Cellular Networks With Vehicular Relay Nodes: Capacity, Coverage, and Scalability

Nadav Lavi, *Member, IEEE*, Hanoch Levy, *Member, IEEE*, and Yuval Shavitt, *Senior Member, IEEE*

***Abstract*—A new deployment scheme proposes to leverage the on-board connectivity capabilities of vehicles and turn them (while they are parked) to vehicular relay nodes (VeRNs). The use of on-street parked vehicles as relays within the cellular network is compelling as it can reduce the investments of the operators in deploying infrastructure while increasing the coverage and capacity of the network. Thus, overcoming the exponential growth in users' data demand in a more cost-effective manner compared to traditional fixed infrastructure based deployments. Due to the dynamic nature of vehicles, there are challenges in the analysis and operations of this network. This is due to the fact that current analysis methods assume that the infrastructure is fixed. Thus, in this paper, we develop analytical tools to investigate the gain of utilizing VeRNs. We derive closed-form approximation expressions for two types of analysis, relevant for cellular deployments: a user focused analysis and a VeRN focused analysis. These results can be used for planning and operations of the system. Our extensive simulations show the accuracy and flexibility of our analysis scheme, as well as the high potential of VeRNs as a layer in cellular deployments. Furthermore, we demonstrate and investigate a unique scalability property of this deployment scheme.**

***Index Terms*—Heterogeneous networks, relay nodes, cellular deployments, vehicular relay nodes.**

I. Introduction

VEHICULAR Relay Nodes (VeRNs) were recently proposed as an additional nomadic layer to cellular deployments [1], [2]. VeRNs are on-street parked vehicles with wireless relaying capabilities, which are activated on demand and only when needed. For example, vehicles located (parked) at city centers can be activated to enhance the cellular network during peak demand periods.

Vehicles can act as VeRNs (relays) due to the fact that nowadays they are already equipped with wireless capabilities such as cellular (LTE) UE and WiFi Access-Point (AP) [3]. Thus, VeRNs can relay data to and from users via a 2-hop link which can be based on various options: cellular link to the basestation and WiFi link to the users, or cellular link to the BS and cellular device-to-device (D2D) [4], [5] to the users. Hence, VeRNs are a viable solution for nowadays 4G-LTE [6] and future 5G deployments [7], [8]. The value of utilizing VeRNs to enhance cellular deployment performance is based on the fact that the vehicle (VeRN) has superior antenna and channel conditions (compared to the user), which translates to better modulation and coding schemes which in turn reduces the amount of resources required to transmit data and thus lower the loading conditions on the BSs.

The VeRN deployment concept also incorporates financial and operational advantages. For example, the operator has no need to invest in additional BSs and their deployment. Moreover, vehicles have their own power source (battery) or, in case of electrical vehicles, are connected directly to the smart-grid. In addition, there is no need for real-estates (such as poles, street-lamps, roofs, etc.) as vehicles are utilizing on-street parking. Furthermore, in case of malfunction, vehicles are served periodically in dealerships and there is no need to dispatch engineering crews.

Another benefit of utilizing vehicles as part of the cellular deployment is the fact that vehicles can be activated when needed, for example in situations in which the fixed infrastructure cannot support the demand. Thus, the operator can rely on "thinner" (sparser) deployment, which is enhanced by VeRNs during peak time. This is in contrast to the case of fixed BS deployments which are planned in accordance to peak demand time, and which operators strive to utilize 100% of the time, due to their financial investment in the deployment. Adding vehicles as a layer within the cellular deployment incorporates some complexity in management, as vehicles are nomadic and may change their state. For example, a vehicle whose VeRN capabilities were activated may leave its parking spot. Another case is when a VeRN stops its operation due to battery constraints (as the vehicle needs to maintain its regular operation despite the usage of the VeRN capabilities). However, due to the plurality of vehicles (especially in dense areas in which users and demand exist), when a specific vehicle cannot continue its operation, another vehicle (or vehicles) in the same area can be activated to maintain the service level.

In this paper we investigate the performance gain of a cellular deployment which incorporates VeRNs. We explore a single BS scenario and investigate the performance via two Figures-of-Merit (FOMs): the link gain, and the throughput gain. We analyze VeRN performance in two scenarios to be useful in network design: a user focused analysis, and a VeRN focused analysis.

In the user focused analysis we investigate the expected gain of the VeRN for a user located at a certain location (distance

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N. Lavi and Y. Shavitt are with the School of Electrical Engineering, Tel Aviv University, Tel Aviv 69978, Israel (e-mail: nadavlavi@mail.tau.ac.il; shavitt@eng.tau.ac.il).

H. Levy is with the Blavatnik School of Computer Science, Tel Aviv University, Tel Aviv 61392, Israel (e-mail: hanoch@cs.tau.ac.il).

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from the BS). This type of analysis can assist in understanding the benefit of operating VeRNs for high demand sub-regions, i.e., regions within a BS where large number of co-located users load the BS. This deployment scenario is also termed as *hotspot*. We explore two types of VeRN selection schemes: random selection and best-VeRN selection. The first scheme illustrates the potential of utilizing VeRN without sophisticated control mechanisms. For this scheme we reach a closed form approximation expression for the VeRN gain compared to the direct link (of the user or users in the specific location). For the latter scheme, we provide analytical expressions for the VeRN gain performance upper and lower bounds. We compare the accuracy of the expressions and approximations (of both schemes) to simulation results. We further demonstrate the viability of our approximation by examining a larger parametric space.

In the VeRN focused analysis we investigate the expected gain of utilizing a specific VeRN (at a certain distance from the BS) to serve users in its vicinity. This form of analysis assists in quantifying the effectiveness of VeRNs spread across a deployment which can be coupled with online VeRN activation management mechanisms, i.e., network operation optimization (for example in which locations VeRNs should be considered for activation). We leverage the approximation methodology developed for the user focused analysis, adapt it for the VeRN focused analysis, and derive a closed form approximation expression. We further compare the approximation accuracy with simulation results.

Our main contributions are: (1) the development of a flexible and accurate approximation scheme for two scenarios encompassing both VeRN coverage and capacity; (2) extensive simulations demonstrating the accuracy of the approximation scheme and VeRN performance; (3) analyzing and illustrating the unique scaling property of VeRN based deployments. Partial results of this work were presented in [9].

The rest of the paper is organized as follows, in Section II we provide background on HetNet and relay based deployments as well as more unique deployment schemes. Afterwards in Section III we described our network model. In Sections IV and V we analyze the gain in using VeRN for the user focused analysis and VeRN focused analysis, respectively. In Section VI we describe the scalability of VeRN based cellular networks. Section VII concludes our paper.

II. Related Work

Relaying and Relay Nodes (RNs) were vastly investigated, especially within cellular networks, e.g., 3GPP-LTE [6] which also adopted RNs into its standards, and in relations to Heterogeneous Networks (HetNet). The drive to adopt new deployment schemes and deploy various types of network entities arise from the growth in data demand [10]. A HetNet deployment does not necessarily include all layers (or network entities), rather it means that the deployment is composed of macro BSs and at least one additional type of layer. Thus, multiple types of HetNet deployments may exist, which usually depends on the location, requirements, and constraints, e.g., cost, real-estate, etc. Clearly the VeRN deployment concept is aligned with the HetNet scheme.

[11] provides a survey on HetNet in 3GPP, and details the features included in LTE to support HetNet. [12] analyzes the advantages of HetNets in terms of capacity and coverage. Both papers also describe how RNs perform as part of a HetNet deployment. [13] presents various relay based deployment concepts and investigates their merits. It shows the coverage and capacity gain when using single-hop and multi-hop relaying schemes (when more than 1 relay is used). Other papers that investigated relay based deployments performance, are [14]–[19], which evaluate the impact of incorporating relays on the performance via detailed deployment simulations in various scenarios, from simple single BS to city wide simulations. [20] analyzes the impact of relays in deployments through both link level analysis and simulations. [21] analyzed the gain in k-tier RN deployment. [22] investigated the benefit in utilizing RNs for the uplink direction via simulations. [23] explored the use of multiple RNs within a single BS deployment example. All of them present the value of relays for cellular deployments.

Relay based deployments were also investigated in other aspects beside capacity and coverage enhancements. [24] analyzed the energy savings when deploying relays in a macro deployment. [25]–[27] investigated the impact of relays on the deployment cost.

While our VeRN based deployment concept is similar to HetNet and relay based deployments, it differs from them as our concept does not rely solely on fixed infrastructure. The similarities with HetNet comes from the fundamental concept of relaying the communication between the users and the BSs. Thus, VeRN can act as an additional layer within a HetNet deployment. Yet, due to the dynamic nature of VeRN, network management must be more agile to cope with the changes in the layer. Therefore, VeRN is obviously a non-traditional deployment concept compared to the aforementioned HetNet deployments.

