Coherence Multiplexing of Fiber-Optic Interferometric Sensors

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Abstract—This paper describes a method of multiplexing several optical signals onto a single spatial channel (e.g., a single-mode fiber) using a short coherence length continuous wave light source. Several system configurations which utilize this technique are proposed, and some design considerations are discussed. Experimental results for a single sensor and receiver are presented and compared with theoretical predictions.

I. INTRODUCTION

FIBER SENSORS have been actively studied and developed during the last few years for application in a wide range of fields [1]. Currently interest is being shown in arranging such sensors in arrays. Ideally, such an array would consist of a fiber input bus which would carry light to a set of sensors. Each sensor would imprint information about the environment onto this optical carrier. An output fiber bus would then collect this information and bring it back to a central processing location. In many applications it would be advantageous if no electrical components were required at sensor sites. The replacement of electrical lines by fiber-optic cable ameliorates electrical pickup, cable weight, and safety hazards.

The primary consideration in designing a sensor array is the method of separating each sensor's information from the single data stream. Previously there have been two approaches to constructing a distributed sensing system which address this problem. The first is time-division multiplexing of the sensor outputs [2]. Typically, the optical input is pulsed and each sensor produces a pulse which as a consequence of the system geometry is separated in time from the other sensor signals. The pulses are multiplexed onto the return fiber bus and are carried back to the central processing location where demultiplexing and signal processing occur. The second approach is to frequency-division multiplex the sensor outputs [3]. This can be accomplished by frequency ramping the optical source and arranging the array geometry so that the transit time of the light from the source to a sensor and back to the central location is unique for each sensor. The delayed array output is mixed with the source's undelayed signal producing a unique heterodyne frequency for each sensor. The environmental information is carried in the sidebands about the several beat frequencies which act as carriers for the sensor responses.

This paper describes a new approach to multiplexing several remote sensors, using control of optical coherence in place of the usual time-domain or frequency-domain techniques. A short coherence length continuous optical source is used instead of the pulsed or ramped coherent source of the aforementioned systems. The idea of using a short coherence length source to separate the signals returning from a series of sensors was first published by Al-Chalabi et al. [4]. However, in addition to introducing new configurations, our systems differ significantly in that they monitor each sensor continuously, separating the signals from the sensors spatially rather than temporarily.

A number of multiplexing configurations have been envisioned. The basic configurations are here referred to as "series," "extrinsic-reference ladder," and "intrinsic-reference ladder" versions. The principle of operation of the "series" configuration is illustrated in Fig. 1, using two sensors for simplicity. Assume that the source has a coherence time $\tau_c$ and coherence length given by $L_c = v_g \tau_c$, where $v_g$ is the group velocity of light in the fiber. Light is launched into a single-mode fiber, where it is sent through a series of Mach Zehnder interferometers which have two path pairs of different length mismatches, with the mismatches designated by $l_1$ and $l_2$. The interferometers are constructed by using directional couplers [5] to split the light, with the coupling constant prescribed by the number of sensors in the system [6]. The optical path length differences $\Delta l_n$ are chosen to be much longer than the source coherence length $L_c$, so that a change in the relative phase between the arms of the interferometer will not be converted into detectable intensity modulation at the sensors' outputs. The information imprinted on the light in each sensor is the difference in the phase between the two arms of the interferometer. After the light has traveled to the central processing location, phase information can be retrieved by the receiving interferometers with path length differences $l_1$ and $l_2$ which match $l_1$ and $l_2$, respectively, to within a fraction of $L_c$. (An equal fraction of the light coming back is diverted to each receiver in order to make all signals equal in strength.) Thus the phase modulation is converted to amplitude modulation at the photodetector.
Fig. 1. Schematic of a two sensor "series" coherence-multiplexed system. $l_1$, $l_2$, $L_1$, and $L_2$ are the path length differences in each Mach Zehnder interferometer. Ideally, $L_1 = l_1$ and $L_2 = l_2$.

Each Mach Zehnder has a free end from which light escapes. This introduces loss, but this is not as serious a problem as one might think. Even when many sensors are multiplexed, power loss can be kept relatively modest by choosing the coupling constants of the directional couplers appropriately [6]. It is possible to avoid this loss by using a system in which the fibers from both output ports of each sensing interferometer are continued to form the next sensor, as shown in Fig. 2. However, in such a system if all couplers are set to 50 percent and there is more than one sensor in the system, then no signal will be produced. This is because in this configuration the $\pi/2$ phase shift which occurs when light couples between two fibers becomes important. Light from one input port of a sensing interferometer enters the longer arm delayed by $\pi/2$ relative to light entering the shorter arm. Light from the second input port, which is incoherent with respect to the light entering the first input port, enters the shorter arm with a relative delay of $\pi/2$ rad. This difference in relative delays leads to cancellation between the signals associated with light entering each of the two input ports. A more complete discussion of this cancellation is given in Appendix A. If coupling constants are adjusted to optimal values, this system can produce slightly stronger signals than those produced by the one shown in Fig. 1. However, this stronger signal is obtained at the expense of making the sensing interferometers include the entire length of fiber between desired sensing sites. One can, of course, use extra interferometers, without corresponding receivers, as links between the more localized sensors at the points of interest, but adding these links will tend to degrade the signal-to-noise ratio associated with each sensor. In addition to providing a reasonable compromise in terms of performance, the discontinuous version of the series system has the practical advantage that the free fiber ends simplify alignment by providing access to the signal present throughout the system.

