\bar{\alpha} - \underline{\alpha})^2}{2} \left( \frac{\theta}{2\theta} - \frac{\theta - 1}{2\theta - 1} \right) = \frac{\theta m(\bar{\alpha} - \underline{\alpha})^2}{4(2\theta - 1)} \geq 0.$$

Furthermore,  $\bar{\alpha} > \underline{\alpha}$  implies that  $\frac{\theta^2(\theta+y-1)m(\bar{\alpha}-\underline{\alpha})^2}{2(2\theta+y-1)^2}$  increases as  $y$  changes from 0 to 1.  $\square$

**Proof of Proposition 3.4.** (1) holds because the consumer surplus and total social welfare are increasing in  $\beta$  and  $f^*$ , and because  $f^*$  is increasing in  $\beta$  by Proposition 3.1. To prove (2), consider the first order condition for maximizing the total social welfare with respect to the frequency offered, given by

$$H_W \equiv -\frac{s^2\theta f}{2m\theta(\theta - 2)} + s\beta \left[ \ln(1 + (1 + l)f) + \frac{(1 + l)f}{1 + (1 + l)f} \right] + s\frac{\bar{\alpha} + \underline{\alpha}}{2} - c = 0.$$

Since  $H_W$  is strictly concave in  $f$  and equals  $s\frac{\bar{\alpha}+\alpha}{2} - c > 0$  at  $f = 0$ , there exists a unique  $f^{W*}$  satisfying this first order condition, thus maximizing the total social welfare. Fixing the value of  $l$ , since  $f^*$  for  $r = 1$  is less than  $f^*$  for  $r = 0$  by Proposition 3.1, and the difference

$$H_W - H = s\beta l(1-r)\frac{f}{1+f(1+l)} + \frac{s^2 f}{2m\theta(\theta-2)}(\theta-1+r) > 0,$$

decreasing  $r$  from 1 to 0 while fixing  $l$  improves the total social welfare. There is an improvement also when  $l$  increases alone or together with decreasing  $r$  because the total social welfare is increasing in  $l$ . This proves the comparison for the first four agreement types, except when comparing no agreement and bilateral codesharing. In this case we show that higher threshold values than those appearing in part (4) of Proposition 3.1 are relevant. To see this, note that a higher frequency offered under the bilateral codesharing implies higher total social welfare because both  $l$  and  $f^*$  increase. However there is a range of values where  $l$  and  $f^*$  move in opposite directions such that no agreement results in a higher frequency offered but lower total social welfare. For sufficiently high values of  $s\frac{\bar{\alpha}+\alpha}{2} - c$ , as the frequencies offered move further apart, no agreement results also in higher total social welfare, thus proving the higher threshold result.

Finally, (3) is verified because  $\bar{\alpha} > \alpha$  implies that  $\frac{\theta(\theta+y-1)(3\theta+y-1)m(\bar{\alpha}-\alpha)^2}{2(2\theta+y-1)^2}$  increases as  $y$  changes from 0 to 1.  $\square$

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