Recent releases of LTE include device-to-device (D2D) communication capabilities [4], [5], i.e., user devices can communicate directly without going through the cellular BS. This form of communication can be conducted using coordination through the BS, i.e, the BS only controls the session, as well as autonomously when the devices are outside of cellular coverage. Clearly, D2D can be leveraged to enable relaying capabilities, as described in [28] for $5G$ cellular networks. Yet, we note that the main focus of this standartization effort is to define the communication requirements enabling LTE devices to communicate directly, and not to enable VeRN link solutions. Yet, it is important to note that VeRN can rely on 3GPP-LTE D2D technology for its operation.

A parallel work stream in LTE standartization includes the support for Vehicle-to-Vehicle (V2V) communication [29]. V2V capabilities rely on the technological foundations of D2D, yet only recent proposal [30] proposes to combine V2V and D2D in a more holistic manner, making a step towards the support of VeRN in 3GPP-LTE.

D2D was investigated as the potential technology to enable a layer of RNs based on user devices [31]. [32] investigated the coverage and capacity of such solution, [33] explored via simulation the use of D2D to support 5G line-of-sight (LoS) network. Clearly this form of deployment is much more dynamic and unreliable compared to VeRNs (which are stationary when in use), and thus resembles more to an ad-hoc type of network.

Vehicles and relays were investigated as part of a solution to support users travelling within the vehicles, i.e., the mobile relay concept. [34] investigated mobility support of mobile RNs, [35] explored the potential of mobile relays in public transportation, and [36] investigated via simulation the performance of vehicle serving its users and sensors.

Papers that investigated other non-traditional deployment concepts include [37], which suggested to use home femtocells to relay data to users and save expensive macro backhaul resources, [38]–[40] explored densification of the deployment, and [41] proposed and investigated a unique deployment scheme in which macro BSs are deployed randomly. We emphasize that even these deployments still rely on static infrastructure.

A unique deployment scheme that does not rely solely on fixed entities was proposed in [42], in which mobile relays, e.g., user devices or vehicle, are used by the deployment to increase coverage. The most challenging problem in this scheme is the probability a destination (user device) will have a relay to connect to. Furthermore, it is unclear what is the amount of time this mobile relay can be used. While our VeRN scheme relies on a mobile platform, the vehicle, it utilizes it only when it is stationary, and by that it ensures continuous service for a loaded target area.

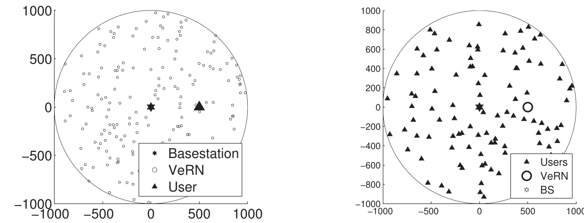
We note that the use of vehicles for networking was previously investigated in many aspects related to V2V communication, such as Delay Tolerant Network (DTN), mesh and ad-hoc network, and gossip based network: [43]–[47] just to name a few. While these types of work indeed leveraged the wireless communication capabilities of the vehicle they were focused only on communication within the mobile vehicular network, i.e., while the vehicles are moving, and the dissemination of data within this network, which is obviously a very different application compared to VeRN. Papers that include support of communication with infrastructure, e.g., [44], do not guarantee delay constraints as the data may traverse through multiple hops. This is obviously unacceptable in cellular networks servicing users data, which emphasizes the uniqueness of our VeRN scheme.

III. Network Model

We focus on a single BS scenario and investigate the gain of VeRNs in the view of our two types of analysis: user focused and VeRN focused. Our model is based on [9], and is extended to support the additional VeRN focused analysis.

A BS covers a circular area of radius R_{BS} and size $\Lambda_{BS} = \pi R_{BS}^2$. In the first type of analysis, i.e., in the user focused analysis, a user is located at distance d_u from the BS, and N_v VeRNs are uniformly distributed at random over the BS coverage area, as illustrated in Fig. 1(a). In the second type of analysis, i.e., in the VeRN focused analysis, a VeRN is located at distance d_v from the BS, and N_u users are uniformly distributed at random over the BS coverage area, as illustrated in Fig. 1(b).

We note that in our network settings there are three types of cellular links between the three network entities (BS, VeRN, and user), as illustrated in Fig. 2. The link between the user and the BS (which we term the direct link), and the VeRN link (between the user and the BS via a VeRN) which is composed of two links: the link between the VeRN and the BS (termed backhaul link), and the link between the VeRN and the user (termed access link). We emphasize that a user is connected to either the BS or the VeRN, i.e., it cannot be connected to both BS and VeRN simultaneously.



(a) User focused ($d_u = 500$m). (b) VeRN focused ($d_v = 500$m).

Fig. 1. Deployment model for the two types of analysis.

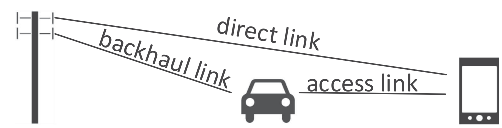


Fig. 2. The model's three wireless links.

We are interested in investigating the gain of the VeRN link versus the user direct link. For this purpose we define two Figures of Merit (FOMs): a link FOM and a throughput (capacity) FOM.

A wireless link FOM, i.e., the link signal to noise ratio (SNR), is defined by the following expression [48] (which is based on [49]):

$$S_i(l_i) = k_i \frac{P_i^{TX} g_i}{\sigma^2 l_i^{\alpha_i}} \tag{1}$$

where, $i \in \{d, a, bh\}$ (d - direct, a - access, bh - backhaul), P_i^{TX} is the transmission power, g_i is the overall antennas gain of both transmitter and receiver ($g = G_{TX} \times G_{RX}$), l_i is the distance between the transmitter and receiver, α_i is the path-loss exponent, σ^2 is the receiver noise level, and k_i is a unitless constant that depends on the antenna characteristics and average channel attenuation.

We express the links characteristics in a second FOM (the throughput FOM), which is based on the link SNR (1), through Shannon's channel capacity limit as follows (extension to Multiple Input Multiple Output (MIMO) is based on [50]):

$$T_i(l_i) = B \min\{Ant_i^{TX}, Ant_i^{RX}\} \log_2(1 + S_i(l_i)) \tag{2}$$

where, B is the bandwidth used by the system, and Ant_i^{TX} and Ant_i^{RX} are the number of transmit and receive antennas, respectively.

As the VeRN link is composed of the combination of the access and backhaul links, the overall two-hops VeRN link FOM is define by the bottleneck between them:

$$S_{VeRN} = \min\{S_{bh}, S_a\} \tag{3}$$

TABLE I
PATH LOSS EXPONENTS PER WINNER II CHANNEL MODEL

Channel	WINNER II Model	α
Direct Link	$C2$	3.71
Backhaul Link	$B5d$	3.68
Access Link	$B1$	3.39

We note that similarly to (3), we can also define VeRN overall throughput FOM as:

$$T_{VeRN} = \min\{T_{bh}, T_a\} \tag{4}$$

In the next sections we will investigate the average gain of utilizing VeRN in several deployment scenarios based on the two FOMs. Meaning, we are interested to calculate the average of S_{VeRN}/S_d and of T_{VeRN}/T_d.

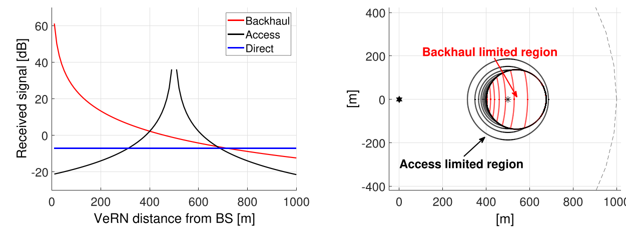
For our link model, we use the well known WINNER II channel models [51], which is widely adopted in the cellular community. For simplicity we define $k_i = 0.001, \forall i$, similarly to [48], and adjust the link parameters, i.e., α_i, such that it follows the appropriate WINNER II links. Table I details the channel model selected for the direct, access, and backhaul links (we note that this approach is similar to [1], [2], [9]), as well as the values used for path-loss exponents.

While in our main analysis we limit ourselves to WINNER II models, we emphasize that our analysis technique, detailed below, can be applied to scenarios with different path-loss exponents. In Section IV-D we will analyze the parametric space in which our technique can be employed.