The coherence-multiplexed "extrinsic-reference ladder" configuration (which is so-called because light from a separate reference arm is mixed with the light from the return bus to generate a signal) is shown in Fig. 3. Light from a short coherence length laser is launched into a single-mode fiber, then split by a directional coupler along two paths. Part of the light enters the input fiber-optic bus and is distributed along the sensors; the rest enters a fiber-optic tapped delay line to act as a reference. Again, at each sensor environmental information is imprinted onto the light in the form of modifications to the optical phase. The difference between the path lengths of the sensing loops (to the point where they rejoin on the fiber buses), including the buses between sensors, is much greater than $L_c$ so that there is no significant intensity modulation due to interference when the light from each sensor is collected onto a fiber-optic return bus.
At the central processing location, light from the sensors is mixed with the light which has been tapped from the delay line. Phase information is retrieved by matching each sensor path length $l_1$ with a reference path length $L_o$ to within a fraction of $L_c$. The reference arm should be shielded from the environment, since in this configuration information consists of the difference in the phase between the light which traveled through the sensor arm and that which traveled through the corresponding delay line. Each detector sees not only the environmental information from each sensor but that from the input and output buses as well. One way to get the desired specific information is to shield the fiber buses. Alternatively, the signals received by adjacent detectors can be electronically subtracted. The resulting differences are independent of phase variations induced on the buses except in the region between the corresponding sensors.

A third system geometry, the “intrinsic-reference ladder” configuration, is shown in Fig. 4. In this case, sensing Mach Zehnder interferometers characterized by path length differences that differ by much more than $L_c$ are arrayed in parallel between input and output buses. Information from the return bus is processed by receivers in the same way as in the series system. The intrinsic-reference ladder system exhibits lead insensitivity like the series system (which also is an intrinsic-reference system, in that all the light necessary for forming signals passes through the sensor network and returns to the receivers along a common path). However, it is also like the extrinsic-reference ladder system in that requirements regarding allowed path length differences are relatively modest, as will be discussed later. Note that the series, extrinsic-reference ladder, and intrinsic-reference ladder coherence-multiplexing systems, although prototypical, do not exhaust the possible configurations for a coherence-multiplexed sensor network; various combinations of these schemes are clearly also possible.

The performance of coherence-multiplexed systems is limited in general by several types of noise. In addition to the shot noise and electronic amplification noise present in any optical sensing system, the proposed systems are subject to noise resulting from interference between light components from nominally noninterfering paths. Such interference can occur either in a signal-dependent or a signal-independent manner. If the difference in optical delays between the two paths is not sufficiently large, then the light components from the two paths in an interferometer are not entirely incoherent, and there are “crosstalk” terms in the detected power, i.e., the detected power depends weakly on the relative phase delays of nominally noninterfering paths. On the other hand, even if the light associated with the two paths is mutually incoherent, instantaneous interference effects will be present. Although such interference effects vanish on average, detection systems with finite bandwidth will not completely average out the resulting intensity fluctuations. From another perspective, this incoherent-interference noise is due to the interferometric conversion of source phase noise into intensity noise—an effect which is known to be important in both recirculating structures [7], [8] and single-arm [9]–[11] and multiple-arm [9] unbalanced interferometers. For a simple Mach Zehnder interferometer with a small path imbalance the “phase-induced intensity noise” is comparatively small, but may still be the dominant type of noise. As the path mismatch increases, this noise initially increases linearly, then reaches a saturation level for optical path mismatches longer than a few coherence lengths. A coherence-multiplexing system intrinsically requires that many paths through the system differ in path length by more than a coherence length. Hence, phase-induced intensity noise will always be present, although it can be minimized by using a source with a very short coherence length. In addition, some cross-talk and phase-induced noise can be eliminated by controlling polarizations so as to prevent paths from interfering; however, this tactic will be only partially effective in systems with more than two paths which are not intended to interfere with one another.

In Section II we discuss some of the theory relevant to coherence multiplexing. Section III describes experimental results achieved to date. Section IV suggests possible directions for future work.

II. Theory

A key consideration for a coherence-multiplexed sensor network is the need to ensure that only the paths intended to interfere are closely matched. This is easy to accomplish in the extrinsic-reference ladder configuration: each successive sensor path length (from the source to the sensor and back to some common point on the return bus) should be longer than the previous by at least an amount $L_o$, where $L_o >> L_c$ is chosen to be sufficiently large to reduce cross-talk to an acceptable level. For the intrinsic-reference ladder geometry, the lengths $l_1, \ldots, l_n$ should be chosen to differ by at least $L_o$, so that if, for example, the smallest of these path length differences is $L_0$, then the longest could be as small as $NL_0$. In addition, the differ-
These differences could, for example, be chosen to be in path lengths between the longest arm of one sensing interferometer and the shortest arm of the next one further out, measured between points where the two paths intersect, should differ from further out, measured between points where the two paths intersect, should differ from.