IV. USER FOCUSED ANALYSIS

As indicated above, our analysis is based on [9], and investigates the gain in utilizing VeRN in the view of a user (which is also applicable to the case of a set of co-located users, as they experience the same channel conditions). This type of analysis is beneficial to understand the potential of leveraging VeRNs to handle difficult scenarios such as crowded locations (squares, events, etc,) also known as deployment hotspots. In these types of scenarios the operator is forced to design the network to support the peak demand. Moreover, in certain cases, such as during events, the operator might decide to send engineering teams to deploy temporary infrastructure. Thus, the use of vehicles is very compelling in such cases, as usually the users are also coming with their cars. We further illustrate the scaling property in utilizing VeRN within cellular deployments in Section VI.

The analysis is focused on the downlink direction as this direction suffers more from congestion. Yet, we emphasize that our analysis can be easily extended to investigate the uplink direction as well. We examine two VeRN selection schemes: random VeRN selection (similarly to [9]), and best VeRN selection. For both schemes we will introduce closed form expressions of the VeRN gain compared to the direct link. Afterwards, in Section V, we address the VeRN focused analysis. Recall that in the user focused analysis we evaluate the gain of the VeRN for a user located at distance d_u from the BS, and N_v VeRNs that are uniformly distributed at random across the BS coverage area, as illustrated in Fig. 1(a) (in which $N_v = 200$, $d_u = 500$ m, and $R_{BS} = 1000$ m).



(a) Links comparison. (b) Coverage regions.

Fig. 3. Coverage regions bounding factor ($d_u = 500$ m).

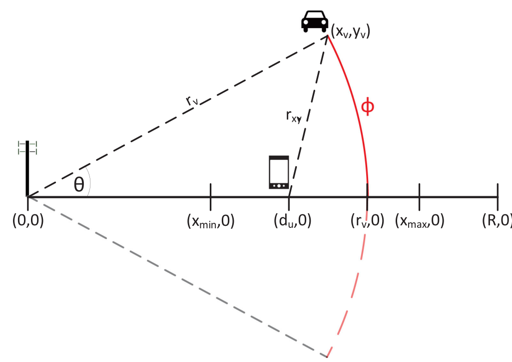


Fig. 4. User focused analysis - backhaul limited coverage region.

We note that in this analysis we are investigating a single user link and comparing its direct link to its VeRN link, as in the case of co-located users they will experience similar link performance. Hence, the results will apply to all users. We first investigate the area in which the VeRN link is superior to the direct link, i.e., the VeRN effective coverage area, and show how to calculate the area size. We then analyze, based on the area's characteristics and other factors, the performance gain of VeRN for two selection schemes.

A. VeRN Effective Coverage

Similarly to [9], in our investigation we focus on the backhaul limited region, i.e., the region in which $S_{VeRN} \geq S_d$ and $S_{bh} \leq S_a$. This is based on the analysis conducted in [1], [2] that showed that in most scenarios (under the WINNER-II channel model) the backhaul link is the bottleneck of the VeRN link, as also illustrated in Fig. 3(b). We also note that the gain in the access limited region is much smaller compared to the backhaul limited region, as can be seen in Fig. 3(a).

By investigating only the backhaul limited region we simplify the analysis (as we only investigate a single region), and achieve an upper bound on the overall VeRN performance, as adding the access limited region will decrease the average VeRN gain (this will be demonstrated in Section IV-B1).

To evaluate the VeRN effective coverage region we note an important characteristic of the region. As the region is defined by the bottleneck, i.e., in our analysis the backhaul link, the region is therefore composed of semicircles (arcs) defined by the radius (distances of the VeRN from the BS), r_v, and a central angle 2θ, as illustrated in Fig. 4 (in the figure Φ is half of the semicircle). The distance r_v is such that $r_v \in [x_{\min}, x_{\max}]$, where $x_{\min}$ and $x_{\max}$ are the points in which the backhaul FOM is equal to the access FOM[1]. Thus, the borders of the effective area are extracted using:

$$S_{bh}(x_{\max}) = S_a(x_{\max} - d_u)$$
$$S_{bh}(x_{\min}) = S_a(d_u - x_{\min})$$

Which then results in:

$$Mx_{\min}^{\alpha_{bh}/\alpha_a} + x_{\min} - d_u = 0 \tag{5}$$

$$Mx_{\max}^{\alpha_{bh}/\alpha_a} - x_{\max} + d_u = 0 \tag{6}$$

where, $M = \sqrt[\alpha_a]{\frac{P_a^{TX} g_a}{P_{bh}^{TX} g_{bh}}}$.

We now calculate θ. We note that θ is a function of r_v, i.e., $\theta(r_v)$, and in this paper we use both notations interchangeably. Note that $\sin(\theta) = y_v/r_v$. To calculate x_v and y_v (which indicate the location in which $S_a = S_{bh}$ over the arc), we note that:

$$r_v^2 = x_v^2 + y_v^2$$
$$S_{bh}(r_v) = S_a(r_{xy})$$
$$r_{xy}^2 = (x_v - d_u)^2 + y_v^2$$

Thus, we get:

$$x_v = \frac{1}{2d_u}\left(d_u^2 + r_v^2 - M^2 r_v^{2\alpha_{bh}/\alpha_1}\right) \tag{7}$$

$$y_v = \frac{1}{2d_u}\sqrt{2d_u^2\left(1+M^2\right) r_v^2 - d_u^4 - \left(1-M^2\right)^2 r_v^4} \tag{8}$$

Combining (7) and (8):

$$\sin(\theta) = \frac{\sqrt{4d_u^2 r_v^2 - (d_u^2 + r_v^2 - M^2 r_v^{2\alpha_{bh}/\alpha_a})^2}}{2d_u r_v} \tag{9}$$

We note that in order to extract a closed-form formulation we need to conduct few approximations. To approximate $x_{\min}$ and $x_{\max}$ we use $\alpha_{bh}/\alpha_a \sim 1$ (note that $\alpha_{bh}/\alpha_a = 1.08$ according to Table I). Thus:

$$x_{\min}(d_u) \simeq d_u \frac{1}{1+M} \tag{10}$$

$$x_{\max}(d_u) \simeq d_u \frac{1}{1-M} \tag{11}$$

Next we approximate θ, we use the approximation on the path-loss exponents as above ($\alpha_{bh}/\alpha_a \sim 1$), and apply the sine approximation ($\theta \sim \sin(\theta)$). Hence, we are able to formulate an expression from (9).

To calculate the size of the effective area, i.e., the backhaul limited region, denoted as Λ_v, we note that instead of integrating $2\theta r_v$ over $[x_{\min}, x_{\max}]$ we can leverage the fact that the area is circular with diameter $x_{\max} - x_{\min}$. Thus, $\Lambda_v(d) = \pi(x_{\max} - x_{\min})^2/4$. The proof for this appears in Appendix A.

B. Random VeRN Selection

We now analyze the gain of randomly selecting a VeRN out of the available VeRNs within the effective coverage region. We will derive an approximation which will allow closed-form evaluation of the system performance. We note that the random selection scheme is a simple method to activate/select VeRNs, which are randomly available (both in time and space).

We first calculate the probability, $P_v(d_u)$, that at least one VeRN is located in the effective area (Λ_v), when using uniform distribution of N_v VeRNs:

$$P_v(d_u) = 1 - \left(1 - \frac{\Lambda_v(d_u)}{\Lambda_{BS}}\right)^{N_v} \tag{12}$$

We are interested in calculating the average gain achieved by the random selection scheme. With probability $1 - P_v(d_u)$ the gain is equal to 1 (no VeRN is available). Otherwise, the probability of a VeRN residing across an arc 2ϕ needs to be considered, with the gain value $S_{bh}(r_v)/S_d(d_u)$.

For the probability calculation over an arc, we note that given a circular area of size Λ and radius R, the joint PDF of random variables r and ψ, representing the location of a point in Λ is:

$$f_{R,\Psi}(r,\psi) = \begin{cases} \frac{r}{\Lambda} & 0 \le \psi \le 2\pi, 0 \le r \le R \\ 0 & \text{otherwise} \end{cases} \tag{13}$$

With the abuse of notation, from this point on we denote the PDF of a point to be located within Λ_v as $f_{R,\Psi}(r,\psi)$. We now proceed to the random selection scheme investigation.

We now formulate the average link gain of a VeRN within the effective region (BH limited) for a user located at distance d_u from the BS:

$$\overline{G}_{rand}^u(d_u) = (1 - P_v(d_u)) + P_v(d_u)\int_{r_v=x_{\min}}^{x_{\max}}\int_{\theta=-\theta(r_v)}^{\theta(r_v)} f_{R,\Phi}(r_v,\theta)\frac{S_{bh}(r_v)}{S_d(d_u)}\partial\theta\partial r_v \tag{14}$$

Based on (13) the probability that there is at least one VeRN over an arc defined by distance r_v and central angle $2\theta(r_v)$ is $2r_v\theta(r_v)/\Lambda_v(d)$. We further substitute (1), and (12) in (14), and get:

$$\overline{G}_{rand}^u(d_u) = (1 - P_v(d_u)) + P_v(d_u)\frac{2P_{bh}^{TX} g_{bh} d_u^{\alpha_d}}{\Lambda_v(d_u) P_d^{TX} g_d}\int_{r_v=x_{\min}}^{x_{\max}} \frac{\theta}{r_v^{\alpha_{bh}-1}}\partial r_v \tag{15}$$

Due to the fact that α_{bh} is not an integer (see Table I for the path-loss coefficients), (15) can only be solved numerically. In order to reach a closed form analytical expression we develop an approximation on $\overline{G}_{rand}^u(d_u)$ by substituting $\alpha_{bh} = 3$. However, due to this change, we need to adjust α_d in a similar manner (otherwise we will not derive appropriate gain values). To extract the new path-loss exponent for the direct link we require to maintain the following relation:

$$\frac{d_u^{\alpha_d}}{x_{\max}^{\alpha_{bh}}} = \frac{d_u^{\beta}}{x_{\max}^3}.$$

[1] We note that $x_{\min}$ and $x_{\max}$ are functions of d_u, i.e., $x_{\min}(d_u)$ and $x_{\max}(d_u)$. Yet, for ease of reading we simply write $x_{\min}$ and $x_{\max}$.

Meaning, we wish to maintain the same ratio at the $x_{\max}$ location.[2] Thus, we replace α_d by $\beta = \alpha_d - (\alpha_{bh} - 3)/\lg_{x_{\max}}(d_u)$.

Using $\alpha_{bh} = 3$ and β we now continue and investigate (15). We substitute $u = r_v^2$, and use (9) for the approximation of θ. We can now leverage results from [52] (Equations 2.267, 2.266, 2.261) to devise an approximation (16) to $\overline{G}^u_{rand}(d_u)$ (shown at the bottom of this page). In which, M is as defined above, and $K = \frac{g_{bh} P_{BS}}{g_d P_{BS}}$.

By substituting r_v^2 with u (and thus adapt the integral boundaries to $x^2_{\max}$ and $x^2_{\min}$), and further substituting Λ_v, (16) collapses to the following simple expression:

$$\overline{G}^u_{rand}(d_u) \cong (1 - P_v(d_u)) + (1 - M^2)^2 K P_v(d_u) d_u^{\beta-3} \tag{17}$$

This is an important result that significantly simplifies the investigation of VeRN link and VeRN networks. We emphasize that our analysis technique as well as the closed form result can be leveraged in other system settings, as will be demonstrated in Section IV-D.

Calculating the average throughput gain directly (in a similar manner as the average link gain) is very difficult. To overcome that, we leverage (17) and approximate the average throughput link by substituting the average link SNR, $\overline{G}^u_{rand}(d_u) S_d(d_u)$, to replace S_i in (2) (applying the backhaul antenna parameters). This is a common approximation of link throughput.[3] Finally dividing this by $T_d(d_u)$ we get an approximation of the average throughput gain.

1) Approximation Evaluation via Simulations: We evaluate the accuracy of the VeRN gain derived in (17) by comparing it to simulation results. For this purpose we consider a single user at 7 different distances from the BS: $d_u = 100, 200, \ldots, 700$ m. For each distance, we calculate (17) and compare it to the average gain and throughput of 10,000 simulated networks.

A simulated network consists of 30,000 VeRNs that are uniformly distributed at random over a BS coverage region (with 1000 meters radius). For each simulated network and user distance (d_u), we randomly select a VeRN from the set of VeRNs within the user effective coverage area (as detailed above we are interested in the backhaul limited region), and calculate the average link gain and throughput gain based on the FOMs (1) and (2), respectively. Fig. 5 depicts the simulated network setup, where the red vehicle in this example was randomly selected out of the vehicles within the backhaul limited coverage region. We note that if there is no VeRN then we consider the simulation gain as 1, i.e., use of the direct link. We further compare our closed form approximation (17) to the average VeRN gain when considering the entire effective area (both backhaul and access limited regions).

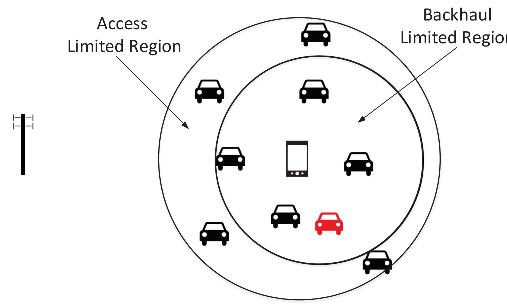
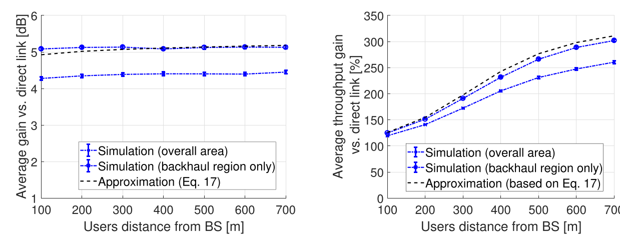


Fig. 5. Simulated network for the random VeRN selection.



(a) Link gain. (b) Throughput gain.

Fig. 6. User focused analysis - random VeRN selection gain.

In our simulations we apply common system parameters: noise level (σ^2) -101 dB, transmission power (P_i^{TX}) of 40 dBm and 25 dBm for the BS and VeRN, respectively, antenna gain (G) of 12, 5, 0 dBi for the BS, VeRN and user, respectively.

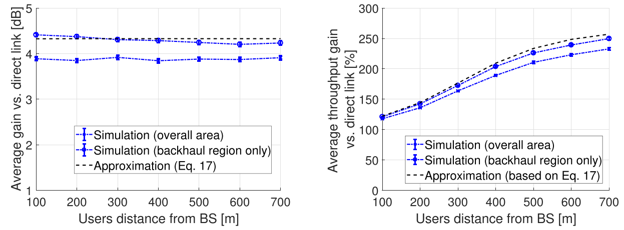
Fig. 6, depicts the gain in random VeRN selection compared to the direct link for the various user distances. Fig. 6(a) depicts the average link gain (in dB), and Fig. 6(b) depicts the average capacity gain, which was calculated using (2).

The results of both evaluations (Figs. 6(a) and 6(b)) demonstrate that our analysis scheme, in particular expression (17), provides a very good approximation of the average VeRN gain for the backhaul limited region. This is an important result emphasizing the accuracy of our approximation scheme. Furthermore, it can be seen that our approximation is an upper bound (as indicated above) on the average gain when considering the entire effective region. The reason for this is that the access limited region contributes VeRNs with lower link performance

[2] We note that this is a conservative location, as using $x_{\min}$ will result in higher gain values.

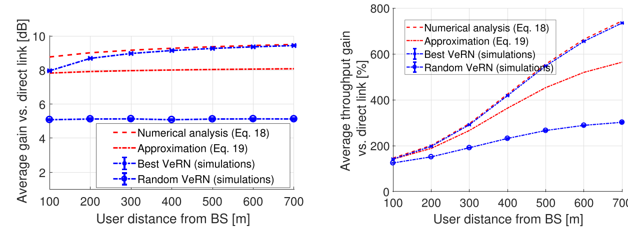
[3] This is true due to Jensen inequality ($\mathbb{E}[\log_2(1+X)] \le \log_2(1+\mathbb{E}[X])$). We note that this is a common practice in approximating channel capacity [53]–[55]. We further note that in the case where $\mathbb{E}[X]^2 > \mathrm{Var}[X]$, which is the case for the link gain (as the backhaul link does not vary significantly within the region of interest), the approximation is very accurate (can be seen via second-order Taylor expansion about $\mathbb{E}[X]$, e.g., [56]).

$$\overline{G}^u_{rand}(d_u) \cong (1 - P_v(d_u)) + K\frac{P_v(d_u) d_u^{\beta-1}}{2\Lambda_v(d_u)} \left[\frac{-\sqrt{-d_u^4 + 2d_u^2(1+M^2)u - (1-M^2)^2 u^2}}{u} + (1+M^2)\arcsin\left(\frac{(1+M^2)u - d_u^2}{2uM}\right) + (1-M^2)\arcsin\left(\frac{(1+M^2)d_u^2 - (1-M^2)^2 u}{2M d_u^2}\right)\right]_{u=x^2_{\min}}^{x^2_{\max}} \tag{16}$$



(a) Link gain. (b) Throughput gain.

Fig. 7. User focused analysis - random VeRN selection gain ($\alpha_{bh} = \alpha_d$).



(a) Link gain. (b) Throughput gain.

Fig. 8. User focused analysis - random vs. best VeRN selection schemes.