For the series scheme, the situation is more complicated. Let \( m_1 L_0, m_2 L_0, m_3 L_0, \ldots \), where \( m_1 \) is an integer, be the differential path delays of the sensing Mach Zehnder interferometers. (The delays may be numbered in any order.) Then \( m_{k+1} \) must satisfy \( m_{k+1} \in C_k \) and \( 2m_{k+1} \in A_k \), where

\[
A_k = \left\{ n: n = \sum_{j=1}^{k} e_j m_j, e_j \in \{0, \pm 1\} \right\}
\]

\[
B_k = \{0, \pm m_1, \ldots, \pm m_k\}
\]

\[
C_k = \{n: n = n_1 + n_2, n_1 \in A_k, n_2 \in B_k\}. \tag{1}
\]

One method of constructing sequences of permissible delays is to start with some particular \( m_1 \), and then select each subsequent series element to be the next smallest number which satisfies the above constraints. For \( m_1 = 1 \), the resulting sequence, 1, 3, 8, 21, 55, \ldots, obeys the recursion relation

\[
m_{k+1} = 1 + m_k + \sum_{j=1}^{k} m_j.
\]

By using z-transforms [12], one can show that this equation has the explicit solution

\[
m_k = \frac{1}{\sqrt{5}} \left[ \left( \frac{3 + \sqrt{5}}{2} \right)^k - \left( \frac{3 - \sqrt{5}}{2} \right)^k \right] ^{2 \cdot 2^{k-1} - 1} \tag{2}
\]

This sequence continues indefinitely, so that for a system with any number of sensors \( N \), one can always choose a subset of the sequence to specify the delays. In general, one can also use a set of delays whose construction depends on \( N \). In particular, one can choose \( m_k \) according to [13]

\[
m_k = m_1 + 2^{k-1} - 1 \tag{3}
\]

provided \( N \geq 4 \) and

\[
m_1 \geq 3 \cdot 2^{N-1} - 4 \cdot 2^{N/2} + 3, \quad N \text{ even}
\]

or, alternatively,

\[
m_k = m_N - 2^{N-k} + 1 \tag{5}
\]

where \( N \geq 1 \) and

\[
m_N \geq 3 \cdot 2^{N-1} - 2 \cdot 2^{N/2} + 1, \quad N \text{ even}
\]

\[
m_N \geq 3 \cdot 2^{N-1} - 3 \cdot 2^{(N-1)/2} + 1, \quad N \text{ odd} \tag{6}
\]

As an example, for \( N = 4 \) the smallest sets of these forms are given by \{11, 12, 14, 18\} and \{10, 14, 16, 17\}, respectively, while for \( N = 5 \) the corresponding sets are \{27, 28, 30, 34, 42\} and \{22, 30, 34, 36, 37\}. Note that the last of the three classes of delays given above exhibits the slowest growth of the maximum delay with increasing \( N \). It is not known whether or not more compact sets of delays are possible. The fact that all known classes of permissible delays have a maximum delay which grows generally exponentially with increasing \( N \) poses a significant limit to the applicability of the series configuration to systems with large numbers of sensors.

Another issue relevant to the design of a coherence-multiplexed sensor array is the proper selection of coupling coefficients for the various directional couplers used in the system. This determination can be based in part on the intuitive requirement that all sensors experiencing equal environmental modulation amplitudes should return signals of comparable strength to the central processing location. For the series system, this means that all sensing Mach Zehnders should be built from identical couplers, providing that all the interferometers have comparable environmental sensitivities. (The order in which sensors appear in the chain has no effect on the nature of the field which reaches the central processing location; thus sensors built from identical couplers will yield signals of equal strength.) What is not entirely obvious is that the coupler at the two ends of each sensing interferometer should also be identical; however, Appendix B shows that this must be the case.

In the extrinsic-reference ladder system, the solution is less trivial. Assume that there are \( N \) sensors in a ladder system. Number the sensors with an index \( j \) running from 1 to \( N \), starting with \( j = 1 \) for the sensor nearest to the light source and receivers. Let magnitude of the amplitude coupling coefficient for the couplers associated with sensor \( j \) be \( \sqrt{\kappa_j} \), so that a fractional power \( \kappa_j \) is transferred between the two fibers in the coupler, and a power \( 1 - \kappa_j \) passes straight through. Note that by the argument in Appendix B, the couplers at either end of the ladder rung associated with a single given sensor should be identical. We will assume for simplicity that light must couple across fibers in the couplers in order to get from the input bus to a sensing ladder rung and back to the return bus (as shown in Fig. 3), although the situation could just as well be reversed. Light returning from sensor \( j \) will have suffered loss from couplers 1 through \( j \) on both the input bus and the return bus. Couplers 1 through \( j - 1 \) will have a transmission \( 1 - \kappa_q \) for both the input and return couplers, and the two couplers at sensor \( j \) will each have a transmission \( \kappa_j \). Hence the power returning from sensor \( j \) to the central processing location is given by