(compared to the backhaul limited region), and thus decreases the average gain performance. Another observation is that our approximation provides a tight upper bound for users located close to the BS (up to 400 m), see Fig. 6(b). The difference between the approximation and the simulation of the entire area increases with the distance, as the effective region increases and hence the average gain decreases.

It is clear that our analysis scheme provides a good approximation for the defined model, which is based on the WINNER II channel model. We now investigate another system setting in which $\alpha_{bh} = \alpha_d = 3.71$, the rest of the simulation parameters are the same as above. This setting is very similar to the WINNER II model, and will provide us with additional insight on the performance of our approximation. Similarly to the comparison above, Fig. 7(a) depicts the average link gain and Fig. 7(b) depicts the average capacity gain. The comparison shows that also in this configuration our closed form approximation provides accurate gain results. This shows the potential of our scheme in different system settings, which will be further investigated in Section IV-D.

C. Best VeRN Selection

The random-VeRN selection scheme, detailed above, provides insight on the potential of utilizing a VeRN link. We emphasize that the random selection scheme does not necessarily select the best VeRN out of the potential VeRNs. We note that in our settings the best VeRN is the VeRN that is closest to the BS (out of the VeRNs within the backhaul limited region). More precisely, the most optimal location is where $S_{bh} = S_a$.

While random selection scheme is simple to implement and does not require sophisticated mechanisms, we are interested in evaluating the benefit of a more complex selection scheme. For this propose we use the best VeRN selection scheme as a potential upper bound on performance.

We note that the solution of (5), denoted as $x_{\min}^{sol}$, is the closest location to the BS within the backhaul limited region. Thus:

$$\overline{G}_{best}^{u}(d_u) \leq \frac{g_{bh} d_u^{\alpha_d}}{g_d (x_{\min}^{sol})^{\alpha_{bh}}} \tag{18}$$

We note that we can solve (5) only numerically, and that (18) is an upper bound on the average gain of the best VeRN selection scheme (as the average gain depends on vehicle availability, and hence on vehicle density). Another approach to calculate the best VeRN gain is via the results of the effective area approximation. Recall, that in Section IV-A we approximated $x_{\min}^{sol}$ using (10) (denoted as in Section IV-A $x_{\min}$). Hence:

$$\overline{G}_{best}^{u}(d_u) \geq \frac{g_{bh} d_u^{\alpha_d}}{g_d (x_{\min})^{\alpha_{bh}}} \tag{19}$$

Clearly, due to the method of approximation ($\alpha_{bh}/\alpha_a \sim 1$), $x_{\min} > x_{\min}^{sol}$ and thus the gain based on (19) is lower than the gain based on (18).

1) Evaluation via Simulations: We evaluate the accuracy of (18) (which is a closed form expression which requires numerical calculations) and (19) (which is a closed form analytical expression) via simulations. We follow the same methodology as in Section IV-B1. For each simulated network and user distance, d_u, we select the best VeRN (out of the VeRNs within the effective coverage area) and calculate the average gain via the two FOMs. We further examine the potential benefit of best VeRN selection by comparing the results to the simulation results of the random VeRN selection (as descried in Section IV-B1).

Fig. 8 depicts the average link (Fig. 8(a)) and the average capacity (Fig. 8(b)) gains of best VeRN selection and compares them to the random selection scheme. Clearly the best VeRN selection scheme achieves higher gain performance. The difference between the selection schemes increases as the distance of the user from the BS increases. The reason is that as the user is farther away from the BS the VeRN effective area (and the backhaul limited region) increases, which in turn means that the best VeRN is becoming much closer to the BS.

We further note that the upper bound of (18) practically coincides with the simulation starting at $d_u = 300$ m. This is due to the increase in the probability of a vehicle to exist at $x_{\min}$ (or its vicinity). As indicated above (19) is a lower bound to the simulation results.

We conclude that the difference between the schemes emphasizes the potential in implementing a more sophisticated VeRN activation mechanism, which takes into account various parameters, such as the VeRN location relative to the user and the BS.

D. Expanding the Parametric Space

So far in our analysis we based our system settings on the WINNER II cellular link model. Yet, our analysis and approximation scheme can span across many other channel configurations. We now evaluate the configuration space in which one can employ our analysis scheme, and evaluate its accuracy. We note that we still base our system on the link FOM (1), and examine the relations between the path-loss exponents α_d, α_{bh}, and α_a.

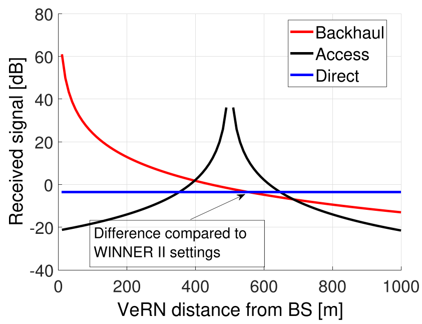


Fig. 9. Another manner of links relation (non WINNER II parameters).

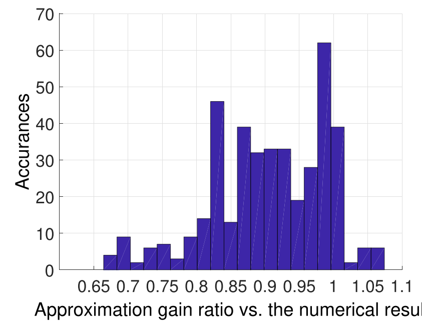


Fig. 10. Histogram comparing the approximation to numerical analysis.

First, we note that in our analysis we are focused on the backhaul limited region. Hence, we examine the conditions in which the links behave similarly to the WINNER II settings, as illustrated in Fig. 3(a). Meaning, the region boundaries are defined by the intersections between the backhaul and access links. Yet, once exploring a new configuration space we indicate that different behavior of the links may potentially be observed, as illustrated in Fig. 9, i.e., $x_{\max}$ may lie on the intersection between the backhaul and direct links.

Recall that the boundaries of the backhaul limited region are defined by (10) and (11) (when we use the approximation $\alpha_{bh}/\alpha_a \sim 1$). Hence, to be able to leverage our technique, we are interested in ensuring that:

$$S_{bh}(x_{\min}) > S_d(d_u) \tag{20}$$

This will imply that the VeRN link is superior to the direct link and a backhaul limited region exists. This, in turn, means that our technique can be employed.

From (20) we can now derive the conditions on the relations between the path-loss exponents of the direct and backhaul links:

$$\alpha_d > \alpha_{bh} - \log_{d_u}\left(\frac{g_{bh}(1+M)^{\alpha_{bh}}}{g_d}\right) \tag{21}$$

This means that given a set of path-loss exponents, one can deduce if our approximation scheme can be applied using (21).

We next evaluate the performance of our scheme in various system settings. We use similar system settings as detailed above in the WINNER II based analysis: R_{BS}, d_u, P_i, g_i. Yet, now we investigate a set of path-loss exponents, and compare our gain approximation to the numerical calculation based on (15) using the region boundaries ((5) for $x_{\min}$, and for $x_{\max}$ depending on the setup). We note that our approximation is based on two elements: (1) the use of $\alpha_{bh}/\alpha_a \sim 1$; and (2) the setting of α_{bh} to 3 to conduct the integration. Thus, to examine our scheme's accuracy, we select α_{bh} from the set $\{3.1, 3.3, 3.5, 3.7, 3.9\}$, and set α_a such that $\alpha_{bh}/\alpha_a \in \{0.9, 0.95, 1, 1.05, 1.1\}$. Then we calculate the minimal α_d ($\alpha_d^{\min}$) based on (21), and select α_d from the set $\{\alpha_d^{\min} + (0, 0.1, \ldots, 0.5)\}$, which result in more than 400 system settings.

Fig. 10 depicts a histogram of the ratio between our approximation to the gain calculated numerically for the investigated settings. We can see that our approximation technique is mostly accurate, where most of the scenarios are located in the area between 85% and 105%. More specifically, 90% of the scenarios reach up to 20% error, and about 50% of the scenarios up to 10% error. We further can see that in most cases the approximation predicts lower gains than the exact analysis. We conclude that our scheme can provide good approximation to most of the relevant system settings, yet our scheme is most accurate in settings that behave similarly to the WINNER II model.

V. VeRN Focused Analysis

In this section, we leverage the technique developed in the previous section and demonstrate how it can be utilized in a different system settings. This transformation of our approximation scheme to tackle a different type of analysis emphasizes the technique flexibility.

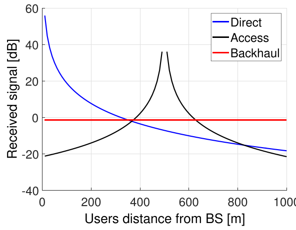
In this setting, we explore the expected gain provided by a VeRN for the benefit of multiple users in its vicinity. This analysis is streamlined with the operational view of the cellular operator, in which the operator evaluates the gain of activating a VeRN for the benefit of users around the VeRN. Thus, the outcomes of this investigation can assist the cellular operator in both deployment planning and operation. For example, the VeRN real-time activation algorithm can leverage these analysis results in the decision of which VeRN to activate. Furthermore, based on known parking locations the operator can plan the hybrid (BSs and VeRNs) deployment coverage and capacity.

We note that the approximation technique relies on the same steps as of the user focused analysis, yet they require some modifications. Hence, we first investigate the VeRN effective coverage, i.e., the region in which the VeRN (under investigation) link is superior to the direct link of users around it, and afterwards calculate the overall gain of the VeRN compared to those direct links.

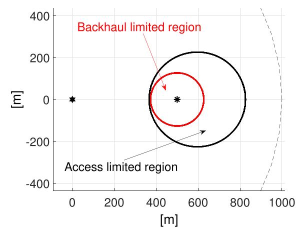
Recall, that in this type of analysis a VeRN is located at distance d_v from the BS, and N_u users are uniformly distributed at random within the BS coverage area Λ_{BS}. The FOMs are as defined in Section III.

A. VeRN Effective Area

Similarly to Section IV-A, we investigate the boundaries of the VeRN effective area, i.e., the area around the VeRN in which users can increase their performance by connecting to the VeRN. Fig. 11(a) illustrates the relations between the three different links in the new settings for $d_v = 500$ m, and Fig. 11(b) shows the coverage regions for the same scenario. We note that the



(a) Links comparison.



(b) Coverage regions.

Fig. 11. Coverage regions bounding factor ($d_v = 500$ m).

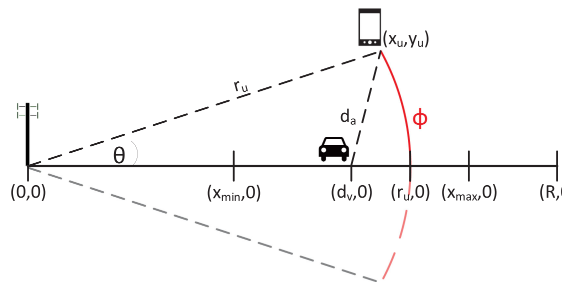


Fig. 12. VeRN focused analysis - backhaul limited coverage region.

difference in the links and the formation of the backhaul and access limited regions compared to the user focused analysis, is due to the fact that now the VeRN is stationary and hence its link, i.e., the backhaul link, is fixed (as depicted in Fig. 11(a)).

Also in this type of analysis we focus on the backhaul limited region. This region is bounded by the intersection of the backhaul link with the access link. Hence, we are interested in x_{min} such that:

$$S_{bh}(d_v) = S_a(d_v - x_{\text{min}})$$

And x_{max} such that:

$$S_{bh}(d_v) = S_a(x_{\text{max}} - d_v)$$

Which results in:

$$x_{\text{min}}(d_v) = d_v - M d_v^{\alpha_{bh}/\alpha_a} \tag{22}$$

$$x_{\text{max}}(d_v) = d_v + M d_v^{\alpha_{bh}/\alpha_a} \tag{23}$$

where, M is as defined in Section IV-A.

It is clear that due to the fact that in these settings S_{bh} is fixed, and S_a is symmetrical around d_v then the area of the backhaul limited region is circular with radius $d_a = (x_{\text{max}} - x_{\text{min}})/2$, and its area equals to: $\Omega_v(d_v) = \pi d_a^2$.

B. VeRN Gain Analysis

We now analyze the gain in utilizing VeRN to connect users within the VeRN effective coverage area (recall we focus on the backhaul limited region). These users are distributed uniformly at random within the area of interest (and across the entire BS coverage area). Thus, and similarly to the calculation in (12), we can calculate the probability that at least one user (out of the N_u users distributed across the BS) reside within the area of interest:

$$P_u(d_v) = 1 - \left(1 - \frac{\Omega_v(d_v)}{\Lambda_{BS}}\right)^{N_u} \tag{24}$$

We note that the backhaul limited region is composed of semicircles (arcs), which is similar to the case above. Yet, while in the user focused analysis the semicircles were related to the backhaul link, in this case they relate to the direct link of the users to the BS, and defined by the angle 2θ and radius r_u ($r_u \in [x_{\text{min}}, x_{\text{max}}]$) as illustrated in Fig. 12. We note that in the figure Φ is half of the semicircle.

We now calculate θ in a similar manner as done above in the user focused analysis. We note that $\sin(\theta) = y_u/r_u$, and $r_u^2 = x_u^2 + y_u^2$. In addition, we note that $d_a^2 = (d_v - x_u)^2 + y_u^2$. Thus, we get:

$$y_u = \frac{\sqrt{-r_u^4 + (2d_v^2 + 2d_a^2)r_u^2 - (d_a^2 - d_v^2)^2}}{2d_v} \tag{25}$$

Using the sine approximation and (25), we get that $\theta \sim y_u/r_u$.

Calculating the average gain of VeRN compared to the direct link of randomly distributed users over the backhaul limited area is similar to (14), as we can also utilize the PDF of a point within a circular area (13):

$$\overline{G}^v_{rand}(d_v) = (1 - P_u(d_v)) + P_u(d_v) \int_{r_u = x_{\text{min}}}^{x_{\text{max}}} \int_{\theta = -\theta(r_u)}^{\theta(r_u)} \frac{\theta r_u}{\Omega_v} \frac{S_{bh}(d_v)}{S_d(r_u)} \partial\theta \partial r_u \tag{26}$$

Substituting θ, (25), (22), (23), and the relevant links FOM into (26), we get:

$$\overline{G}^v_{rand}(d_v) = (1 - P_u(d_v)) + \frac{P_u(d_v) g_{bh}}{g_d \Omega_v d_v^{\alpha_{bh}+1}} \cdot \int_{r_u = x_{\text{min}}}^{x_{\text{max}}} r_u^{\alpha_d} \sqrt{-r_u^4 + 2(d_v^2 + d_a^2)r_u^2 - (d_a^2 - d_v^2)^2} \partial r_u \tag{27}$$

We now take three additional steps which are similar to the previous user focused analysis. First, we define $x = r_u^2$, which results in:

$$\overline{G}^v_{rand}(d_v) = (1 - P_u(d_v)) + \frac{P_u(d_v) g_{bh}}{2 g_d \Omega_v d_v^{\alpha_{bh}+1}} \cdot \int_{x = x_{\text{min}}^2}^{x_{\text{max}}^2} x^{(\alpha_d - 1)/2} \sqrt{-x^2 + 2(d_v^2 + d_a^2)x - (d_a^2 - d_v^2)^2} \partial x \tag{28}$$

Next, to reach a closed form solution for the integral, we need to approximate α_d using an integer number. Thus, we set $\alpha_d = 3$. Recall that as we deal with the ratio between the direct and backhaul links we need to adjust the backhaul link path-loss exponent as well. We note that in this type of analysis the backhaul limited region is symmetrical around d_v. Thus, we wish to find a new path-loss exponent, β, for which we require that:

$$\frac{d_v^{\alpha_d}}{d_v^{\alpha_{bh}}} = \frac{d_v^{\beta}}{d_v^3} \tag{29}$$

From (29) we derive that:

$$\beta = \alpha_{bh} - \alpha_d + 3 \tag{30}$$

With $\alpha_d = 3$ and β, we can now use [52] (Equations 2.262 and 2.261), simplify (28), and derive (31) which is a closed-form approximation (shown at the bottom of this page). The ability to reach a closed form also in this case emphasizes the flexibility of our scheme. Furthermore, due to the fact that the VeRN is static, we note that this analysis technique can be employed in various RN based deployment investigations.

We note that while (31) is a closed-form solution, it can be simplified even further. Recall that in Section IV we used the approximation $\alpha_{bh}/\alpha_a \sim 1$. By applying this approximation also here, we can derive from (22) and (23):

$$x_{\max} \cong (1+M)d_v \tag{32}$$

$$x_{\min} \cong (1-M)d_v \tag{33}$$

Recall that $d_a = (x_{\max} - x_{\min})/2$, thus $d_a = Md_v$.

We can now substitute d_a, (32) and (33) into (31), which collapses to:

$$\overline{G}^{v}_{rand}(d_v) \cong (1 - P_u(d_v)) + \frac{g_{bh}}{g_d} P_u(d_v) d_v^{3-\beta}(1+M^2) \tag{34}$$

(34) simplifies the closed-form approximation for the VeRN gain even further, and enables a simple tool for the purpose of the deployment analysis. In the next section we compare the two closed-form approximations to simulation results. We further note that the average throughput gain can be calculated using (34) in a similar manner to the one explained in Section IV-B.