\[
P_{j, \text{ret}} = P_{j, \text{in}} \kappa_j \prod_{q=1}^{j-1} (1 - \kappa_q)^2 \tag{7}
\]

where \( P_{j, \text{in}} \) is the power being sent to the sensor array. Setting \( P_{j+1, \text{ret}} = P_{j, \text{ret}} \), we find that the coupling coefficients are related by \( \kappa_{j+1} = \kappa_j/(1 - \kappa_j) \), or equivalently

\[
\kappa_j = \frac{\kappa_{j+1}}{1 + \kappa_{j+1}}. \tag{8}
\]
The last sensor does not really require any couplers since no power is needed for subsequent sensors; hence we can set $\kappa_N = 1$. Together with the recursion relation just derived, this implies that the coupling coefficient for the couplers of sensor $j$ is just

$$\kappa_j = \frac{1}{N - j + 1}. \quad (9)$$

This, in turn, means that the total transmission $P_{\text{rel}}/P_{\text{in}}$ is the same for every sensor, as expected, and is equal to $1/N^2$. One of the factors of $1/N$ appears because the input power has to be split up among $N$ sensors. The other factor of $1/N$ is a result of the unavoidable loss which occurs when signals from two fibers (the sensing ladder rung and the return bus) are combined by a passive linear coupler to form a single-mode signal (i.e., the signal on the return bus).

Coupling to the input and output buses in the intrinsic-reference ladder configuration can be specified in the same way as for the extrinsic-reference ladder scheme. In addition, the couplers that form the various sensing interferometers should be chosen to be equal to ensure comparable signal powers from each sensor. Note that for all configurations, coupling constants which are not determined by the requirement of equal sensor sensitivities can be chosen so as to maximize the signal-to-noise performance of each sensor. This optimization is beyond the scope of this paper; however a detailed theory of the noise expected in general coherence-multiplexed systems will be presented in a subsequent paper [6].

Having specified the structure of a coherence-multiplexed system, we shall now turn our attention to the signal obtained from one such configuration. Consider a simplified version of the system shown in Fig. 1, such that there is only one sensor and one receiver. Although this is not truly a multiplexed system, it serves to illustrate some characteristics of coherence-multiplexed systems.

Let the optical field present at the input of the system be given by $\sqrt{2}P_{\text{in}}u(t)e^{i\phi}$ where $P_{\text{in}}$ is the optical power and $u(t)e^{i\phi}$ is the stochastic analytic signal describing the field (see, for example, [14]), normalized so that its mean square value $\langle |u(t)|^2 \rangle$ is unity. If we assume a lossless system in which all couplers are set to 50-percent coupling and all paths through the system result in the same final polarization, then the optical power $P(t)$ incident on the detector is given by

$$P(t) = \frac{P_{\text{in}}}{16} |u(t - T_0) + (e^{-i\Delta\phi_s} + e^{-i\Delta\phi_r})u(t - T_0 - T)
+ e^{-i(\Delta\phi_s + \Delta\phi_r)}u(t - T_0 - 2T)|^2 \quad (10)$$

where $T_0$ is the minimum delay through the system, $T$ is the differential delay in each Mach Zehnder, and $\Delta\phi_s$ and $\Delta\phi_r$ are the differential phase delays in the sensing and receiving Mach Zehnders, respectively.

Taking the expected value of this expression and using the normalized self-coherence function $\Gamma_s(\tau) = \langle u(t + \tau)u^*(t) \rangle$, the mean detected power can be written as

$$\langle P(t) \rangle = \frac{P_{\text{in}}}{8} \left[ 2 + \cos(\Delta\phi_s - \Delta\phi_r) + \text{Re}[2\Gamma_s(T)
\times (e^{i\Delta\phi_s} + e^{i\Delta\phi_r}) + \Gamma_s(2T)e^{i(\Delta\phi_s + \Delta\phi_r)}] \right] \quad (11)$$

It can be shown that the self-coherence function $\Gamma_s(\tau)$ is the Fourier transform of the single-sided optical power spectral density, appropriately normalized and shifted to the origin. Consequently, if the light produced by the source has a Lorentzian lineshape with a full width at half maximum (FWHM) given by $(\pi\tau_c)^{-1}$, then the self-coherence function will be $\Gamma_s(\tau) = e^{-|\tau|/\tau_c}$. This implies that if the Mach Zehnder mismatch $T$ is chosen to be much greater than the coherence time $\tau_c$, then $\Gamma_s(T)$ and $\Gamma_s(2T)$ become negligibly small, so that

$$\langle P(t) \rangle \approx \frac{P_{\text{in}}}{8} \left[ 2 + \cos(\Delta\phi_s - \Delta\phi_r) \right]. \quad (12)$$

Thus the received power is given by a mean level equal to one quarter of the input power, together with a modulation dependent on the signal phase $\Delta\phi_s - \Delta\phi_r$. The modulation depth is only 50 percent because only two of the four paths from the source to the detector interfere. The other two paths just add to the mean received power.