C. Approximation Evaluation via Simulations

We now evaluate the VeRN link gain closed form approximation when serving users within its backhaul limited coverage region. We calculate the closed form approximations (31) and (34) for a set of distances of a VeRN from the BS: $d_v = 100, 200, \ldots, 700$ m, and compare them to simulations results.

The simulations are conducted similarly to Section IV-B1. For each distance d_v we simulate 10 networks. In each network we distribute 30,000 users in a uniform manner at random over the BS coverage. We used the same system parameters as in Section IV-B1. For each simulation we calculate the average gain of users within the backhaul limited region, and use it to derive the overall average gain.

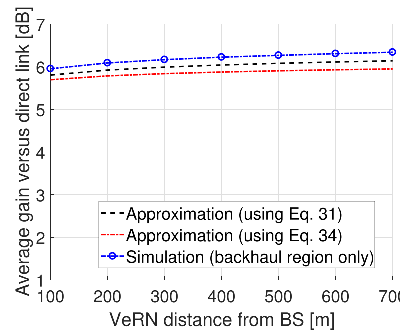


Fig. 13. VeRN focused analysis - link gain.

Fig. 13 depicts the average link gain of the VeRN for both the simulation and the two approximations. It can be clearly seen that our closed form approximations closely follow the simulation results. We note that the approximation using (31) achieved slightly better results, which is reasonable as it does not include the further approximation step based on $\alpha_{bh}/\alpha_a \sim 1$. However, (34) proposes an approximation solution with balanced tradeoff between accuracy and complexity (as it has much simpler representation). These results emphasize the flexibility of our analysis scheme to adapt to other relay/VeRN related scenarios and different system settings, and the ability to reach closed-form approximations that closely follow the actual system performance.

VI. VeRN: A Scalable Solution

Traditional cellular deployments are planned to support peak demand (or high percentage of that peak, e.g., 95th percentile), which in turn requires the cellular operator to deploy sufficient number of BSs. However, when utilizing vehicles as part of the cellular network the operator can reduce the number of deployed BSs due to the presence of vehicles. An economically important and advantageous property of the system is that not only vehicles are co-located in areas and times in which users (and hence also cellular demand) exist, but further, the numbers of vehicles and users can possibly be correlated. Intuitively speaking, areas which at certain time experience many users and therefore are subject to high demand, are expected to benefit from the presence of many vehicles, since some of the users reach the area using their personal car. The relations between the supply (vehicles) and the demand (users) can also be deduced from statistical surveys of cellular data (such as [57]) and parking utilization surveys (such as [58]).

$$\begin{aligned}\overline{G}^{v}_{rand}(d_v) \cong (1 - P_u(d_v)) + \frac{P_u(d_v) g_{bh}}{2\Omega_v(d_v) g_d d_v^{\beta+1}} \Bigg[&-\frac{\sqrt{(-x^2 + (2d_v^2 + 2d_a^2)x - (d_a^2 - d_v^2)^2)^3}}{3} \\ &- \frac{((2d_v^2+2d_a^2) - 2x)(2d_v^2+2d_a^2)\sqrt{(-x^2 + (2d_v^2 + 2d_a^2)x - (d_a^2 - d_v^2)^2)^3}}{8} \\ &- \frac{(2d_v^2+2d_a^2)(4 + (2d_v^2+2d_a^2)^2)}{16} \arcsin\left(\frac{-2x + (2d_v^2 + 2d_a^2)}{4 d_a d_v}\right) \Bigg]_{x=x_{\min}^2}^{x_{\max}^2}\end{aligned} \tag{31}$$

It should be noted that the relation between the demand and vehicle distributions may depend on the region characteristics, i.e., residential, downtown, offices, etc., and the time of day. For example, an office area will be loaded with cellular traffic during work hours and unloaded afterwards, similarly the utilization of on-street parking places. On the other hand, residential area will be highly loaded from late afternoon (once people are back from work) until the next morning. Also in this case parking utilization will behave similarly.

To illustrate the value of such correlation, we investigate a simple example with two settings. In one the number of users and number of vehicles are un-correlated, i.e., independent, and in the other the number of users and the number of vehicles are correlated. We denote the random variable of the number of users and the random variable of number of vehicles in the region as N_u and N_v, respectively. We assume that $N_u, N_v \leq N_{\max}$, and that N_u is Poisson distributed[4] with mean λ.

In the correlated case, we assume that with probability p ($0 \leq p \leq 1$) a user arrives to the area of interest with his/her vehicle (and with probability $1-p$ without a vehicle). Assuming (for the sake of simplicity) there is no additional source of vehicles, i.e., vehicles parked in the area only belong to users arriving to the area. Then, the conditional probability $P(N_v = m | N_u = l)$ is a binomial distribution:

$$P(N_v = m | N_u = l) = \binom{l}{m} p^m (1-p)^l. \tag{35}$$

We note that as vehicles only belong to users in the region then in the case above we must have $m \leq l$.

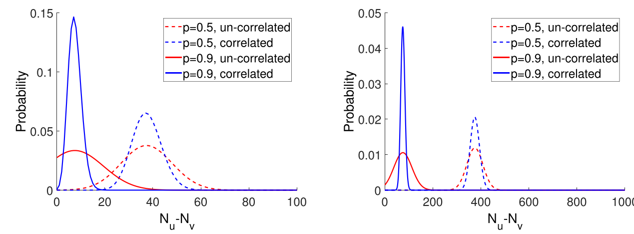
We are interested in the investigation of $N_u - N_v$, i.e., the random variable representing the residual number of users without vehicles. This variable provides insights on the excessive demand the network will need to handle using BS resources, and will assist in comparing the correlated and non-correlated scenarios.

It is well known that in the correlated case N_v is distributed $Poiss(p\lambda)$ (also truncated[5]). Thus, in the correlated case $N_u - N_v$ is also Poisson distributed (for large $N_{\max}$, which is our case) with mean $(1-p)\lambda$, and hence the variance is $\mathrm{Var}(N_u - N_v) = (1-p)\lambda$. In order to evaluate the scaling value of the VeRN network, we consider the un-correlated case under similar conditions to those of the correlated case. To this end we assume that N_v is Poisson (truncated) distributed with mean $p\lambda$, just like in the correlated case. We then note that in the un-correlated case $N_u - N_v$ is Skellam distributed with $\mathbb{E}[N_u - N_v] = (1-p)\lambda$, and $\mathrm{Var}(N_u - N_v) = (1+p)\lambda$. In our comparison between the correlated and uncorrelated cases the selection of mean $p\lambda$ for N_v for both cases targets a "fair" comparison between the cases and emphasizes the benefit in a correlated system due to the lower variance of $N_u - N_v$.

This difference in the variance will have impact on the probability distribution and hence on the deployment planning based on the peak load. The reason for this is that the operator plans

[4]Due to the physical constraint $N_{\max}$ on the maximum number of users and vehicles, the distributions of N_u and N_v will be truncated at $N_{\max}$. Yet this has minor influence on the probability function.

[5]The truncation provides a good approximation.



(a) $N_{max} = 100$. (b) $N_{max} = 1000$.

Fig. 14. Comparison of correlated and uncorrelated systems.

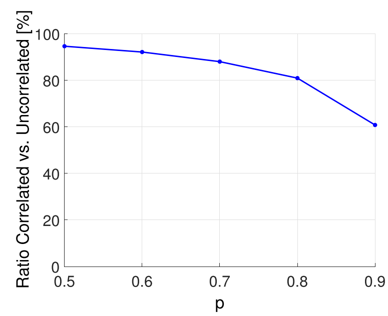


Fig. 15. Ratio between the number of residual users (which indicates the required number of BS resources) of the correlated and uncorrelated systems.

the deployment to minimize a certain expected cost, e.g., the amount of resources needed to be deployed which obviously depends on the residual demand. Large variance means, to the operator, higher uncertainty on residual demand which will force it to deploy more resources. Furthermore, we note that another benefit in the correlated system is that as the number of users increases (for example in very large events or venues) the VeRN value will become significant as it will scale together with the number of users. This is in contrast to the un-correlated case in which the variance increases.

Fig. 14 depicts the probability of $N_u - N_v$ for $N_{\max} = 100$ and $N_{\max} = 1000$, and four system settings. In all systems the number of users is Poisson (truncated) distributed with mean $0.75N_{\max}$. We explore two values of p: 0.5 and 0.9. Hence, we have correlated and uncorrelated systems for two values of p.