In taking the expected value of $\langle P(t) \rangle$ to obtain the detected signal, we have averaged out the signal intensity noise $P(t) - \langle P(t) \rangle$ which will be present in practice. If one assumes that the light at the output of the laser has a random phase which may be modeled as a Wiener-Levy stochastic process, together with negligible intensity noise, then one can show that the two-sided power spectral density of this noise is given by [6]

$$G_\delta(f) = \left( \frac{P_{\text{in}}^2}{128} \right) \frac{\tau_c}{1 + (\pi f \tau_c)^2} \left[ 1 + 4 \left[ 1 + (\cos(\Delta\phi_s - \Delta\phi_r)) \right] \left[ 1 + \cos(2\pi f T) \right] \right] \quad (13)$$

where the signal phase $\Delta\phi_s - \Delta\phi_r$ must be averaged since it is now being treated as a stochastic quantity. Thus the spectrum of the phase-induced intensity noise is characterized by a Lorentzian envelope, with a width equal to twice the source linewidth, and a height that depends on the signal phase. Within the envelope, there is a sinusoidal modulation which peaks at zero frequency and has a period $1/T$. For comparison, if one were to inject a power $P_{\text{in}}/4$ into a single strongly mismatched Mach Zehnder interferometer, the phase-induced noise power spectral density would be [7]

$$G_\delta(f) = \left( \frac{P_{\text{in}}^2}{128} \right) \frac{\tau_c}{1 + (\pi f \tau_c)^2}. \quad (14)$$

This normalization is convenient since it also allows this expression to be interpreted as giving the noise power spectral density that results when the polarizations in the single sensor serial system are adjusted to allow only two pairs of paths to interfere incoherently. Comparison of the
two expressions for $G_{\phi}(f)$ reveals that the modulated signal-dependent part of the spectrum in the double Mach Zehnder case results from interference between the signal-bearing paths and the other two paths, while the unmodulated portion of the spectrum results from interference between the two paths which do not contribute to the signal.

Since phase-induced intensity noise is the dominant type of noise in coherence-multiplexing systems with a relatively small number of sensors, knowledge of its power spectrum allows one to predict the phase modulation sensitivity of a sensor/receiver pair. However, before this can be done the system must be specified further, since in general both the small signal phase modulation sensitivity and the noise level depend on the phase of the system, which undergoes constant change due to low frequency environmental noise.

This signal fading is a significant problem for all Mach Zehnder-type sensors, and coherence-multiplexed sensors are equally susceptible. One solution to this problem is to heterodyne the signal by introducing a frequency shifter into one arm of the receiver. In this case the phase $\Delta \phi_s - \Delta \phi_e$ acquires the form $\Delta \phi_e$ + $2\pi f_\ell t$ + $\Delta \phi_e$ sin $2\pi f_\ell t$ where $\Delta \phi_e$ is a slowly changing environmental phase bias, $f_\ell$ is the heterodyne frequency, and $\Delta \phi_s$ sin $2\pi f_\ell t$ is the higher frequency environmental signal detected by the sensor. If $\Delta \phi_e$ is small, then only the first sidebands will be appreciable, and the heterodyned signal will have a power spectrum given by

$$G_{(s)}(f) = \frac{P^2_0}{16} \delta(f) + \frac{P^2_0}{2^{10}} \sum_{\varepsilon = \pm 1} (4\delta(\varepsilon f - f_\ell) + (\Delta \phi_e)^2 [\delta(\varepsilon f - f_\ell - f_c) + \delta(\varepsilon f - f_\ell + f_c)]) \tag{15}$$

where $\delta(\cdot)$ represents the Dirac delta function. Comparing this to the noise power spectral density $G_{\phi}(f)$ and noting that $\langle \cos(\Delta \phi_s - \Delta \phi_e) \rangle = 0$ and $f_c, f_\ell << 1/T$, we see that

$$\langle \Delta \phi_e \rangle_{S/N=1} = 6 \sqrt{2B\sigma_c} \tag{16}$$

where $\langle \Delta \phi_e \rangle_{S/N=1}$ is the magnitude of $\Delta \phi_e$ for which the signal and noise levels are equal, and $B$ is the bandwidth of the detection electronics. For convenience in comparing to our experimental results, the “signal power” is defined here as the electrical power present at a single modulation sideband frequency, i.e., either $f_\ell + f_c$ or $f_\ell - f_c$.

While heterodyning provides one method for allowing sensitive detection of signals in the desired frequency range in the presence of lower frequency environmental effects, this approach has the disadvantage that it requires the use of a frequency shifter. For the extrinsic-reference ladder system only a single frequency shifter placed at the beginning of the tapped delay line would be required to heterodyne all of the signals, but in the series or intrinsic-reference ladder systems a frequency shifter would be required in one arm of each receiver. An easier and less expensive method to avoid signal fading is the synthetic-heterodyne technique which was developed for the fiber-optic gyroscope [15]–[17]. The equipment necessary for this technique is minimal: a phase modulator, a frequency generator, and an electronic gate. The output of the frequency source is used to drive both the phase modulator and the gate, which is placed immediately after the detection and amplification electronics. In a series system with a single sensor the power seen by the detector is

$$P(t) = \frac{P^2_0}{8} [2 + \cos(\Delta \phi_m \sin \omega_{mf} + \Delta \phi_e \sin \omega_c + \Delta \phi_b)] \tag{17}$$

where $\Delta \phi_m$ sin $\omega_{mf}$ is the modulation imposed on the receiving arm and $\omega_c = 2\pi f_c$. We choose $\omega_m$ to be much greater than $\omega_c$ so that upper and lower sidebands of harmonics of $\omega_m$ do not overlap. The gate following the detector effectively multiplies the received signal (which is proportional to $P(t)$) by a dc biased square wave at the frequency $\omega_m$, thus mixing the harmonics of $\omega_m$ in the signal.

If the modulation amplitude $\Delta \phi_m$ and gate phase (relative to that of the phase modulator) are chosen appropriately, then a bandpass filter centered on $2\omega_m$ placed on the output of the gate will pass a signal of the form

$$V(t) = k \times \left\{ J_0 (\Delta \phi_e) \cos(2\omega_{mf} - \Delta \phi_b) + \sum_{n=1}^{\infty} J_{2n} (\Delta \phi_e) [\cos(2(\omega_m - n\omega_c)t - \Delta \phi_b)] + \cos(2(\omega_m + n\omega_c)t - \Delta \phi_b) \right\} \tag{18}$$

where $k$ is an unimportant constant and $J_n(\cdot)$ is the Bessel function of the first kind of order $n$. Note that $V(t)$ is identical in form to the signal produced by a conventional heterodyne system operating with a heterodyne frequency $2\omega_m$. By putting the heterodyne signal into a spectrum analyzer the height of the Bessel function sidebands around the second harmonic of the modulation frequency can be measured to give the phase modulation amplitude in the sensor at a particular frequency. Alternatively, for a complicated signal such as those sensed in practice, a phase demodulator can be used to recover the environmental signal.

Detailed calculation shows that for a double channel implementation [15], [17] of the synthetic-heterodyne technique, one can expect the minimum detectable phase modulation amplitude $\langle \Delta \phi_e \rangle_{S/N=1}$ to be roughly the same as that for a true heterodyne system. For the single channel approach [15], [16] which we used in our experiments, $\langle \Delta \phi_e \rangle_{S/N=1}$ is expected to be larger by about a factor of $\sqrt{2}J_2(2.8)^{-1} \approx 1.4.$
III. EXPERIMENTAL SYSTEM AND RESULTS

Recent research has been directed toward demonstrating the coherence multiplexing scheme with a single sensor. This minimal configuration is sufficient to demonstrate some of the principles and limitations of the coherence-multiplexing technique. The experimental system is shown in Fig. 5. ITT-1601 fiber, which is designed for use at 633 nm but guides a single mode loosely at 790 nm, was used to construct two Mach Zehnder single-mode fiber interferometers having nearly the same differential paths lengths (i.e. $l_1 = L_1$). Splices in the system (denoted by an x in Fig. 5) were made with a fusion splicer. Since splicing technology for this wavelength and fiber was not well developed, we were unable to match the lengths of the Mach Zehnder arms exactly. In addition, the loss at each splice was quite large, on the order of 1-2 dB. The splicing technology for this wavelength and fiber was not well developed, we were unable to match the lengths of the Mach Zehnder arms exactly. In addition, the loss at each splice was quite large, on the order of 1-2 dB. The difference in path lengths in each individual interferometer was approximately 21 m. The differential lengths of the two Mach Zehnders ($l_1$ and $L_1$) were matched to within 8 cm by using 120 ps (FWHM) pulses to probe the delta function response of each interferometer separately. The path mismatch of each interferometer was estimated by using a high-speed avalanche photodiode and 1-GHz amplifier to detect the pulses at the output, then digitizing the waveform and calculating the delay between the peaks of the received pulses. Measurements were repeated several times and averaged to find the delay time in each interferometer. The resolution of the measurement was limited primarily by the bandwidth of the oscilloscope amplifier and the digitization. When the pulse was applied to the entire system, the bandwidth of the measurement system was insufficient to resolve the two pulses which would be coincident if the paths were exactly matched.