It can be seen that even for $p = 0.5$, in which the difference in the variance is not large, still the correlated system is favorable. For $p = 0.9$, it is obvious that the correlated system is strongly preferable as the variance is very small. This emphasizes that systems with correlated users and vehicles will be preferable to the operator, as it will be required to support the user via a leaner layer of BSs. For example, for 1000 users and $p = 0.5$, considering a target of 95th percentile service, the operator needs to handle 430 residual users in the uncorrelated case versus only 407 residual users in the correlated case. For $p = 0.9$ this difference increases significantly: 148 versus 90. Fig. 15 depicts the ratio between the residual users (for a 95th service approach) of the two systems (correlated system divided by the uncorrelated system) for $p = 0.5, 0.6, \ldots, 0.9$. Clearly this ratio decreases significantly as p increases, which emphasize the merits in VeRN based deployment.

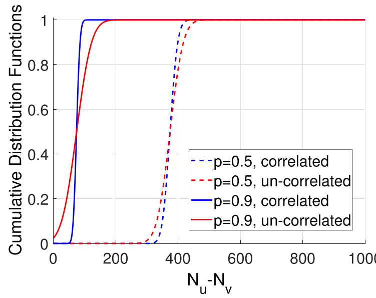


Fig. 16. CDF of residual users for system with $N_{\max} = 1000$ and $\mathbb{E}[N_u] = 750$.

Statistically we can show that the correlated system is second-order stochastically dominating the uncorrelated system [59], [60]. This can be easily seen by investigating the difference between the Cumulative Distribution Functions (CDF) of the residual users of the two systems, as depicted in Fig. 16. Meaning, denoting the CDF of the correlated system and the uncorrelated system as F_c and F_{uc}, respectively, then: $\int_{-N_{\max}}^{x} (F_{uc}(x) - F_c(x))dx \geq 0, -N_{\max} \leq x \leq N_{\max}$.

Moreover, with high correlation the system scale well when the number of users (and hence the demand) increases. Furthermore, comparing between Fig. 14(a) and Fig. 14(b) it can be clearly seen how the network scale with the number of users (load) in the correlated case. This shows the economy-of-scale of the VeRN concept, which is a very unique characteristic of this deployment scheme.

VII. Conclusion

In this paper we investigated a VeRN deployment concept through two types of analysis, which are important to cellular networks. In the first analysis, the user focused analysis, we explored the gain of a user to connect to a BS via a VeRN, and in the second analysis, the VeRN focused analysis, we investigated the gain of a VeRN to cover and boost the performance of an area with multiple users. We based our analysis on a technique first presented in [9], and developed an approximation scheme for calculating the gain in utilizing the relay link of the VeRN (versus the direct link) for the two analyses. This approximation is based on detailed analysis combining both spatial geometry and link-level formulation. Overall, we reached a closed-form expression in both analyses.

For the user focused analysis, our closed-form expression to the random VeRN selection scheme closely follows actual performance results attained from exhaustive system simulations. Furthermore, we compared our results of the random selection scheme with an upper bound on the performance, i.e., the best VeRN selection scheme. This comparison emphasized the potential in implementing a more sophisticated VeRN activation mechanism, to increase the value of incorporating VeRNs into the cellular deployment. We further showed how our scheme can be extended and employed in other system settings, i.e., in other channel models beside the WINNER II model. Our investigation revealed that our scheme can accurately approximate many other channel models.

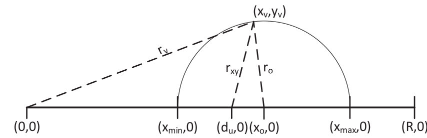


Fig. 17. Analysis of the VeRN coverage area.

We then adapted the approximation scheme to another type of analysis, the VeRN focused analysis, in which a VeRN serves multiple users within its surrounding. By following similar steps as the approximation scheme developed in the user focused analysis, we successfully reached a closed-form approximation here as well. Using simulations, we showed that also this approximation result closely follows actual performance. This result is important as it emphasizes the flexibility of our scheme to be applied in various systems (not necessarily VeRN based).

Our analysis illustrated the gain in utilizing VeRN in cellular deployment, to boost performance in areas and times with high demand. For the user focused analysis with random VeRN selection, a VeRN adds ~5 dB gain, and for the VeRN focused analysis it adds ~6 dB gain. Furthermore, the developed analysis techniques can serve as the foundation of VeRN deployment planning tools. Moreover, the difference in performance between the random VeRN selection scheme and the best VeRN selection scheme, emphasizes the potential in smart activation policy for VeRNs. Thus, our results can also assist in the development of online VeRN activation schemes.

We further showed another merit of VeRN based deployments which is the economy of scale of the network. Meaning, cellular deployments with VeRNs have a unique scaling characteristics which enables the network to self-maintain itself, i.e., the network scale in correlation to the demand. We illustrated this behavior using numerical example and its statistical analysis. Overall, we conclude that VeRN can significantly enhance the performance of cellular networks.

Appendix A
VeRN Coverage Area

We now prove that the VeRN backhaul limited region for the user focused analysis is circular with $r_o = (x_{\max} - x_{\min})/2$ and central point $x_o = (x_{\max} - x_{\min})/2$, as illustrated in Fig. 17.

We wish to prove that for all (x_v, y_v) over the area's edge, the relation $y_v^2 + (x_v - x_o)^2 = r_o^2$ holds (x_o and r_o are defined above). To prove this property we first note the following relations:

$$r_v^2 = x_v^2 + y_v^2 \tag{36}$$

$$r_{xy}^2 = (x_v - d_u)^2 + y_v^2 \tag{37}$$

Next we recall that the edge of the region is defined by $S_{bh}(r_v) = S_a(d_v)$, which means that:

$$\frac{P_a^{TX} g_a}{r_{xy}^{\alpha_a}} = \frac{P_{bh}^{TX} g_{bh}}{r_v^{\alpha_{bh}}}$$

Which we can write as:

$$r_{xy} = \sqrt[\alpha_a]{\frac{P_a^{TX} g_a}{P_{bh}^{TX} g_{bh}}}\ r_v^{\alpha_{bh}/\alpha_a} \tag{38}$$

Substituting (36) and (37) into (38), and set $\alpha_{bh}/\alpha_a = 1$ (as in the approximation), we get:

$$(x_v - d_u)^2 + y_v^2 = M^2(x_v^2 + y_v^2) \tag{39}$$

Manipulating (39) we get:

$$\left(x_v - d_u\frac{1}{1-M^2}\right)^2 + y_v^2 = \left(\frac{d_u M}{1-M^2}\right)^2 \tag{40}$$

Which means that the coverage region is indeed circular with x_o as the center point and r_o as radius.

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Nadav Lavi received the B.Sc. and M.Sc. degrees in electrical engineering from the Technion—Israel Institute of Technology, Haifa, Israel, in 2002 and 2005, respectively, and the Ph.D. degree in electrical engineering from Tel Aviv University, Tel Aviv, Israel, in 2019. His industrial experience spans across multiple domains: from 2005 to 2010, he managed system engineering and research groups in Alvarion, a 4G infrastructure developer and manufacturer, and from 2011 to 2019, he was a Staff Researcher and Group Manager with General Motors Research, Herzliya, Israel. His research interests include wireless networks, vehicular networks, layer 2 and beyond protocols, and stochastic analysis.

Hanoch Levy received the B.A. degree in computer science with distinctions from the Technion—Israel Institute of Technology, Haifa, Israel, in 1980 and the M.Sc. and Ph.D. degrees in computer science from the University of California–Los Angeles, Los Angeles, CA, USA, in 1982 and 1984, respectively. From 1984 to 1987, he was a member of technical staff with the Department of Teletraffic Theory, AT&T Bell Laboratories. Since 1987, he has been with the School of Computer Science, Tel Aviv University, Tel Aviv, Israel, where he was the Department Chairman from 1998 to 2000 and the Head of the School from 2016 to 2018. From 1992 to 1996, he was with the School of Business and RUTCOR, Rutgers University, New Brunswick, NJ, USA, and from 2007 to 2008, he was with the Communication Systems Group, ETH Zurich, Zurich, Switzerland, both on leave from Tel Aviv University. His research interests include computer communications networks, wireless networks, performance evaluation of computer systems and queuing theory.

Yuval Shavitt received the B.Sc., M.Sc., and D.Sc. degrees from the Technion—Israel Institute of Technology, Haifa, Israel, in 1986, 1992, and 1996, respectively. After one year as a Postdoctoral Fellow at Johns Hopkins University, Baltimore, MD, USA, he joined the networking center at Bell Labs, Lucent Technologies, Holmdel, NJ, USA, where he worked on network management, QoS routing, and wireless networks. Since 2000, he has been a Faculty Member with the School of Electrical Engineering, Tel Aviv University, Tel Aviv, Israel, where he is currently a Full Professor. In 2004, he incepted the DIMES project for mapping the Internet infrastructure, which revolutionized the field of Internet measurement and mapping. In addition to his work on network measurements and analysis, he works on algorithms for the analysis of large graphs, IP geolocations, and protection of the Internet routing infrastructure.