The CW source used on this system was an AlGaAs laser diode (NEC model NDL 3000) emitting approximately 790-nm light. Previous spectral measurements on this laser had revealed a strong central longitudinal mode and a second mode having approximately 5 dB less power. We determined the coherence length of this source indirectly: using (14) and measuring the full width at half maximum of the spectrum of the light from our laser diode after it had passed through a single mismatched Mach Zehnder, we estimated the coherence time $\tau_c$ to be approximately 22 ns. This corresponds to a coherence length of around 4.5 m in fiber. This agrees well with the estimate obtained by using the rule-of-thumb [18] which says that for a single-mode diode laser, the coherence length in fiber in meters is approximately equal to 80 percent of the output power of the laser in milliwatts. For our laser this gives a coherence length of approximately 4.8 m. An attempt was made to use a multimode laser as a source for this system, but the effective coherence length was on the order of only a few centimeters. Thus the path mismatch between $l_1$ and $L_1$ caused the modulation depth of the signal to be extremely small with the multimode laser.

Each Mach Zehnder interferometer consisted of two directional couplers, a phase modulator, and a polarization controller [19]. One polarization controller allowed the polarizations of the two paths carrying the signal (paths 2 and 3 in Fig. 6) to be aligned so that the modulation depth was maximized. The other polarization controller allowed the shortest and longest paths to both have polarization either parallel or perpendicular to that of the signal paths. The phase modulator in the first interferometer was used to simulate an acoustic frequency environmental signal. In the receiver, the phase modulator generated a signal for the synthetic-heterodyne demodulation technique described in Section II.

The coupling ratios were set using the impulse response of the system, with the optimal response being a 1:2:1 ratio of the pulses so that the power from all four paths were equal. Due to the slight mismatch of the path lengths $l_1$ and $L_1$, the two pulses from similar length paths did not interfere. However, the delay between these pulses was too short to allow resolution, as mentioned previously. Therefore, the two pulses were detected as a single pulse of twice the original power of each individual pulse.

Figure 6 shows the four possible paths that light can take through the single sensor and receiver of the experimental system. Paths 2 and 3 are nominally the same length and will produce the signal.

![Fig. 5. The experimental system. The upper phase modulator was used for the signal and the lower for the demodulation technique based on a previously published open-loop heterodyne-like scheme.](image-url)
into the system. Light was detected at the unconnected port of the first interferometer. No significant signal was observed when phase modulation was applied, regardless of polarization orientation. This demonstrates that the system was in fact coherence multiplexed, and not simply "polarization multiplexed". A small signal (roughly 2 percent for a phase modulation on the order of 1 rad) was observed regardless of the orientation of the polarization controller. Since this modulation was insensitive to polarization, it was probably due to bending loss occurring at the phase modulator in our loosely guiding fiber. Our interference after 21 m of path mismatch was thus no more than 2 percent.

The spectrum of the phase-induced intensity noise (with no signal applied and all paths in the same polarization) measured at the output of the second interferometer is shown in Fig. 7. The expected cosinusoidal spectrum is seen (in decibels), with minima occurring approximately every 10 MHz. The periodicity corresponds to the time delay of the interferometers, which was roughly 105 ns. At acoustic frequencies the spectrum of the noise is relatively flat and at its maximum value.

Measurements were made on the output signal from the system, as shown in Fig. 5. By comparing the heights $J_1(\Delta \phi_e)$ and $J_2(\Delta \phi_e)$ of the signal sidebands displayed on the spectrum analyzer for known single input frequencies, an estimate of the phase modulation amplitude $\Delta \phi_e$ was made. A measurement of the noise level without the signal then provided the minimum detectable phase modulation amplitude ($\Delta \phi_e$ such that $S/N = 1$, with $J_1(\Delta \phi_e)$ the signal). We measured the variation of phase-induced intensity noise with respect to polarization by adjusting the polarization controllers in the sensor to minimize and then to maximize this noise. If the paths not contributing to the signal were made orthogonal to the signal-bearing paths, then noise was minimized, while if all paths were parallel the noise was maximized. The results are plotted in Fig. 8. Error bars in the figure reflect reading error due to the resolution of the spectrum analyzer. The geometrical mean of the minimum detectable phase modulation amplitude with the phase-induced noise minimized was 1.2 mrad/√Hz. When phase-induced noise was maximized the minimum measurable $\Delta \phi_e$ went to an average of 4.1 mrad/√Hz. For comparison, the predicted minimum detectable modulation when noise is maximized is approximately 1.8 mrad/√Hz. The experimental ratio of the maximized and minimized minimum detectable phase modulations was 3.5, which agrees with our theoretical prediction of 3 (c.f., (13) and (14)) to within the experimental uncertainty.

Measurements of the laser intensity noise and electronic noise were made to determine whether these were affecting the minimum detectable phase modulation measurement. These noise levels were found to be approximately 30 and 35 dB, respectively, below the limiting noise in the system when the polarization controllers were set to minimize the noise.

IV. CONCLUSIONS

We have presented three systems which can be used for coherence multiplexing, together with design criteria. Noise level predictions have been presented for the simple case of a system involving a single sensing Mach Zehnder interferometer in series with a receiving interferometer. Our experimental results are fairly consistent with theoretical predictions. We might note that in the present system the fiber and components were originally designed for use at another wavelength, and so introduced extra loss at the operating wavelength.

In a system with few sensors the limiting factor appears to be phase-induced intensity noise, an intrinsic problem of the multiplexing method proposed here. It is believed that by using a broad-band source, such as a superluminescent diode (SLD), phase-induced intensity noise could be greatly reduced. The feasibility of using an SLD is under consideration, but there are several practical problems with this approach. First, the matching of tens of meters of fiber to fractions of a millimeter in length is a formi-
dable task. Second, environmental effects may cause long lengths of fiber to change by more than a coherence length, with the result that the reference arms will no longer match the sensor arms. While feedback could be employed to compensate for such drifts [20], [21], such an approach greatly increases the complexity of the sensor system.

Coherence multiplexing may have its greatest potential in the possibility of combining this method with other multiplexing methods. For example, several coherence-multiplexed systems could be time-division or frequency-division multiplexed to enable more sensors to be incorporated into a system. In this way the performance of either individual multiplexed system may be surpassed.

**APPENDIX A**

**Signal Cancellation**

Consider a balanced Mach Zehnder interferometer constructed by using two directional couplers, each with a 50-percent power coupling coefficient, to couple light between two pieces of optical fiber. If mutually incoherent optical fields $E_1$ and $E_2$ are injected into the two input ports of the first directional coupler, then the field in one arm of the interferometer will be $(E_1 + iE_2)/\sqrt{2}$ and the field in the other arm will be $(E_1 - iE_2)/\sqrt{2}$. Assuming the light in the latter arm experiences a phase delay $\psi$ relative to light in the former arm, the field at one output of the second directional coupler will be $[(1 - e^{i\psi})E_1 + i(1 + e^{i\psi})E_2]/\sqrt{2}$, while the field at the other output port is $[(1 + e^{i\psi})E_1 - (1 - e^{i\psi})E_2]/\sqrt{2}$. Hence the average optical power present at these two ports will be

$$P_{\pm} = \frac{1}{2} [\langle |E_1|^2 \rangle + \langle |E_2|^2 \rangle] \pm \frac{1}{2} [\langle |E_1|^2 \rangle - \langle |E_2|^2 \rangle] \cos \psi \quad (A1)$$

where the upper sign corresponds to the former output port, and where $\langle |E_2|^2 \rangle$ has been taken to be zero since $E_1$ and $E_2$ are mutually incoherent. (A factor of $1/2$ appears because we take power to be one half of the magnitude squared of the field analytic signal.) Note that power associated with $E_1$ tends to cancel power associated with $E_2$ in the term that bears information about $\psi$. In fact, this term vanishes if $\langle |E_1|^2 \rangle = \langle |E_2|^2 \rangle$.

The interferometric configuration illustrated in Fig. 2 is more complicated than the one just analyzed, but similar arguments apply. In this system there are two ways cancellation can occur. First, if equal amounts of light enter the two input ports of a sensing interferometer, then complete signal cancellation occurs for that sensor. For this reason, any sensors downstream from a coupler to 50-percent coupling fail to produce a signal. The second way that cancellation can occur relates to the fact that light which passes from the upper (lower) output port of sensing interferometer to the lower (upper) input port of the corresponding receiving interferometer produces a signal opposite in sign to the signal produced by light passing from the upper (lower) output port to the upper (lower) input port. This implies that if any coupler between the sensing and receiving interferometers is set to 50-percent coupling, then complete cancellation occurs, and no net signal results. From these arguments, one concludes that if this type of system is to include more than one sensor, then no couplers in the sensor chain may be set to 50-percent coupling.

**APPENDIX B**

**An Optimal Coupling Theorem**

Consider a portion of a fiber-optic sensing or signal processing system in which light is split by one directional coupler, passes through two optical subsystems, then is recombined by a second directional coupler, as shown in Fig. 9. Suppose the two couplers in question have power coupling coefficients $k_1$ and $k_2$, and the two subsystems have amplitude transmission coefficients $\Lambda_1$ and $\Lambda_2$. (Any phase associated with the coupling process will be assumed to be included in $\Lambda_1$ and $\Lambda_2$.) It is clear that the general structure of the optical signal after the combining coupler can depend only on the ratio $[k_1 k_2/(1 - k_1)(1 - k_2)]^{1/2}$ of the amplitude transmission coefficients associated with the two paths. This ratio does not depend on $k_1$ or $k_2$ alone, but only on the two of them together. If we maximize the total power $[k_1 k_2/(1 - k_1)(1 - k_2)]^{1/2} \Lambda_1 + [(1 - k_1)(1 - k_2)]^{1/2} \Lambda_2^2$ subject to the constraint that the above ratio be constant, we find that the power is maximal for $k_1 = k_2$. Hence we conclude that for any system of this sort, one should always make the coupling constants of the splitting and recombining couplers identical, so long as it is desirable to maximize the power of the final signal.

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**References**


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